

Ad Bibliothecam
Canonice SS. Apost. Petri & Pauli
in Weyarn

comparavit

P. VALENTINUS HANSCH Conventualis ibid.

TITULUS SCRINII.

SERIES.

NUMERUS.

3.

THE HISTORY OF THE
BY CHARLES Y Y G E
New York, and London: W. B. E. & Co.
Barrington Street.

AND D. O. N.
New York, and London: W. B. E. & Co.
Barrington Street.

Math. F. 629

Mathesis pura. Arithmetica. Systemata
et methodi 207.

R

THE
KEY
TO THE
TUTOR'S GUIDE:
OR, THE
ARITHMETICIANS REPOSITORY.

CONTAINING
The SOLUTIONS of the QUESTIONS, &c.
in the TUTOR'S GUIDE.

With the REFERENCES as they stand in the Second Edition.

To which is added (where necessary)

Some USEFUL RULES, &c.

As those for the attaining a thorough Knowledge of
CIRCULATING NUMBERS.

Likewise

A N A P P E N D I X,

Shewing the Combination of Quantities; the different
Ways they may be varied; with the Method of filling
the Magic Squares, &c.

The whole being principally designed for the Ease of School-
masters, and, with the GUIDE, furnishes a more Complete and
Extensive SYSTEM of ARITHMETIC, than any extant; and
will enable all those who are acquainted with the first Princi-
ples, to attain a competent Knowledge of the several Rules,
with Ease and Precision.

The SECOND EDITION, Corrected.

By CHARLES VYSE,

Teacher of the Mathematics, and Master of Westham-Abbey
Boarding-School.

L O N D O N,

Printed for G. ROBINSON, Pater-noster Row. 1775.



Bayerische
Staatsbibliothek
München



THE
P R E F A C E.

WHAT gave rise to the following Sheets, was my receiving Letters from several eminent Mathematicians, and School-masters, wherein they expressed great Desire for such a Performance, mentioning the Utility such a Work would be to School-masters in general ; as the TUTOR'S GUIDE contained such a Variety of Questions, suitable to all Capacities, and adapted for the Use of the Gentleman and Scholar as well as

for the Man of Business—But as many of the Questions are long and difficult, made it impossible for Schools in general to make use of the GUIDE, without the KEY ; the Time in Schools not admitting: therefore they gave me great Hopes of Encouragement, by promising to make Use of the TUTOR's GUIDE in their own Schools ; beside recommending it to all School-masters in Great Britain, as the most complete Epitome of Arithmetic, or Question Book, extant ; and with the KEY, would enable the Tutor to instruct ten Pupils with greater Ease, more correctly, and with less Perplexity both to himself and Scholars, than one by any other Book. Thus encouraged, and at the same Time being sensible of the favourable Reception the GUIDE has met with, gives me great Hopes that the following Pages will meet with that Encouragement, due to so useful and laborious a Work.

The

The Title Page gives a short Account of what the following Pages contain, which I think needless to enlarge upon, therefore shall leave the Book to speak for itself; and if it does not give Satisfaction to the Reader, I am sure all I can say in its Behalf will never recommend it: but this may be justly said, whoever reads it over, will find the Solutions of a greater Variety of Questions than in any other Treatise on the same Subject; and are performed in as intelligible, and comprehensive a Manner, as my Bounds would admit of, or even as is necessary.

How well I have performed this Work, must be left to proper Judges; and as I am not sensible of any fundamental Error in the following Pages, yet I cannot pretend to say it is without Imperfections; which I hope the good-natured Reader will excuse and pass over with the like Candour and Good-will.

will, with which it was composed for his
Use, by

Waltham-Abbey,
Oct. 24, 1774.

CHARLES VYSE.

N. B. As I am anxious of making this
Book as perfect and complete as possible, I
most earnestly request, that if any of my
Readers should discover any Defect in it, that
they will be so obliging as to favour me with
a Line, which shall be carefully attended to
in the next Impression.

To

To Mr. CHARLES VYSE.

S I R,

YOUR valuable Treatise on Arithmetic, as soon as it came from the Press, was immediately introduced into my School, where I have continued to use it ever since; and look upon it as exceedingly well calculated to facilitate the Business of a School, in those Branches of Learning of which it treats.

A second Edition falling into my Hands, it gave me great Pleasure to find by your Preface thereto, that you had some Thoughts of giving us (in a separate Publication) the Solutions to all the Questions at large; which would be in my Opinion of great Use to the Master, and what I wish of all things to hear you have done. I immediately wrote to my Bookseller, for the Key to your Book, but received for answer, it was not in Print; which has induced me to give you this Trouble, to know whether that be really the Case; and whether I may expect ever to see so excellent a system of School Instructions rendered complete, by the Publication of the Key, &c.

I am, Sir,

Your most obedient humble Servant,

Sept. 16, 1773.

CHRISTOPHER CAVE.

Master of the Free-School at Caister, in Lincolnshire.

To Mr. CHARLES VYSE,

S I R,

I Purchased your second Edition of the Tutor's Guide some Time ago, with a view of being furnished with a complete System of Arithmetic, when the Key should be published; and have been very impatient for so useful a Performance, being sensible of the utility such a Work must be to the masters in general, particularly to all those who have the Care of
a nu-

a numerous School ; I beg you will inform me whether the Key is (already, or intended to be) published, and you will highly oblige

Your most humble Servant,

and well wisher,

Plumton,
June 12, 1773.

ISAAC SLEE.

School-master at Plumton in Cornwall.

To Mr. CHARLES VYSE.

S I R,

I Cannot help thanking you for the pains you have so happily employed for the facilitating of the Science of Numbers.

Solutions of Mr. Clare's very ingenious Questions have been much wanted ; and it must certainly afford great satisfaction to every Lover of Figures to find that Task so well executed.

I am, Sir,

Your obliged humble Servant,

July 14, 1774.

SAMPSON WRIGHT.

Master of the Free-Grammar-School of Bradley,
near Stafford.

WE have also received Letters to the same purport from the following gentlemen, viz. Mr. Rigge, Writing-master and Land-surveyor, at Cambridge ; Mr. Ross, Author of the Instructor's Assistant, at Portsmouth ; Mr. Boome, School-master, at Putney ; Mr. —, Master of the Boarding-school, at St. Edmund's Bury ; Mr. Lloyd, Master of the Boarding-school, in Kennington-lane, near Vauxhall ; Mr. Applegarth, Master of the Free Grammar-school, in Castle-street, St. Martins in the Fields ; the Rev. Mr. Holiday, of Linconshire ; likewise from several others, whose Letters being mislaid, their Names are forgot.

THE

T H E
K E Y
T O T H E
T U T O R ' S G U I D E :
O R , T H E
A R I T H M E T I C I A N ' S R E P O S I T O R Y .

B O O K I . P A R T I .

I. N U M E R A T I O N .

NUMBERS expressed in WORDS.

NINETY FOUR ; seven hundred and sixty-two ; thirty-thousand and twenty-four ; thirty-seven thousand four hundred and sixty ; one hundred forty-two thousand, six hundred and thirteen ; six million, forty thousand, three hundred and ninety ; forty-seven million, six hundred thirty-nine thousand, one hundred and twenty-one ; seven hundred ninety million, four hundred one thousand, nine hundred and fifty ; seventy-nine million, forty-one thousand, nine hundred and fifty-five.

WORDS expressed in FIGURES.

77, 490, 6055, 17709, 800002, 7044074, 694400060.

Roman Numerical LETTERS expressed in FIGURES.

19, 200, 600, 560, 1001, 1750, 70000, 110000, 1500000, 1600000.

Common FIGURES expressed in Numerical LETTERS.

XXIX. CIV. CCCCXIX. MDCCXLI. MMVII.
XVIIDCLXXVIII. XIV. DCLXXIVLXXXIV.

I N T E G E R S.

2. A D D I T I O N.

To add these Examples I begin with the first, and say 6 and 7 is 13, and 6 is 19, and 4 is 23, and 2 is 25, and 9 is 34; this is the Amount of the first Row, or Unit's Place, which contains 3 Tens, and 4 over; this 4 I set under the Row, and carry 3 to the next Row or Tens Place, saying 3 that I carry and 4 is 7, and 1 is 8, and 9 is 17, and 1 is 18, and 4 is 22, and 7 is 29, this being 2 Tens and 9 over, therefore I set down 9 and carry 2 to the next Row or Hundred's Place, saying 2 and 7 is 9, and 4 is 13, and 1 is 14, and 9 is 23, and 2 is 25, which is 5 to set down, and I carry 2 to the fourth Row, or Place of Thousands, saying 2 and 4 is 6, and 7 is 13, and 2 is 15, and 6 is 21, and 4 is 25, and 7 is 32, which is 2 to set down, and I carry 3 to the fifth Row, or Place of Tens of Thousands, and find that it also amounts to 32, then I set down the odd 2, and carry 3 to the sixth and last Row, or Place of Hundreds of Thousands, and find that it amounts to 22, and as this is the last Row, I set down the whole 22, that is, the 2 under the Row and the 2 Tens I carry to the Left Hand, and in the same Manner proceed with the rest of the Examples.

- | | | | |
|--------------------|-------------------|-------------------|---------------------|
| (1) <u>2222594</u> | (2) <u>696237</u> | (3) <u>433968</u> | (4) <u>225879</u> |
| (5) <u>2181162</u> | (6) <u>326034</u> | (7) <u>341780</u> | (8) <u>88622054</u> |
| (9) <u>1148103</u> | | | |

3. S U B T R A C T I O N.

When the upper Figure is greater than the lower, as in the first Example, I say 0 from 2 there remains 2; then 2 from 4 there remains 2, and 6 from 7 there remains 1; in this manner proceed to the end of this Example. But when the lower Figure is larger than the top one, as in the second Example, I say 7 out of 1 I cannot have, therefore I take 7 out of 10, (which I borrow) and there remains 3, and the top Figure 1 makes 4, which I place under the Unit's Place, and carry 1 to the next lower Figure, saying that 1 I carry to 6 makes 7, then 7 from 4 I cannot, but 7 from 10, there remains

remains 3, and the top Figure 4 makes 7 to set down, and 1 to carry to the next lower Figure, which is 4, makes 5; then say 5 from 9 there remains 4; but now I do not carry any to the next Figure, because I did not borrow, but only say 7 from 4 I cannot, but 7 from 10 there remains 3, and the top Figure 4 makes 7 to set down, and 1 I carry to 0 is 1, from 0 I cannot, but 1 from 10 there remains 9, and carry 1 to 8 is 9, from 7 I cannot, but 9 from 10 there remains 1, and the top figure 7 makes 8; lastly, 1 I carry from 1, and there remains 0, which being the last Figure, I do not set it down; and in this manner proceed with the other Examples.

- | | | | |
|--------------------|----------------------|-----------------------|--------------------|
| (1) <u>221132</u> | (2) <u>3160990</u> | (3) <u>897474</u> | (4) <u>313992</u> |
| (5) <u>689796</u> | (6) <u>2708970</u> | (7) <u>8960066</u> | (8) <u>3320801</u> |
| (9) <u>1069897</u> | (10) <u>31981794</u> | (11) <u>106799280</u> | |

4. MULTIPLICATION.

CASE I.

I begin with the first Example, and say, twice 3, or 2 Times 3, is 6 to set down; then twice 5 is 10, that is 0 to set down, and carry 1; then twice 8 is 16, and 1 I carry is 17, that is 7 to set down, and I carry 1; then twice 9 is 18, and 1 I carry is 19, that is 9 to set down, and 1 to carry to the next Figure; and so I proceed to the End of this Example, and likewise with all the rest, always remembering to carry 1 for every 10 to the next Figure on the Left Hand, as in Addition.

- | | | |
|------------------------|-----------------------|-----------------------|
| (1) <u>835259706</u> | (2) <u>1028158758</u> | (3) <u>1030735656</u> |
| † (4) <u>871478415</u> | (5) <u>1649792286</u> | (6) <u>5244299361</u> |
| (7) <u>2102067152</u> | (8) <u>3386626605</u> | |

† This is the Product of 5.

Multiplication.

C A S E II.

$$\begin{array}{r} (9) \quad 856424376 \\ 142737396 \\ \hline \end{array}$$

$$\begin{array}{r} 2283798336 \\ \hline \end{array}$$

$$\begin{array}{r} (12) \quad 1237835 \\ 1732969 \\ 990268 \\ \hline \end{array}$$

$$\begin{array}{r} 117594325 \\ \hline \end{array}$$

$$\begin{array}{r} (15) \quad 295356 \\ 1181424 \\ 886068 \\ 738390 \\ \hline \end{array}$$

$$\begin{array}{r} 839106396 \\ \hline \end{array}$$

$$\begin{array}{r} (18) \quad 4585548 \\ 6878322 \\ 2292774 \\ 5349806 \\ 764258 \\ 3057032 \\ \hline \end{array}$$

$$\begin{array}{r} 318998232168 \\ \hline \end{array}$$

$$\begin{array}{r} (10) \quad 172859776 \\ 43214944 \\ \hline \end{array}$$

$$\begin{array}{r} 605009216 \\ \hline \end{array}$$

$$\begin{array}{r} (13) \quad 2858841 \\ 2541192 \\ 1905894 \\ \hline \end{array}$$

$$\begin{array}{r} 218860161 \\ \hline \end{array}$$

$$\begin{array}{r} (16) \quad 47299 \\ 378392 \\ 236495 \\ 141897 \\ 331093 \\ \hline \end{array}$$

$$\begin{array}{r} 3480307719 \\ \hline \end{array}$$

$$\begin{array}{r} (11) \quad 84537943 \\ 108691641 \\ \hline \end{array}$$

$$\begin{array}{r} 1171454353 \\ \hline \end{array}$$

$$\begin{array}{r} (14) \quad 16241460 \\ 10827640 \\ 18948370 \\ 8120730 \\ \hline \end{array}$$

$$\begin{array}{r} 10140084860 \\ \hline \end{array}$$

$$\begin{array}{r} (17) \quad 662229 \\ 441486 \\ 147162 \\ 515067 \\ 294324 \\ \hline \end{array}$$

$$\begin{array}{r} 3478100289 \\ \hline \end{array}$$

$$\begin{array}{r} (19) \quad 3339168 \\ 2086980 \\ 834792 \\ 1669584 \\ 2504376 \\ 2921772 \\ \hline \end{array}$$

$$\begin{array}{r} 318998232168 \\ \hline \end{array}$$

$$\begin{array}{r} (20) \quad 16314384 \\ 8157192 \\ 19033448 \\ 21752512 \\ 10876256 \\ 16314384 \\ 2719064 \\ \hline \end{array}$$

$$\begin{array}{r} 4483018703104 \\ \hline \end{array}$$

CASE III.

$$\begin{array}{r} (21) \quad 3365222 \\ 2884476 \\ 4326714 \\ \hline 432963212822 \end{array}$$

$$\begin{array}{r} (22) \quad 85968376 \\ 10746047 \\ 10746047 \\ 42984188 \\ \hline 430917645273076 \end{array}$$

$$\begin{array}{r} (23) \quad 7242305625 \\ 6437605000 \\ 5632904375 \\ 1609401250 \\ \hline 166579474222305625 \end{array}$$

CASE IV.

$$\begin{array}{r} (24) \quad 24714 \\ 5492 \\ \hline 79634000 \end{array}$$

$$\begin{array}{r} (25) \quad 102263 \\ 116872 \\ \hline 12709830000 \end{array}$$

$$\begin{array}{r} (26) \quad 16608 \\ 11072 \\ 5536 \\ \hline 68092800000 \end{array}$$

CASE V.

$$\begin{array}{r} (27) \quad 24674 \\ 4 \times 4 = 16 \\ \hline 98696 \\ 4 \\ \hline 394784 \end{array}$$

$$\begin{array}{r} (28) \quad 340764 \\ 7 \times 4 = 28 \\ \hline 2385348 \\ 4 \\ \hline 9541392 \end{array}$$

$$\begin{array}{r} (29) \quad 142395 \\ 8 \times 7 = 56 \\ \hline 1139160 \\ 7 \\ \hline 7974120 \end{array}$$

$$\begin{array}{r} (30) \quad 176848 \\ 9 \times 7 = 63 \\ \hline 1591632 \\ 7 \\ \hline 11141424 \end{array}$$

$$(31) \quad 420746$$

$$9 \times 8 = 72$$

$$\begin{array}{r} 3786714 \\ 8 \end{array}$$

$$\begin{array}{r} 30293712 \end{array}$$

$$(32) \quad 17093$$

$$9 \times 9 = 81$$

$$\begin{array}{r} 153837 \\ 9 \end{array}$$

$$\begin{array}{r} 1384533 \end{array}$$

$$(33) \quad 43074$$

$$12 \times 12 = 144$$

$$\begin{array}{r} 516888 \\ 12 \end{array}$$

$$\begin{array}{r} 6202656 \end{array}$$

$$(34) \quad 14068$$

$$12 \times 11 = 132$$

$$\begin{array}{r} 168816 \\ 11 \end{array}$$

$$\begin{array}{r} 1856976 \end{array}$$

C A S E VI.

To perform the Examples in this Case, I begin with the last, and say, 9 times 8 is 72, put down 2 and carry 7; then 9 times 6 is 54, and 7 I carry is 61, and the Right Hand (or back) Figure 8 is 69, put down 9 and carry 6; then 9 times 7 is 63, and 6 I carry is 69, and 6 the back Figure is 75, put down 5 and carry 7; then 9 times 2 is 18, and 7 I carry is 25, and 7 the back Figure is 32, put down 2 and carry 3; then 9 times 4 is 36, and 3 I carry is 39, and 2 the back Figure is 41, put down 1 and carry 4; then 9 times 1 is 9, and 4 I carry is 13, and 4 the back Figure is 17, put down 7 and carry 1. Now as the Multiplication by the 9 (the Unit's Figure) is ended, I add the 1 I carry to the last Figure in the Multiplicand, and it makes 2, which I put down, and the work is ended; and in this Manner proceed with all the rest of the Examples in this Case.

$$(35) \quad 1569876 \quad (36) \quad 171312 \quad (37) \quad 1899222 \quad (38) \quad 338464$$

$$(39) \quad 542130 \quad (40) \quad 2822784 \quad (41) \quad 248353 \quad (42) \quad 335286$$

$$(43) \quad 2712592$$

5. S E C T. V. D I V I S I O N.

C A S E I.

In the first Example I begin and say, How oft 2 in 17? Answer, 8 times 2 is 16, and 1 over, which is 10, added to 4 the next Figure, makes 14; then I say how oft 2 in 14? Answ. 7 times 2 is 14, and 0 over; how often 2 in 2? Answer, 1 and 0 over; how oft 2 in 6? Answ. 3 times 2 is 6, and 0 over; how often 2 in 3? Answ. 1 and 1 over, which is 10, and the next Figure 6 is 16; how oft 2 in 16? Answ. 8 times 2 is 16, and nothing remains; and in this Manner proceed with all the rest of the Examples in this Case.

(1) Quotient $\begin{array}{r} 871318 \\ 2 \end{array}$	(2) $\begin{array}{r} 921354-2 \\ \hline \end{array}$	(3) $\begin{array}{r} 540185-2 \\ \hline \end{array}$
Proof $\begin{array}{r} 1742636 \\ \hline \end{array}$	(4) $\begin{array}{r} 215285-1 \\ \hline \end{array}$	(5) $\begin{array}{r} 11903492-2 \\ \hline \end{array}$
(6) $\begin{array}{r} 667751-6 \\ \hline \end{array}$	(7) $\begin{array}{r} 346012 \\ \hline \end{array}$	(8) $\begin{array}{r} 752010-4 \\ \hline \end{array}$
(9) $\begin{array}{r} 251160-4 \\ \hline \end{array}$	(10) $\begin{array}{r} 23040-4 \\ \hline \end{array}$	

C A S E II.

To perform this Case I begin with the first Example, and say, How oft is 25 in 73; or, which is better, how oft 2, the first Figure in the Divisor, in 7, the first in the Dividend, and I find (after the Allowance is made for what I shall have to carry) it will only go 2 times; wherefore I place 2 in the Quotient, and multiply 25 the Divisor thereby, the Product (viz. 50) set under 73, and subtract; then to the Remainder 23 I bring down the next Figure 6 in the Dividend; then I say (as before) how oft 2 in 23? Answer, 9 times, which I place in the Quotient, and multiply (25) the Divisor thereby, the Product, viz. 225 subtracted from 236 leaves 11, to which bring the next Figure in the Dividend, viz. 4, then proceed as before, till you have brought down all the Figures in the Dividend, and the Work will finished.

Divisor.

Divisor Divid. (11) Quot.

25)736473575(29458943

50

5

236

147294715

225

5

.114

Pr. 736473575

100

.147

125

.223

200

.235

225

.107

100

..75

..75

..

(13)

648)272357640(420305

2592

.1315

1296

..1976

1944

..3240

3240

.....

(12)

84)35730972(425368⁶⁰/₈₄

336

.213

168

.450

420

.309

252

.577

504

.732

672

Remains 60

(14)

759)30891829676(40700697

3036

..5318

5313

...5296

4554

.7427

6831

.5966

5313

Remains .653

Division.

9

$$\begin{array}{r} (15) \\ 3065 \overline{) 63463902247} (20706003 \\ \underline{6130} \end{array}$$

$$\begin{array}{r} .21639 \\ \underline{21455} \end{array}$$

$$\begin{array}{r} ..18402 \\ \underline{18390} \end{array}$$

$$\begin{array}{r} ...12247 \\ \underline{9195} \end{array}$$

$$\text{Remains } \underline{3052}$$

$$\begin{array}{r} (16) \\ 7489 \overline{) 1204530760} (160839 \\ \underline{7489} \end{array}$$

$$\begin{array}{r} .45563 \\ \underline{44934} \end{array}$$

$$\begin{array}{r} 62907 \\ \underline{59912} \end{array}$$

$$\begin{array}{r} 29956 \\ \underline{22467} \end{array}$$

$$\begin{array}{r} 74890 \\ \underline{67401} \end{array}$$

$$\text{Remains } \underline{.7489}$$

$$\begin{array}{r} (17) \\ 42163 \overline{) 112737328} (2673 \\ \underline{84326} \end{array}$$

$$\begin{array}{r} .284113 \\ \underline{252978} \end{array}$$

$$\begin{array}{r} .311352 \\ \underline{295141} \end{array}$$

$$\begin{array}{r} .162118 \\ \underline{126489} \end{array}$$

$$\text{Remains } \underline{.35629}$$

$$\begin{array}{r} (18) \\ 61745 \overline{) 392628787} (6358 \\ \underline{370470} \end{array}$$

$$\begin{array}{r} 221587 \\ \underline{185235} \end{array}$$

$$\begin{array}{r} .363528 \\ \underline{308725} \end{array}$$

$$\begin{array}{r} 548037 \\ \underline{493960} \end{array}$$

$$\text{Remains } \underline{.54077}$$

(19)

Division.

(19)	(20)
684573)3233238699(4723	476085)98839054780(207608
2738292	952170
4949466	3622054
4792011	3332595
• 1574559	• 2894597
1369146	2856510
• 2054139	• 3808780
2053719	3808680
Remains 420	Remains 100

(21)
4728395)27750950255(5869
23641975
• 41089752
37827160
• 32625925
28370370
• 42555555
42555555
.....

(22)	CASE III.	(23)
28,00)119282,48(4260	172,000)247004,674(1436	12674
112	2800	172
72	3408248	• 750
56	852	688
168	11928248 Proof.	• 620
168		516
Rem. . . . 248		1044
		1032
		Remains . . 12674

Division.

11

$$\begin{array}{r}
 (24) \\
 473,000 \overline{) 351858,000} (743\frac{419}{73} \quad 697,0000 \overline{) 59943,0000} (86\frac{393}{697} \\
 \underline{3311} \\
 2075 \\
 \underline{1892} \\
 1838 \\
 \underline{1419} \\
 \text{Remains } 419|000 \\
 \underline{473|000}
 \end{array}$$

$$\begin{array}{r}
 (25) \\
 5576 \\
 \underline{4183} \\
 4182 \\
 \underline{\hspace{1cm}} \\
 \text{Remains } 10000 \\
 \underline{697|0000}
 \end{array}$$

CASE IV.

$$\begin{array}{l}
 (26) \\
 16 \left\{ \begin{array}{l} 4) 1206816 \\ 4) 301704 \end{array} \right. \\
 \text{Quot. } 75426
 \end{array}$$

$$\begin{array}{l}
 (27) \\
 48 \left\{ \begin{array}{l} 6) 42768 \\ 8) 7128 \end{array} \right. \\
 \underline{891}
 \end{array}$$

$$\begin{array}{l}
 (28) \\
 72 \left\{ \begin{array}{l} 8) 74682 \\ 9) 9335-2 \\ 1037-2 \end{array} \right\} 18 R.
 \end{array}$$

$$\begin{array}{l}
 (29) \\
 144 \left\{ \begin{array}{l} 12) 14276 \\ 12) 1189-8 \\ 99-1 \end{array} \right\} 20 \text{ Rem.}
 \end{array}$$

$$\begin{array}{l}
 (30) \\
 28 \left\{ \begin{array}{l} 4) 247684 \\ 7) 61921 \end{array} \right. \\
 \underline{8845-6=24 R.}
 \end{array}$$

$$\begin{array}{l}
 (31) \\
 64 \left\{ \begin{array}{l} 8) 14652 \\ 8) 1831-4 \\ 228-7 \end{array} \right\} 60 \text{ Rem.}
 \end{array}$$

$$\begin{array}{l}
 (32) \\
 81 \left\{ \begin{array}{l} 9) 417681 \\ 9) 46409 \end{array} \right. \\
 \underline{5156-5=45 \text{ Rem.}}
 \end{array}$$

$$\begin{array}{r}
 (33) \\
 132 \left\{ \begin{array}{l} 11) 307684 \\ 12) 27971-3 \\ 2330-11 \end{array} \right\} 124 \text{ Remains.}
 \end{array}$$

C A S E V.

$$\begin{array}{r}
 (34) \\
 17) 690489(40617 \\
 \hline
 104 \\
 \hline
 28 \\
 \hline
 119 \\
 \hline
 \dots
 \end{array}$$

$$\begin{array}{r}
 (35) \\
 86) 5343698(62136 \\
 \hline
 183 \\
 \hline
 116 \\
 \hline
 309 \\
 \hline
 518 \\
 \hline
 \text{Remains } 2
 \end{array}$$

$$\begin{array}{r}
 (36) \\
 467) 2148686(4601 \\
 \hline
 .2806 \\
 \hline
 486 \\
 \hline
 \text{Remains } 19
 \end{array}$$

$$\begin{array}{r}
 (37) \\
 6074) 24939844(4106 \\
 \hline
 6438 \\
 \hline
 36444 \\
 \hline
 \dots
 \end{array}$$

6. R E D U C T I O N.
M O N E Y.

$$\begin{array}{r}
 (1) \quad \text{£.} \\
 27 \\
 20 \\
 \hline
 540 \text{ Shillings.} \\
 12 \\
 \hline
 6480 \text{ Pence.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 12) 6480 \text{ Pence.} \\
 \hline
 2,0) 54,0 \text{ Shillings.} \\
 \hline
 27 \text{ £.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (3) \quad \begin{array}{r} \text{£. } s. \\ 40 \quad 10 \\ 20 \\ \hline 810 \text{ Shillings.} \\ 12 \\ \hline 9720 \text{ Pence.} \\ 4 \\ \hline 38880 \text{ Farthings.} \end{array}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad \begin{array}{r} \text{qrs.} \\ 4) 38880 \\ \hline 12) 9720 \\ \hline 2,0) 81,0 \\ \hline \text{£. } 40 \quad 10 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (5) \quad \begin{array}{r} \text{£. } s. \quad d. \\ 104 \quad 17 \quad 6\frac{1}{4} \\ 20 \\ \hline 2097 \text{ Shillings.} \\ 12 \\ \hline 25170 \text{ Pence.} \\ 4 \\ \hline 100683 \text{ Farthings.} \end{array}
 \end{array}$$

$$\begin{array}{r}
 (6) \quad \begin{array}{r} \text{qrs.} \\ 4) 100683 \\ \hline 12) 25170\frac{3}{4} \\ \hline 2,0) 209,7-6 \\ \hline \text{£. } 104 \quad 17 \quad 6\frac{1}{4} \end{array}
 \end{array}$$

Guineas.

$$\begin{array}{r}
 (7) \quad \begin{array}{r} 21 \\ 7 \times 3 = 21s. = 1 \text{ Guin.} \\ \hline 147 \\ 3 \\ \hline 441 \text{ Shillings.} \\ 12 \\ \hline 5292 \text{ Pence.} \\ 4 \\ \hline 21168 \text{ Farthings.} \end{array}
 \end{array}$$

$$\begin{array}{r}
 (8) \quad \begin{array}{r} \text{qrs.} \\ 4) 21168 \\ \hline 12) 5292 \\ \hline 21 \left\{ \begin{array}{l} 3) 441 \\ \hline 7) 147 \\ \hline \end{array} \right. \\ \hline 21 \text{ Guineas} \end{array}
 \end{array}$$

- (9) 42 Moidores.
 $9 \times 3 = 27s. = 1 \text{ Moid.}$

$$\begin{array}{r} \text{---} \\ 378 \\ 3 \\ \text{---} \\ 1134 \text{ Shillings.} \\ 12 \\ \text{---} \\ 13608 \text{ Pence.} \\ 4 \\ \text{---} \\ 54432 \text{ Farthings.} \\ \text{---} \end{array}$$

- (10) 4)54432
 $12)13608 \text{ Pence.}$
 $27 \left\{ \begin{array}{l} 3)1134 \text{ Shill.} \\ 9)378 \end{array} \right.$
 42 Moid.

WEIGHTS, MEASURES, &c.
 TROY-WEIGHT.

- (1) 24 lb.
 12

$$\begin{array}{r} \text{---} \\ 288 \text{ oz.} \\ 20 \end{array}$$

$$\begin{array}{r} 5760 \text{ dwts.} \\ 6 \times 4 = 24 \end{array}$$

$$\begin{array}{r} \text{---} \\ 34560 \\ 4 \\ \text{---} \\ 138240 \text{ grs.} \\ \text{---} \end{array}$$

- (3) lb. oz. dw. grs.
 12 10 0 22
 12

$$\begin{array}{r} \text{---} \\ 154 \\ 20 \end{array}$$

$$\begin{array}{r} 3080 \\ 6 \times 4 = 24 \end{array}$$

$$\begin{array}{r} \text{---} \\ 18480 \\ 4 \end{array}$$

$$\begin{array}{r} \text{---} \\ 73942 \text{ grs.} \end{array}$$

- (2) 24 { 4)138240 grs.
 6)34560

$$\begin{array}{r} 2,0)576,0 \text{ dwts.} \\ \text{---} \end{array}$$

$$\begin{array}{r} 12)288 \text{ oz.} \\ \text{---} \end{array}$$

$$\begin{array}{r} 24 \text{ lb.} \\ \text{---} \end{array}$$

- (4) 24 { grs.
 4)73942
 6)18485—2
 2,0)308,0—5 } 22 grs.
 12)154
 lb. 12 10 0 22

APOTHECARIES WEIGHTS.

$$\begin{array}{r} \text{lb.} \\ (1) \quad 14 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 168 \overline{3} \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 1344 \overline{3} \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4032 \overline{3} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 8064 \overline{3} \text{ grs.} \\ \hline \end{array}$$

$$(2) \quad 2,0)8064,0$$

$$\begin{array}{r} 3)4032 \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 8)1344 \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 12)168 \overline{3} \\ \hline \end{array}$$

$$14 \text{ lb.}$$

$$\begin{array}{r} \text{lb.} \quad \overline{3} \quad \overline{3} \quad \overline{3} \text{ grs.} \\ (3) \quad 4 \quad 11 \quad 0 \quad 2 \quad 17 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 59 \overline{3} \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 472 \overline{3} \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 1418 \overline{3} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 28377 \text{ grs.} \\ \hline \end{array}$$

$$(4) \quad 2,0)2837,7$$

$$\begin{array}{r} 3)1418 \text{—} 17 \text{ grs.} \\ \hline \end{array}$$

$$\begin{array}{r} 8)472 \text{—} 2 \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 12)59 \\ \hline \end{array}$$

$$\text{lb.} \quad 4 \quad 11 \quad 0 \quad 2 \quad 17$$

C 2

AVOIR

CLOTH MEASURE.

<i>Yds.</i>		<i>Nails.</i>		<i>E.E. qrs. na.</i>
(1) 24		(2) 4)384		(3) 72 4 2
4				5
—		—		—
96 qrs.		4)96 qrs.		364 qrs.
4		—		4
—		24 Yards.		—
384 Nails.		—		1458 Nails.
—				—
(4) 4)1458		(5) 12 F		(6) 4)1452
—		3		—
5)364—2		—		3)363
—		363		—
E. Ells 72 4 2		4		121 E. F.
—		—		—
		1452 Nails.		
		—		

<i>F.E. qrs.</i>
(7) 42 5
6
—
257 qrs.
4
—
1028 Nails.
—

<i>Nails.</i>
(8) 4)1028
—
6)257 qrs.
—
Fl. E. 42 5 0
—

LONG MEASURE.

<i>M. F. P.</i>		<i>Poles.</i>
(1) 176 0 30		(2) 4,0)5635,0
8		—
—		8)1408—30 P.
1408 Fur.		—
40		M. 176 0 30 P.
—		—
56350 Poles.		
—		

Reduction.

(3) 200 M.
1760

352000 Yards.
3

1056000 Feet.
12

12672000 Inches.

Lea. m. fur. p. y.
 (3) 12 1 6 29 4
 3
 —
 37 Miles.
 8
 —
 302 Fur.
 40
 —
 12109 Poles.
 11 Half Yds. in a Pole.
 —
 133207 Half Yards.
 18 Inches = $\frac{1}{2}$ Yd.
 —
 2397726 Inches.
 3
 —
 7193178 Barley Corns.

(4) 12) 12672000 Inches.

 3) 1056000 Feet.

 176,0) 35200,0 (200 Miles.
 352

 ...00

B. C.

(6) $3 \overline{) 7193178}$

$\left\{ \begin{array}{l} 3 \overline{) 2397726} \text{ Inches.} \\ 6 \overline{) 799242} \end{array} \right.$

$11 \overline{) 133207} \text{ half Yds.}$

$4,0 \overline{) 1210,9} - 8 = 4 \text{ Yds.}$

$8 \overline{) 302} - 29 \text{ P.}$

$3 \overline{) 37} - 6 \text{ Fur.}$

Lea. 12 1 6 29 4

LAND MEASURE.

(1) $\begin{array}{r} A. \\ 42 \\ 4 \\ \hline 168 \text{ Roods.} \\ 40 \\ \hline 6720 \text{ Poles.} \end{array}$

(2) $\begin{array}{r} P. \\ 4,0)672,0 \\ \hline 4)168 \text{ Roods.} \\ \hline 42 \text{ Acres.} \end{array}$

(3)

$$\begin{array}{r}
 \text{(3)} \quad A. \ r. \ p. \\
 12 \ 3 \ 29 \\
 \underline{4} \\
 51 \text{ Roods.} \\
 \underline{40} \\
 2069 \text{ Poles.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(4)} \quad P. \\
 4,0)206,9 \\
 \underline{\hspace{1cm}} \\
 4)51-29 \text{ P.} \\
 \underline{\hspace{1cm}} \\
 A. \ 12 \ 3 \ 29 \\
 \underline{\hspace{1cm}}
 \end{array}$$

WINE MEASURE.

$$\begin{array}{r}
 \text{(1)} \quad A. \\
 4 \\
 \underline{10} \\
 40 \text{ Gal.} \\
 \underline{4} \\
 160 \text{ Qts.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(2)} \quad 4)160 \\
 \underline{\hspace{1cm}} \\
 1,0)4,0 \text{ Gal.} \\
 \underline{\hspace{1cm}} \\
 4 \text{ Anchors.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(3)} \quad Hbds. \\
 4 \\
 \underline{63} \\
 252 \text{ Gall.} \\
 \underline{8} \\
 2016 \text{ Pints.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(4)} \quad 8)2016 \\
 \underline{\hspace{1cm}} \\
 63 \left\{ \begin{array}{l} 7)252 \\ \underline{\hspace{1cm}} \\ 9)36 \\ \underline{\hspace{1cm}} \end{array} \right. \\
 4 \text{ Hhds.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \quad Tierce \ gal. \\
 42 \ 24 \\
 7 \times 6 = 42 \\
 \underline{\hspace{1cm}} \\
 294 \\
 \underline{6} \\
 1788 \text{ Gal.} \\
 \underline{8} \\
 14304 \text{ Pints.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 \text{(6)} \quad Pints. \\
 8)14304 \\
 \underline{\hspace{1cm}} \\
 42 \left\{ \begin{array}{l} 6)1788 \text{ Gal.} \\ \underline{\hspace{1cm}} \\ 7)298 \\ \underline{\hspace{1cm}} \end{array} \right. \\
 Tierce \ 42 \ 24 \\
 \underline{\hspace{1cm}}
 \end{array}$$

(9) 42 Moidores.
 $9 \times 3 = 27s. = 1 \text{ Moid.}$

$$\begin{array}{r} \text{---} \\ 378 \\ 3 \\ \text{---} \\ 1134 \text{ Shillings.} \\ 12 \end{array}$$

$$\begin{array}{r} \text{---} \\ 13608 \text{ Pence.} \\ 4 \end{array}$$

$$\begin{array}{r} \text{---} \\ 54432 \text{ Farthings.} \\ \text{---} \end{array}$$

(10) 4)54432
 $12)13608 \text{ Pence.}$

$$27 \left\{ \begin{array}{l} 3)1134 \text{ Shill.} \\ \text{---} \\ 9)378 \end{array} \right.$$

$$\begin{array}{r} \text{---} \\ 42 \text{ Moid.} \\ \text{---} \end{array}$$

WEIGHTS, MEASURES, &c.
 TROY-WEIGHT.

(1) 24 lb.

$$\begin{array}{r} 12 \\ \text{---} \\ 288 \text{ oz.} \end{array}$$

$$20$$

$$5760 \text{ dwts.}$$

$$6 \times 4 = 24$$

$$\begin{array}{r} \text{---} \\ 34560 \\ 4 \end{array}$$

$$\begin{array}{r} \text{---} \\ 138240 \text{ grs.} \\ \text{---} \end{array}$$

lb. oz. dw. grs.

(3) 12 10 0 22

$$12$$

$$\begin{array}{r} \text{---} \\ 154 \\ 20 \end{array}$$

$$3080$$

$$6 \times 4 = 24$$

$$\begin{array}{r} \text{---} \\ 18480 \\ 4 \end{array}$$

$$73942 \text{ grs.}$$

(2) 4)138240 grs.

$$24 \left\{ \begin{array}{l} 6)34560 \end{array} \right.$$

$$2,0)576,0 \text{ dwts.}$$

$$12)288 \text{ oz.}$$

$$\begin{array}{r} \text{---} \\ 24 \text{ lb.} \\ \text{---} \end{array}$$

(4) 4)73942

$$24 \left\{ \begin{array}{l} 6)18485-2 \end{array} \right.$$

$$2,0)308,0-5$$

$$12)154$$

$$\begin{array}{r} \text{---} \\ \text{lb. } 12 \ 10 \ 0 \ 22 \end{array}$$

APOTHECARIES WEIGHTS.

$$\begin{array}{r} \text{lb.} \\ (1) \quad 14 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 168 \text{ } \overline{3} \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 1344 \text{ } \overline{3} \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 4032 \text{ } \overline{3} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 8064 \text{ } \overline{3} \text{ grs.} \\ \hline \end{array}$$

$$\begin{array}{r} \text{lb. } \overline{3} \text{ } \overline{3} \text{ } \overline{3} \text{ grs.} \\ (3) \quad 4 \text{ } 11 \text{ } 0 \text{ } 2 \text{ } 17 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 59 \text{ } \overline{3} \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 472 \text{ } \overline{3} \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 1418 \text{ } \overline{3} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 28377 \text{ grs.} \\ \hline \end{array}$$

$$(2) \quad 2,0)8064,0$$

$$\begin{array}{r} 3)4032 \text{ } \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 8)1344 \text{ } \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 12)168 \text{ } \overline{3} \\ \hline \end{array}$$

$$14 \text{ lb.}$$

$$(4) \quad 2,0)2837,7$$

$$\begin{array}{r} 3)1418 \text{ } \overline{17} \text{ grs.} \\ \hline \end{array}$$

$$\begin{array}{r} 8)472 \text{ } \overline{2} \text{ } \overline{3} \\ \hline \end{array}$$

$$\begin{array}{r} 12)59 \\ \hline \end{array}$$

$$\text{lb. } 4 \text{ } 11 \text{ } 0 \text{ } 2 \text{ } 17$$

A VO I R D U P O I S E W E I G H T.

(1) 20 C.

4

80 qrs.

28

2240 lb.

16

35840 oz.

16

573440 drs.

lb. oz. drs.

(3) 27 12 11

16

444

16

7115 drs.

T. C. qrs. lb. oz. drs.

(5) 12 10 0 14 11 15

20

250 C.

4

1000 qrs.

28

28014 lb.

16

448235 oz.

16

7171775 drs.

(2)

16 { 4) 573440 drs.
4) 14336016 { 4) 35840 oz.
4) 896028 { 4) 2240 lb.
7) 5604) 80 qrs.
20 C.

(4)

16 { 4) 7115
4) 1778-3 } 11 drs.
16 { 4) 444-2 }
4) 111
lb. 27 12 11

drs.

16 { 4) 7171775
4) 1792943-3 } 15 drs.16 { 4) 448235-3 } 11 oz.
4) 112058-3 }28 { 4) 28014-2 } 14 lb.
7) 7003-2 }

4) 1000-3 }

2,0) 25,0

Tons 12 10 0 14 11 15

CLOTH MEASURE.

<i>Yds.</i>		<i>Nails.</i>		<i>E. E. qrs. na.</i>
(1) 24		(2) 4) 384		(3) 72 4 2
4				5
—		4) 96 qrs.		—
96 qrs.		—		364 qrs.
4		24 Yards.		4
—		—		—
384 Nails.				1458 Nails.
—				—
(4) 4) 1458		(5) 121		(6) 4) 1452
—		3		—
5) 364 — 2		—		3) 363
—		363		—
E. Ells 72 4 2		4		121 E. F.
—		—		—
		1452 Nails.		
		—		

<i>F. E. qrs.</i>
(7) 42 5
6
—
257 qrs.
4
—
1028 Nails.
—

<i>Nails.</i>
(8) 4) 1028
—
6) 257 qrs.
—
Fl. E. 42 5 0
—

LONG MEASURE.

<i>M. F. P.</i>
(1) 176 0 30
8
—
1408 Fur.
40
—
56350 Poles.
—

<i>Poles.</i>
(2) 4,0) 5635,0
—
8) 1408 — 30 P.
—
M. 176 0 30 P.
—

$$\begin{array}{r}
 (3) \quad 200 \text{ M.} \\
 1760 \\
 \hline
 352000 \text{ Yards.} \\
 3 \\
 \hline
 1056000 \text{ Feet.} \\
 12 \\
 \hline
 12672000 \text{ Inches.}
 \end{array}$$

$$\begin{array}{r}
 \text{Lea. m. fur. p. y.} \\
 (3) \quad 12 \quad 1 \quad 6 \quad 29 \quad 4 \\
 3 \\
 \hline
 37 \text{ Miles.} \\
 8 \\
 \hline
 302 \text{ Fur.} \\
 40 \\
 \hline
 12109 \text{ Poles.} \\
 11 \text{ Half Yds. in a Pole.} \\
 \hline
 133207 \text{ Half Yards.} \\
 18 \text{ Inches} = \frac{1}{2} \text{ Yd.} \\
 \hline
 2397726 \text{ Inches.} \\
 3 \\
 \hline
 7193178 \text{ Barley Corns.}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad 12) 12672000 \text{ Inches.} \\
 \hline
 3) 1056000 \text{ Feet.} \\
 \hline
 176,0) 35200,0 (200 \text{ Miles.} \\
 352 \\
 \hline
 \dots 00
 \end{array}$$

$$\begin{array}{r}
 \text{B. C.} \\
 (6) \quad 3) 7193178 \\
 \hline
 18 \left\{ \begin{array}{l} 3) 2397726 \text{ Inches.} \\ \hline 6) 799242 \end{array} \right. \\
 \hline
 11) 133207 \text{ half Yds.} \\
 \hline
 4,0) 1210,9 - 8 = 4 \text{ Yds.} \\
 \hline
 8) 302 - 29 \text{ P.} \\
 \hline
 3) 37 - 6 \text{ Fur.} \\
 \hline
 \text{Lea. } 12 \quad 1 \quad 6 \quad 29 \quad 4
 \end{array}$$

LAND MEASURE.

$$\begin{array}{r}
 A. \\
 (1) \quad 42 \\
 4 \\
 \hline
 168 \text{ Roods.} \\
 40 \\
 \hline
 6720 \text{ Poles.}
 \end{array}$$

$$\begin{array}{r}
 P. \\
 (2) \quad 4,0) 672,0 \\
 \hline
 4) 168 \text{ Roods.} \\
 \hline
 42 \text{ Acres.}
 \end{array}$$

$$\begin{array}{r}
 (3) \quad A. \ r. \ p. \\
 12 \ 3 \ 29 \\
 \underline{4} \\
 51 \text{ Roods.} \\
 \underline{40} \\
 2069 \text{ Poles.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad P. \\
 4,0)206,9 \\
 \underline{\hspace{1cm}} \\
 4)51-29 \text{ P.} \\
 \underline{\hspace{1cm}} \\
 A. \ 12 \ 3 \ 29 \\
 \underline{\hspace{1cm}}
 \end{array}$$

WINE MEASURE.

$$\begin{array}{r}
 (1) \quad A. \\
 4 \\
 10 \\
 \underline{\hspace{1cm}} \\
 40 \text{ Gal.} \\
 4 \\
 \underline{\hspace{1cm}} \\
 160 \text{ Qts.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 4)160 \\
 \underline{\hspace{1cm}} \\
 1,0)4,0 \text{ Gal.} \\
 \underline{\hspace{1cm}} \\
 4 \text{ Anchors.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (3) \quad Hbds. \\
 4 \\
 63 \\
 \underline{\hspace{1cm}} \\
 252 \text{ Gall.} \\
 8 \\
 \underline{\hspace{1cm}} \\
 2016 \text{ Pints.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad 8)2016 \\
 \underline{\hspace{1cm}} \\
 63 \left\{ \begin{array}{l} 7)252 \\ \underline{\hspace{1cm}} \\ 9) \ 36 \\ \underline{\hspace{1cm}} \end{array} \right. \\
 4 \text{ Hhds.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (5) \quad Tierce \ gal. \\
 42 \ 24 \\
 7 \times 6 = 42 \\
 \underline{\hspace{1cm}} \\
 294 \\
 6 \\
 \underline{\hspace{1cm}} \\
 1788 \text{ Gal.} \\
 8 \\
 \underline{\hspace{1cm}} \\
 14304 \text{ Pints.} \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r}
 (6) \quad Pints. \\
 8)14304 \\
 \underline{\hspace{1cm}} \\
 42 \left\{ \begin{array}{l} 6)1788 \text{ Gal.} \\ \underline{\hspace{1cm}} \\ 7) \ 298 \\ \underline{\hspace{1cm}} \end{array} \right. \\
 Tierce \ 42 \ 24 \\
 \underline{\hspace{1cm}}
 \end{array}$$

$$\begin{array}{r} T. p. hd. gal. pts. \\ (7) \quad 4 \quad 1 \quad 1 \quad 42 \quad 6 \\ \quad \quad 2 \end{array}$$

9 Pipes.

$$\begin{array}{r} \quad \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 19 \text{ Hhds.} \\ 63 \end{array}$$

$$\begin{array}{r} \hline 1239 \\ 8 \end{array}$$

$$\begin{array}{r} \hline 9918 \text{ Pints.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

WINCHESTER MEASURE.

$$(1) \quad 12 \text{ B. A.}$$

$$\begin{array}{r} 32 \\ \hline 384 \text{ Gal.} \\ 4 \end{array}$$

$$\begin{array}{r} \hline 1536 \text{ Quarts.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$(3) \quad 42 \text{ B. B.}$$

$$9 \times 4 = 36$$

$$\begin{array}{r} 378 \\ 4 \end{array}$$

$$\begin{array}{r} \hline 1512 \text{ Gal.} \\ 8 \end{array}$$

$$\begin{array}{r} \hline 12096 \text{ Pints.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$(5) \quad 6 \quad 27 \quad 6$$

$$\begin{array}{r} 48 \\ \hline 315 \text{ Gal.} \\ 8 \end{array}$$

$$\begin{array}{r} \hline 2526 \text{ Pints.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$(8) \quad 8)9918$$

$$\begin{array}{r} \hline \end{array}$$

$$63 \left\{ \begin{array}{l} 7)1239-6 \text{ Pints.} \\ \hline 9)177 \end{array} \right.$$

$$\begin{array}{r} \hline 2)19-42 \text{ Gal.} \end{array}$$

$$\begin{array}{r} \hline 2)9-1 \text{ Hhd.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$\text{Tuns. } 4 \quad 1 \quad 1 \quad 42 \quad 6$$

$$\begin{array}{r} \hline \end{array}$$

$$(2) \quad 4)1536 \text{ Qts.}$$

$$\begin{array}{r} \hline \end{array}$$

$$32 \left\{ \begin{array}{l} 4)384 \text{ Gal.} \\ \hline 8)96 \end{array} \right.$$

$$\begin{array}{r} \hline 12 \text{ Barrels.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$(4) \quad 8)12096$$

$$\begin{array}{r} \hline \end{array}$$

$$36 \left\{ \begin{array}{l} 4)1512 \text{ Gal.} \\ \hline 9)378 \end{array} \right.$$

$$\begin{array}{r} \hline 42 \text{ Barrels.} \end{array}$$

$$\begin{array}{r} \hline \end{array}$$

$$(6) \quad 8)2526$$

$$\begin{array}{r} \hline \end{array}$$

$$48 \left\{ \begin{array}{l} 6)315-6 \text{ Pints.} \\ \hline 8)52-3 \end{array} \right.$$

$$\begin{array}{r} \hline 6-4 \end{array}$$

$$\text{Hhds. } 6 \quad 27 \quad 6$$

Reduction.

21

$$\begin{array}{r}
 \text{B. bhd. gal.} \\
 (7) \quad 14 \quad 47 \\
 \quad 54 \\
 \hline
 803 \text{ Gal.} \\
 \quad 8 \\
 \hline
 6424 \text{ Pints.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{T. b. bd. gal.} \\
 (9) \quad 6 \quad 1 \quad 0 \quad 42 \\
 \quad 2 \\
 \hline
 13 \text{ B.} \\
 \quad 2 \\
 \hline
 26 \text{ Hhds.} \\
 9 \times 6 = 54 \\
 \hline
 234 \\
 \quad 6 \\
 \hline
 1446 \text{ Gal.} \\
 \quad 4 \\
 \hline
 5784 \text{ Qts.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Pints.} \\
 (8) \quad 8) 6424 \\
 \hline
 54 \left\{ \begin{array}{l} 6) 803 \text{ Gal.} \\ \hline 9) 133-5 \\ \hline 14-7 \end{array} \right\} 47 \text{ Gal.}
 \end{array}$$

Ans. 14 Hhds. 47 Gal.

$$\begin{array}{r}
 (10) \quad 4) 5784 \\
 \hline
 54 \left\{ \begin{array}{l} 6) 1446 \\ \hline 9) 241 \\ \hline 2) 26-7 \\ \hline 2) 13 \end{array} \right\} 42 \text{ Gal.} \\
 \hline
 \text{T. 6 1 0 42} \\
 \hline
 \end{array}$$

DRY MEASURE.

$$\begin{array}{r}
 \text{Qrs.} \\
 (1) \quad 24 \\
 \quad 8 \\
 \hline
 192 \text{ Bush.} \\
 \quad 4 \\
 \hline
 768 \text{ Pecks.} \\
 \quad 2 \\
 \hline
 1536 \text{ Gal.} \\
 \quad 4 \\
 \hline
 6144 \text{ Qts.}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 4) 6144 \text{ Qts.} \\
 \hline
 2) 1536 \text{ Gal.} \\
 \hline
 4) 768 \text{ Pecks.} \\
 \hline
 8) 192 \text{ Bush.} \\
 \hline
 24 \text{ Qrs.} \\
 \hline
 \end{array}$$

Cha

Cha. bu.
 (3) $36 \times 26 = 36$

216
 6

1322 Bush.

4

5288 Pecks.

Lasts.

(5) 64
 2

128 Weyes.

5

640 Qrs.

8

5120 Bush.

4

20480 Pecks.

Pecks.

(4) $4)5288$

$36 \left\{ \begin{array}{l} 6)1322 \text{ Bush.} \\ 6)220-2 \\ 36-4 \end{array} \right\} 26 \text{ Bush.}$

Ans. 36 Ch. 26 Bush.

Pecks.

(6) $4)20480$

$8)5120$

$1,0)64,0$ Qrs.

64 Lasts.

T I M E.

Days.

(1) 7
 24

168 Hours.

60

10080 Min.

60

604800 Seconds.

(2) $6,0)60480,0$ Seconds.

$6,0)1008,0$ Min.

$24 \left\{ \begin{array}{l} 4)168 \text{ Hours.} \\ 6)42 \end{array} \right.$

7 Days.

M.

Reduction.

23

(3) *M. d.*

$$\begin{array}{r} 6 \ 4 \\ 28 \\ \hline 172 \text{ Days.} \\ 6 \times 4 = 24 \\ \hline 1032 \\ 4 \\ \hline 4128 \text{ H.} \\ 60 \\ \hline 247680 \text{ M.} \\ 60 \\ \hline 14860800 \text{ Sec.} \end{array}$$

(4) *Seconds.*

$$\begin{array}{r} 6,0)1486080,0 \\ \hline 6,0)24768,0 \text{ M.} \\ \hline 24 \left\{ \begin{array}{l} 4)4128 \text{ H.} \\ \hline 6)1032 \\ \hline 7)172 \text{ D.} \\ \hline 4)24-4 \\ \hline \text{M. } 6 \ 4 \text{ D.} \end{array} \right. \end{array}$$

(5) *D. H.*

$$\begin{array}{r} 365 \ 6 \\ 6 \times 4 = 24 \\ \hline 2190 \\ 4 \\ \hline 8766 \text{ H.} \\ 60 \\ \hline 525960 \text{ M.} \\ 60 \\ \hline 31557600 \end{array}$$

(6)
$$\begin{array}{r} 6,0)3155760,0 \text{ Sec.} \\ \hline 6,0)52596,0 \text{ M.} \\ \hline 24 \left\{ \begin{array}{l} 4)8766 \\ \hline 6)2190-6 \\ \hline 365 \text{ D. } 6 \text{ H.} \end{array} \right. \end{array}$$

D.

$$\begin{array}{r} \text{D. h. m. " '"} \\ (7) \quad 365 \ 5 \ 48 \ 57 \ 39 \\ \quad 6 \times 4 = 24 \\ \hline \end{array}$$

2190

4

8765 H.

60

525948 M.

60

31556937 Sec.

60

1893416259 Thirds.

Thirds.

$$(8) \quad 6,0) 189341625,9$$

$$6,0) 3155693,7 - 39'''$$

$$6,0) 52594,8 - 57''$$

$$24 \left\{ \begin{array}{l} 4) 8765 - 48 \text{ M.} \\ 6) 2190 - 5 \text{ D.} \end{array} \right.$$

Days 365 5 48 57 39

SQUARE or SUPERFICIAL MEASURE:

Yds.

$$(1) \quad 42$$

9

378 Feet.

$$12 \times 12 = 144$$

4536

12

54432 Inches.

Sq. f. in.

$$(3) \quad 3 \ 42 \ 64$$

100

342 Feet.

$$12 \times 12 = 144$$

4104

12

49312 Inches.

Inches.

$$(2) \quad 144 \left\{ \begin{array}{l} 12) 54432 \\ 12) 4536 \end{array} \right.$$

9) 378 Feet.

42 Square Yards.

Sq. f.

$$(4) \quad 144 \left\{ \begin{array}{l} 12) 49312 \\ 12) 4109 - 4 \end{array} \right.$$

$$1,00) 3,42 - 5$$

64 Inch.

3 Sq. 42 F. 64 In.

CUBIC

CUBIC or SOLID MEASURE.

<i>Yards.</i>		<i>Inches.</i>
(1) 27	(2) 1728)	1259712 (729 Feet.
9 × 3 = 27		12096
—		—
243		•• 5011
3		3456
—		—
729 Feet.		15552
1728 Inches = 1 Foot.		15552
—		—
15552		•••••
3456		
12096		
—		
1259712 Inches	27 {	Feet.
	3) 729	—
	9) 243	—
	—	
	27 Solid Yards.	—

<i>T. F.</i>	<i>Inches.</i>	<i>Feet.</i>
(3) 4 24	(4) 1728)	387072 (5,012 24
50		3456
—		—
224 Feet.		• 4147
1728		3456
—		—
6912		• 6912
3456		6912
3456		—
—		•••••
387072 Inches.		

4 T. 24 Ft.

END of BOOK I.

THE
K E Y
 TO THE
TUTOR'S GUIDE:
 OR, THE
ARITHMETICIAN'S REPOSITORY.

BOOK II. PART I.

NUMERATION.

- (1) $1000000 + 500000 = 1500000$ South Sea Bonds.
 (2) $60000 + 12000 + 1300 = 73300$ Lead.
 (3) $150000000000 + 800000000 = 150800000000$ Stivers.
 (4) 120206070707 Rials of Plate.
 (5) 3033030 Pieces of Eight.
 (6) £. 404000 34 $15\frac{5}{4} =$ £. 404001 15 $3\frac{1}{4}$.

A D D I T I O N.
 I N T E G E R S.

140724	27460	867
296	176	317
42	2900	69
6740	274	1720
64167	1004	276842
20	64	49
2687	596	426074
2684	41	60
	6104	
217360		705998
	38619	

MONEY.

M O N E Y.

To add these Examples, I begin with the first, the Farthings, going up, saying 1 and 3 is 4, and 2 is 6, and 3 is 9, and 1 is 10 Farthings, that is $2\frac{1}{2}$ d. or 4 in 10 is 2 Times and 2 over, which is $\frac{2}{4}$ or $\frac{1}{2}$ to put down under the Row of Farthings, and carry 2 Pence to the Units Place of Pence, saying 2 I carry and 1 is 3, and 6 is 9, and 1 is 10, and 9 is 19, and 6 is 25, and 4 is 29, and 1 is 30, then coming down the Place of Tens, saying, and 10 is 40, and 10 is 50, and 10 is 60, and 10 is 70 Pence, (which by the Pence Table are 5s. 10d.) or the 12's in 70 are 5 Times and 10 over, which is 10 Pence to set down under the Row of Pence, and carry 5 Shillings to the Unit's Place of Shillings, saying 5 I carry and 8 is 13, and 2 is 15, and 4 is 19, and 7 is 26, and 1 is 27, and 6 is 33, and 7 is 40; I set down 0, and carry 4 to the Place of Tens, saying 4 I carry and 1 is 5, and 1 is 6, and 1 is 7, and 1 is 8, and 1 is 9, Half of which (or the 2's in 9) is 4, and 1 over, which I set down under the Place of Tens, in the Row of Shillings, and carry the Half (viz. 4) to the Unit's Place of Pounds, and proceed then as in Integers.

But the common Method (tho' not so concise nor yet so easy) is to go up the Unit's Place, and come down the Tens, as in the Row of Pence, which (in this Example) comes to 90 Shillings, or the 20's in 90 are 4 times and 1 over, thus, 2,0)9,0(4l. 10s. so that it plainly appears as the Unit's Figure in the Divisor, or, what I stop at is, 0, so that the Unit's Figure in the Dividend, or what the Unit's Row of Shillings comes to above 10 will be to set down, and to carry 1 for every 10 (as in Integers) to the Place of Tens, and what that Sum is divide it by that Figure which is in the Place of Tens in the Divisor, which in this Case is by 2, consequently is the Reason of my halving the Ten's Place of Shillings: Likewise when you stop at 40, 60, or &c. proceed as above, only, instead of dividing by 2, you must divide by 4, 6, or, &c. observing always to set down what remains under the Place of Tens.—See Case III. Sect. V.

$$(1) \begin{array}{r} \text{£. } 36 \ 19 \ 10\frac{1}{2} \\ \hline \end{array} \quad (2) \begin{array}{r} \text{£. } 227 \ 18 \ 9 \\ \hline \end{array} \quad (3) \begin{array}{r} \text{£. } 472 \ 11 \ 3\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r}
 \text{(4)} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 270 \quad 16 \quad 6\frac{1}{2} \\ 60 \quad 0 \quad 10\frac{1}{2} \\ 60 \quad 10 \quad 0\frac{3}{4} \\ 96 \quad 6 \quad 10 \\ 176 \quad 6 \quad 6\frac{1}{2} \\ 2 \quad 2 \quad 0 \\ 16 \quad 17 \quad 6\frac{1}{2} \\ 100 \quad 0 \quad 0 \\ \hline 783 \quad 0 \quad 4\frac{1}{2} \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(5)} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 260 \quad 17 \quad 0 \\ 67 \quad 0 \quad 10\frac{3}{4} \\ 170 \quad 10 \quad 0\frac{1}{2} \\ 100 \quad 10 \quad 6\frac{1}{4} \\ 4 \quad 16 \quad 6\frac{1}{2} \\ 0 \quad 19 \quad 0\frac{3}{4} \\ 37 \quad 11 \quad 11\frac{1}{2} \\ 600 \quad 10 \quad 0 \\ 220 \quad 0 \quad 6\frac{1}{2} \\ \hline 1462 \quad 16 \quad 6\frac{3}{4} \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(6)} \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 276 \quad 17 \quad 0 \\ 16 \quad 0 \quad 10\frac{1}{4} \\ 269 \quad 11 \quad 11\frac{1}{2} \\ 107 \quad 19 \quad 0 \\ 10 \quad 0 \quad 6 \\ 0 \quad 14 \quad 11 \\ 367 \quad 17 \quad 6\frac{1}{4} \\ 0 \quad 12 \quad 4\frac{1}{2} \\ 20 \quad 10 \quad 6 \\ 1000 \quad 0 \quad 0 \\ \hline 2070 \quad 4 \quad 8 \end{array}
 \end{array}$$

WEIGHTS, MEASURES, &c.

Here I begin with the first Example, going up the Unit's Place in Grains, and find it comes up to 23, which, on a Slate or waste Paper, I put down the 3, and carry 2 to the Place of Tens, and find it comes to 11, which I set down to the 3, and the Whole of the Row of Grains make 113, which I divide by as many Grains as make 1 Penny-weight, viz. 24, and find it will go 4 Times and 17 over, thus 24)113(4 Times, and 17 remains; put down 17 Grains under the Row of Grains, and I carry 4 Penny-weights to the Row of Penny-weights, which I proceed with as in Shillings, as I stop at the same Number, viz. 20: Likewise, wherever I stop at 4, 12, or any Number under, I proceed as in Farthings and Pence, according to what Number I stop at; but when I stop at any Number exceeding 12, as 16, 28, 54, &c. then I proceed as with the Grains above, so that it is impossible (without greatly perplexing the Learner) to perform these Examples before he has learned Division: And I am very sorry to see this Method not practised more than what it is, that so much of that precious Time of Youth should be lavished with perplexing Methods of the Teachers, as it is as improper to learn a Scholar to add these Examples before he knows the primary Rules in Integers, as it is to learn a Child to read before he knows his Letters.

$$\text{(1)} \quad \begin{array}{r} \text{oz.} \quad 170 \quad \text{dwt.} \quad 5 \quad \text{grs.} \quad 17 \\ \hline \end{array}$$

$$\text{(2)} \quad \begin{array}{r} \text{lb.} \quad 114 \quad 4 \quad 0 \quad 9 \\ \hline \end{array}$$

(3)

- | | |
|--|--|
| (3) $\begin{array}{r} 3\ 62\ 5\ 1\ 1 \\ \hline \end{array}$ | (4) $\begin{array}{r} 86\ \text{lb.}\ 9\ 0\ 0\ 8 \\ \hline \end{array}$ |
| (5) $\begin{array}{r} \text{Tons}\ 731\ 14\ 0\ 23 \\ \hline \end{array}$ | (6) $\begin{array}{r} \text{lb.}\ 121\ 11\ 17 \\ \hline \end{array}$ |
| (7) $\begin{array}{r} \text{Yards}\ 581\ 0\ 1 \\ \hline \end{array}$ | (8) $\begin{array}{r} \text{Eng. E.}\ 599\ 4\ 0 \\ \hline \end{array}$ |
| (9) $\begin{array}{r} \text{E. Fl.}\ 575\ 2\ 1 \\ \hline \end{array}$ | (10) $\begin{array}{r} \text{Lea.}\ 528\ 2\ 3\ 10 \\ \hline \end{array}$ |
| (11) $\begin{array}{r} \text{Yards}\ 976\ 0\ 4\ 0 \\ \hline \end{array}$ | (12) $\begin{array}{r} \text{A.}\ 668\ 0\ 6 \\ \hline \end{array}$ |
| (13) $\begin{array}{r} \text{Tuns}\ 57\ 0\ 1\ 25\ 0 \\ \hline \end{array}$ | (14) $\begin{array}{r} \text{Pun.}\ 155\ 62\ 2\ 1 \\ \hline \end{array}$ |
| (15) $\begin{array}{r} \text{Tier.}\ 127\ 28\ 7 \\ \hline \end{array}$ | (16) $\begin{array}{r} \text{An.}\ 88\ 9\ 3 \\ \hline \end{array}$ |
| (17) $\begin{array}{r} \text{A. hhds.}\ 118\ 3\ 0 \\ \hline \end{array}$ | (18) $\begin{array}{r} \text{B. hhds.}\ 125\ 53\ 3 \\ \hline \end{array}$ |
| (19) $\begin{array}{r} \text{A.B.}\ 124\ 1\ 0\ 06 \\ \hline \end{array}$ | (20) $\begin{array}{r} \text{B. Fir.}\ 125\ 4\ 0\ 0 \\ \hline \end{array}$ |
| (21) $\begin{array}{r} \text{Qrs.}\ 378\ 4\ 0\ 0 \\ \hline \end{array}$ | (22) $\begin{array}{r} \text{Cha.}\ 138\ 35\ 1 \\ \hline \end{array}$ |
| (23) $\begin{array}{r} \text{La.}\ 78\ 0\ 2\ 1\ 0 \\ \hline \end{array}$ | (24) $\begin{array}{r} \text{Mo.}\ 110\ 1\ 6\ 10 \\ \hline \end{array}$ |
| (25) $\begin{array}{r} \text{D.}\ 513\ 18\ 0\ 0 \\ \hline \end{array}$ | |

QUESTIONS.

- | | | |
|----------------|--|--|
| (1) <i>D.</i> | (2) $\begin{array}{r} 1771 \\ 60 \\ \hline \end{array}$ | (3) $\begin{array}{r} \text{£.} \\ 11000 \\ 1100 \\ 11 \\ \hline 12111 \\ +1000 \\ \hline 13111 \end{array}$ |
| June 29 | | |
| July 31 | | |
| Aug. 31 | | |
| Sept. 30 | Anf. A.D. 1831 | |
| Oct. 31 | | |
| Nov. 30 | | Youngest's |
| Dec. 31 | | [Fortune. |
| Jan. 27 | | |
| | | Eldest's |
| | | [Fortune. |
| Anf. 240 Days. | Then $12111 + 13111 = 25222$ £. what the Father left them. | |

Addition.

(4) D.	(5)	£.	s.	d.	(6)	£.	s.	d.
Jan. 31	Pays	17	17	6		474	17	11½
Feb. 29	Rem. unpaid	82	2	6		467	0	0
Mar. 31								
Apr. 30	Ans. 100	0	0		A. £.	941	17	11½
May 31								
June 30								
July 31								
Aug. 31								
Sept. 30								
Oct. 31								
Nov. 30								
Dec. 31								

Ans. 366 Days.

(7)	£.	s.	d.	(8)	£.	s.	d.
Corn Chandler,	123	19	0	Oats,	-	46	7 6
Brewer,	-	41	10 0	Beans,	-	100	0 0
Butcher,	-	212	0 6	Pease,	-	16	16 0
Baker,	-	24	0 0	Barley,	-	73	0 8
Tallow Chandler,	13	8	0	Wheat,	-	56	9 10
Taylor,	-	137	9 9	Rye,	-	4	4 6
Draper,	-	74	13 6	Water carriage,	-	13	2 7
Coach-maker,	214	16	6	Charges,	-	1	13 0
Wine-merchant,	68	12	0	Gains,	-	18	18 0
Confectioner,	16	2	0				
Rent 82l. + 82s. = 86	2	0		Ans. £.	330	12	1
Servants Wages,	46	5	0				
Ex. on the Road,	50	0	0				
Ans. £.	1108	18	3				

Addition.

31

(9)	£.	s.	d.
A paid	13	2	6
B	2	13	0
C	0	14	4
D	1	9	8½
E	11	0	6¼
F	0	17	6
G	0	12	2
H	1	10	6
I 27s. + 13s. =	2	0	0
K 23 + 23 + 25s. } + 1s. =	3	12	0
L	9	13	4
M	12	12	0
N	15	0	0
O. 5 × 3 + 10s. =	1	5	0

Ans. £. 76 2 6¾

(10)	£.	s.	d.
To Merc. Wares,	418	2	6
Cheshire Cheese,	52	18	0
Broad Cloth,	317	12	10
Lead,	320	0	0
Bar Iron,	173	0	3
Copper,	1110	10	1
Accepted a Bill,	88	14	0
Do. on Honour,	50	0	0
Morocco Skins,	28	15	4
Conv. Insur. &c.	43	0	0
Warehouse Room,	5	5	0
&c.	5	5	0
Factorage,	112	6	0

Ans. £. 2720 4 0

(11)	oz.	dwt.
Fourteen Dishes, wt.	193	6
Thirty-six Plates,	421	11
Four Doz. Spoons,	104	6
Six Salts,	32	0
Knives and Forks,	83	9
Four Presenters,	113	4
Mugs, Tumblers, } Beakers, &c. }	264	18
Tea-kettle & Lamp,	126	9
And the rest of the } Equipage, }	93	2

Ans. 1432 5

(12)	No.	Cwt.	qr.	lb.
Bags	1	2	2	10
	2	2	1	16
	3	2	0	24
	4	0	1	16
Pocket,	58½lb. =	0	2	2½
Ditto,	-	0	2	2½

Ans. wt. 8 2 15

(13)	M.	f.	p.
From hence to B,	39	6	0
thence to C,	46	0	24
- - D,	60	4	39
- - E,	37	6	0

A. from A to E is 184 1 23

(14)	Years.	m.	d.
	28	0	0
	2	10	16
	1	11	0
	3	7	25
	16	9	27

Ans. Years 53 3 12

(15) (16)
 A is now 16 Years old, 17+29+23=69 Years the
 B born 14 Years hence, Answer.

Answ. 30 Years.

(17) Years. (18)
 When the Eldest was } 24 20 left.
 born the Father was } 21
 Then 19 Years and 19 } 42 } stolen { 3 }
 Half-years = $19 + 9\frac{1}{2} =$ } 28 $\frac{1}{2}$ } the { 2 } Night.
 And the Youngest is now 21 } 84 }
 A. 167 Sheep at first.
 Answ. Father's Age, 73 $\frac{1}{2}$

(19)
 When Seth was born Adam was — 130 Years old,
 Enos Seth — 105
 Cainan Enos — 90
 Mahaleel Cainan — 70
 Jared Mahaleel — 65
 Enoch Jared — 162
 Methuselah Enoch — 65
 Lamech Methuselah — 187
 Noah Lamech — 182
 And when the Flood happened Noah was 600
 Answ. 1656 Years.

S U B T R A C T I O N.

I N T E G E R S.

From 476004	276000	40106
Take 120706	106019	17109
Remains 355298	169981	22997

MONEY.

M O N E Y.

As the upper Figures in the first Example are all larger than those which are underneath, so it is only taking the lower Line from the upper without borrowing at all; but in the second Example I begin and say, 2 Farthings from 1 I cannot, but 2 Farthings from a Penny, or 4 Farthings, there remains 2, and 1 the top Farthing makes 3 Farthings, which I set down thus, $\frac{3}{4}$; then I carry 1 to 11 is 12 Pence, which from 10 Pence I cannot, but 12 from 12 there remains 0, but 10 the upper Figure is to put down, and carry 1 to 11 is 12, from 10 I cannot, but 12 from 20 there remains 8, and the top Figure 10 is 18 Shillings to put down, and carry 1 to the Unit's Place of Pounds, which proceed with as in Integers.

Note, When the Learner is pretty ready at subtracting, he need not make use of the Words *I cannot*, as it is always seen at Sight whether he can or cannot; so that when he sees the lower Figure is greater than that which stands over it, always take the lower Figure from what you borrow, and to the Remainder add the top Figure, and their Sum set down; thus, in the 2d Example, I say 2 from 4, 2 and 1 is $\frac{3}{4}$ to set down, and carry 1 to 11 is 12, from 12 0, but 10 to set down, and carry 1 to 11 is 12, from 20, 8, and 10 is 18 to set down, and carry 1, &c.

(1) $\begin{array}{r} \text{£. s. d.} \\ 21 \ 13 \ 5\frac{1}{4} \\ \hline \end{array}$	(2) $\begin{array}{r} \text{£. s. d.} \\ 33 \ 18 \ 10\frac{3}{4} \\ \hline \end{array}$	(3) $\begin{array}{r} \text{£. s. d.} \\ 87 \ 7 \ 5\frac{1}{2} \\ \hline \end{array}$
--	---	---

(4) $\begin{array}{r} \text{£. s. d.} \\ 77 \ 14 \ 5\frac{3}{4} \\ \hline \end{array}$	(5) $\begin{array}{r} \text{£. s. d.} \\ 10 \ 9 \ 11\frac{3}{4} \\ \hline \end{array}$	(6) $\begin{array}{r} \text{£. s. d.} \\ 103 \ 12 \ 11\frac{1}{4} \\ \hline \end{array}$
--	--	--

(7) $\begin{array}{r} \text{£. s. d.} \\ \text{Paid} \ 370 \ 17 \ 1 \\ \hline \text{Rem. unpaid} \ 105 \ 2 \ 11 \\ \hline \end{array}$	(8) $\begin{array}{r} \text{£. s. d.} \\ \text{Received} \ 1450 \ 4 \ 9 \\ \hline \text{Rem. unpaid} \ 26 \ 9 \ 9 \\ \hline \end{array}$
--	--

(9) $\begin{array}{r} \text{£. s. d.} \\ \text{Borrowed} \ 2648 \ 19 \ 10\frac{1}{2} \\ \text{Paid} \ 1843 \ 14 \ 3 \\ \hline \text{Bal.} \ 805 \ 5 \ 6\frac{1}{2} \end{array}$

WEIGHTS

WEIGHTS, MEASURES, &c.

- (1) lb. 2 10 17 20 (2) oz. 3 13 7 (3) lb. 3 0 5 0 14
 (4) T. 1 19 3 21 (5) lb. 2 11 15 (6) Yds. 66 3 1
 (7) E. Eng. 2 4 3 (8) E. Fl. 58 0 1 (9) Lea. 39 1 4 27
 (10) Yds. 50 2 11 (11) A. 67 1 16 (12) T. 1 0 0 43 1
 (13) Pun. 3 75 3 0 (14) Tier. 3 40 5 (15) An. 6 6 5
 (16) A. hds. 6 41 3 (17) B. hd. 1 48 5 (18) B. fir. 3 4 6
 (19) A. Bar. 27 2 5 0 0 (20) Qrs. 6 6 3 (21) Cha. 6 25 2
 (22) La. 4 0 4 4 3 (23) Mo. 2 3 4 23
 (24) D. 156 16 42 45

QUESTIONS.

- (1) $\begin{array}{r} 1772 \\ 1735 \\ \hline \end{array}$ (2) $\begin{array}{r} 102 \\ 72 \\ \hline \end{array}$ $\begin{array}{r} 102 \\ 72 \\ \hline \end{array}$
 Answ. 37 Years. Diff. 30 Sum 174
 (3) $\begin{array}{r} \text{£. s. d.} \\ \text{A's Sum } 74 \ 17 \ 0 \\ \text{Diff. } 49 \ 15 \ 6 \\ \hline \end{array}$ (4) $\begin{array}{r} \text{£. s. d.} \\ \text{Borrowed } 100 \ 0 \ 0 \\ \text{Paid } 41 \ 17 \ 6 \\ \hline \end{array}$
 Answ. B's Sum £. 25 1 6 Answ. Rem. £. 58 2 6
 (5) $\begin{array}{r} \text{£. s. d.} \\ \text{Taxes } 140 \ 0 \ 0 \\ \text{Repairs } 94 \ 17 \ 6 \\ \hline \end{array}$ $\begin{array}{r} \text{£. s. d.} \\ \text{Income } 600 \ 0 \ 0 \\ \text{Pays } 234 \ 17 \ 6 \\ \hline \end{array}$
 Pays in all 234 17 6 Answ. neat £. 365 2 6
 (6)

Subtraction.

35

(6)	£.		£.	s.	d.
	11000		Left	13111	10 6
	1100	Daughter had	12111	0 0	
	11				
		Answ. Son had	£.	1000	10 6
Daughter had	12111				

(7)	£.	s.	£.	s.
Both together are worth	35	10	then	22 18
Horse alone	12	12		12 12
Furniture	22	18	Answ. £.	10 6

(8) Dr.	£.	s.	d.	Cr.	£.	s.	d.
To A, -	71	12	6	By Cash, -	3	13	6
B, -	34	9	9	Commodities,	23	10	0
C, -	16	8	8	Housh. Furnit.	13	8	6
D, -	44	0	0	Plate, -	7	18	5
E, -	66	7	6	Tenement,	56	15	0
F, -	11	2	3	Book Debts,	87	13	10
G, -	19	19	0				
H, 13s.4d. × 30 =	20	0	0				
	Dr. 283	19	8		Cr. £.	192	19 3
	Cr 192	19	3				
Answ. Bal. lost	£.	91	0 5				

(9)	£.	s.	d.	(10)	£.	s.	d.
First Bond,	114	10	0	Altog. are worth	50	0	0
Interest	19	0	0	Horse & Harness	38	16	6
Amount,	£.	133	10 0	Chaise,	£.	11	3 6
Paid off,	42	0	0				
				Chaise & Harness,	13	13	0
Second Bond,	91	10	0	Chaise,	-	11	3 6
Interest, -	13	4	8				
				Harness,	-	£.	2 9 6
Amt.	£.	104	14 8				
Paid off,	37	14	2	Horse & Harness,	38	16	6
				Harness,	-	2	9 6
Third Bond,	67	0	6				
Interest, -	9	11	3	Horse,	£.	36	7 0
Answ. Amt.	£.	76	11 9				(11)

Subtraction.

(11) Stock, Dr.				Per Contra, Cr.			
	£.	s.	d.		£.	s.	d.
To D. E.	713	13	0	By Brandy,	874	10	6
M. F.	352	10	8	Claret,	754	4	0
L. P.	180	12	0	Corn,	675	17	3
I. B.	57	12	10	Canary Seed,	113	0	0
Insurance,	190	0	0	Indigo,	632	12	0
				Saffron,	253	5	0
	£. 1494	8	6	W. P.	384	10	0
				Wines p. F. G.	1011	10	0
				Pepper p. S. Q.	1552	16	8
				Bond on R. O.	300	0	0
				Note on T. M.	260	14	0
				India Bonds,	459	0	0
				Interest,	25	14	6
				Bank Stock,	2134	4	6
				Banker,	1892	17	6
				Cr.	11324	15	11
				Dr.	1494	8	6
				An. Pref. Worth	9830	7	5

(12) Stock, Dr.				Per Contra, Cr.			
	£.	s.	d.		£.	s.	d.
To Debts,	280	0	0	By Cash, &c.	11505	10	0
				By Trade he } cleared	393	13	1
				Cr.	11899	3	1
				Dr.	280	0	0
				Answ. Bal. £.	11619	3	1

Subtraction.

37

(13) Factor,	Dr.			Per Contra,	Cr.		
	£.	s.	d.		£.	s.	d.
To Tin, -	197	12	0	By Wines,	226	16	6
Bees-wax,	71	7	6	Figs, -	157	11	3
Stockings,	47	3	6	Fruit, -	104	6	0
Tobacco,	943	15	10	Olives, -	136	10	0
Cotton,	123	3	7	Oil, - -	193	17	0
Wheat,	116	5	6	Raisins, -	143	0	4
				Wool, -	75	13	8
				Commission,	71	18	11
Dr. £.	1499	7	11				
Cr.	1109	13	8				
					£.	1109	13 8
Employer } draws for }	£.	389	14 3				

(14)

	Y.	m.	w.	d.	h.	m.
He was to serve	-	-	-	-	-	-
Of which he } accomplished }	11	11	11	11	11	11
	12	1	0	4	11	11
Answer, to serve, Year	1	11	6	2	12	49

(15) A.	Dr.			Per Contra,	Cr.		
	£.	s.	d.		£.	s.	d.
1779.				1779.			
Jan. 21, To Cash,	9	10	0	Jan 11, By Cash,	17	17	0
Feb. 19, Ditto,	5	6	8	Feb. 24, Ditto,	42	0	0
Mar. 17, Ditto,	12	2	6	Mar. 24, Ditto,	11	18	4
19, Ditto,	5	5	0				
				Cr.	71	15	4
£.	32	4	2	Dr.	32	4	2
				Bal. to A.	£.	39	11 2

E

B.

Subtraction.

B.	Dr.	Per Contra,	Cr.
	£. s. d.		£. s. d.
Jan. 24, To Cash,	6 10 0	Jan. 11, By Cash,	34 11 6
Mar. 2, Ditto,	6 6 0	Feb. 12, Ditto,	16 14 0
19, Ditto,	5 5 0	Mar. 14, Ditto,	17 8 8
	<u> </u>	24, Ditto,	11 18 4
	£. 18 1 0		<u> </u>
		Cr. 80 12 6	
		Dr. 18 1 0	
		<u> </u>	
		Bal. to B. £. 62 11 6	
		<u> </u>	

C.	Dr.	Per Contra,	Cr.
	£. s. d.		£. s. d.
1779.		Jan. 11, By Cash,	28 18 10
Jan. 30, To Cash,	19 8 4	21, Ditto,	12 5 0
Mar. 19, Ditto,	5 5 0	Mar. 2, Ditto,	21 0 0
	<u> </u>	24, Ditto,	11 18 4
	£. 24 13 4		<u> </u>
To A. - - -	39 11 2	Cr. 74 2 2	
B. - - -	62 11 6	Dr. 24 13 4	
C. - - -	49 8 10		<u> </u>
	<u> </u>	Bal. to C. £. 49 8 10	
Jointly,	£. 151 11 6		<u> </u>
	<u> </u>		

(16) C. qrs. lb.			C. qrs. lb.
Gross 12 0 19			Gross 8 2 2
Tare 1 3 26			Tare 1 1 15
			<u> </u>
Net { 10 0 21 first			Net. 7 0 15
{ 7 0 15 second }	Venture.		<u> </u>
Answ. C. 3 0 6 Difference.			
<u> </u>			

Subtraction.

39

(17) W.	Dr.	Per Contra,	Cr.
1779.	£. s. d.	1779.	£. s. d.
June 30, To Cash,	66 3 0	June 4, By Cash,	47 18 2
July 5, Ditto,	15 10 9	Note,	200 0 0
12, Ditto,	10 10 0	24, Ditto,	14 12 10
	<u> </u>	July 15, Ditto,	52 0 0
	£. 92 3 9		<u> </u>
		Cr.	314 11 0
		Dr.	92 3 9

B. due to W. £. 222 7 3

X.	Dr.	Per Contra,	Cr.
June 24, To Cash,	47 10 8	June 4, By Bill,	33 14 9
July 7, Ditto,	7 3 1	Cash,	66 5 3
19, Ditto,	38 18 10	July 24, Ditto,	19 19 0
	<u> </u>		<u> </u>
	£. 93 12 7	Cr.	119 19 0
		Dr.	93 12 7

Bal. due to X. £. 26 6 5

Y.	Dr.	Per Contra.	Cr.
June 16, To Cash,	43 12 6	June 4, By Bal.	116 14 10
30, Ditto,	88 13 4	11, Ditto,	120 0 0
July 12, Ditto,	81 19 8	28, Ditto,	18 5 0
	<u> </u>	July 15, Ditto,	42 0 0
	£. 214 5 6		<u> </u>
			296 19 10
			214 5 6

Bal. due to Y. £. 82 14 4

Z.	Dr.	Per Contra,	Cr.
June 20, To Cash,	11 11 0	June 4, By Cash,	70 8 0
July 7, Ditto,	12 8 3	July 15, Ditto,	31 12 4
	<u> </u>	Assignment,	63 4 8
	£. 23 19 3		<u> </u>
		Cr.	165 5 0
		Dr.	23 19 3

Bal. due to Z. £. 141 5 9

Subtraction.

		£.	s.	d.
To W.	-	222	7	3
X.	-	26	6	5
Y.	-	82	14	4
Z.	-	141	5	9

Jointly, £. 472 13 9

	Feet.	Feet.	Years.
(18)	57	127 had at first,	(19) 42
	42	99 let out,	14
	—	—	—
	99	Ans. 28 Feet.	Ans. 28 Years.
	—	—	—

(20)	Grandfire's Age	134 Years.	then	112
	Older than the Son by	93		—41
		—		—
	Son's Age,	41	Father's Age,	71 Y.
		—		—

(21)	When C. was 25 Years old ; then when B. was 25 Years old.	
	B. 14	C. will be + 11
	—	—
	Diff. 11 Years.	Ans. Sum 36 C's Age.
	—	—

(22)	1772	B. born A. D. 1785
	+ 13	A. born A. D. 1693
	—	—
	B. born A. D. 1785	Ans. Diff. . . 92 Years A. older
	—	— [than B.

(23)	In fair Weather,	lb.
	foul,	33905
		30624
		—
	Ans. . 3281 lb. Avoirdupoise.	—

(24)

Subtraction.

41

(24)	Years.	Years.	Years.
Before Christ	200	50	1769
Since -	1769	+ 1769	140
Hip. and Arch. } flourished	1969	Paffidius 1819	and Ptolemy 1629 Y.

(25)	lb.	lb.
Pekin Bell weighs	120000	Nankin Bell, - 50000
From which take	94600	Erford Bell, - 25400
Rem. Erford Bell,	25400	Answ. Diff. 24600

(26)	Years.	Years.
Grandfather's Age,	119	Father's Age is - 65
From which take -	83	From which take - 36
Leaves your Age -	36	Answ. Diff. 29

(27)	Feet.
Goes up the first Day,	8
Comes down at Night,	4
	4 Feet made good the first Day.
Goes up the second Day,	8
	12
Comes down at Night,	4
	8 made good the second Day.
Goes up the third Day,	8
	16
Comes down at Night,	4
	12 made good the third Day.
Goes up the fourth Day,	8
	20 the fourth Day at Night, Answ.
	E 3

(28)

(28) From 1772 then $25 + 24 = 49$ C.'s }
 Take 1747 and $49 + 17 = 66$ A.'s } Age.
 also $66 + 13 = 79$ B.'s }
 Rem. . . 25

(29) Conquest, - - Anno 1066
 To which add $134 + 10 =$ 144
 Edifice finished, - Anno 1210
 The Peace of Utrecht, Anno 1713
 From which deduct - - 70
 Demolition, - - Anno 1643
 From which deduct - 1210
 Answer, Duration, - 433 Years.

	Years.		Years.
(30) Revolution,	Anno 1688	Forfeited in Anno	1551
Forfeited	137	Granted	1239
Forfeited in	Anno 1551	Answ. subsisted	312

(31)

Moses born Anno	2433	Christ born Anno	4000
To which add -	832	From which take	40
Hamer born Anno	3265	Cæsar born Anno	3960
		From which take	312
		Alexander born Anno	3648

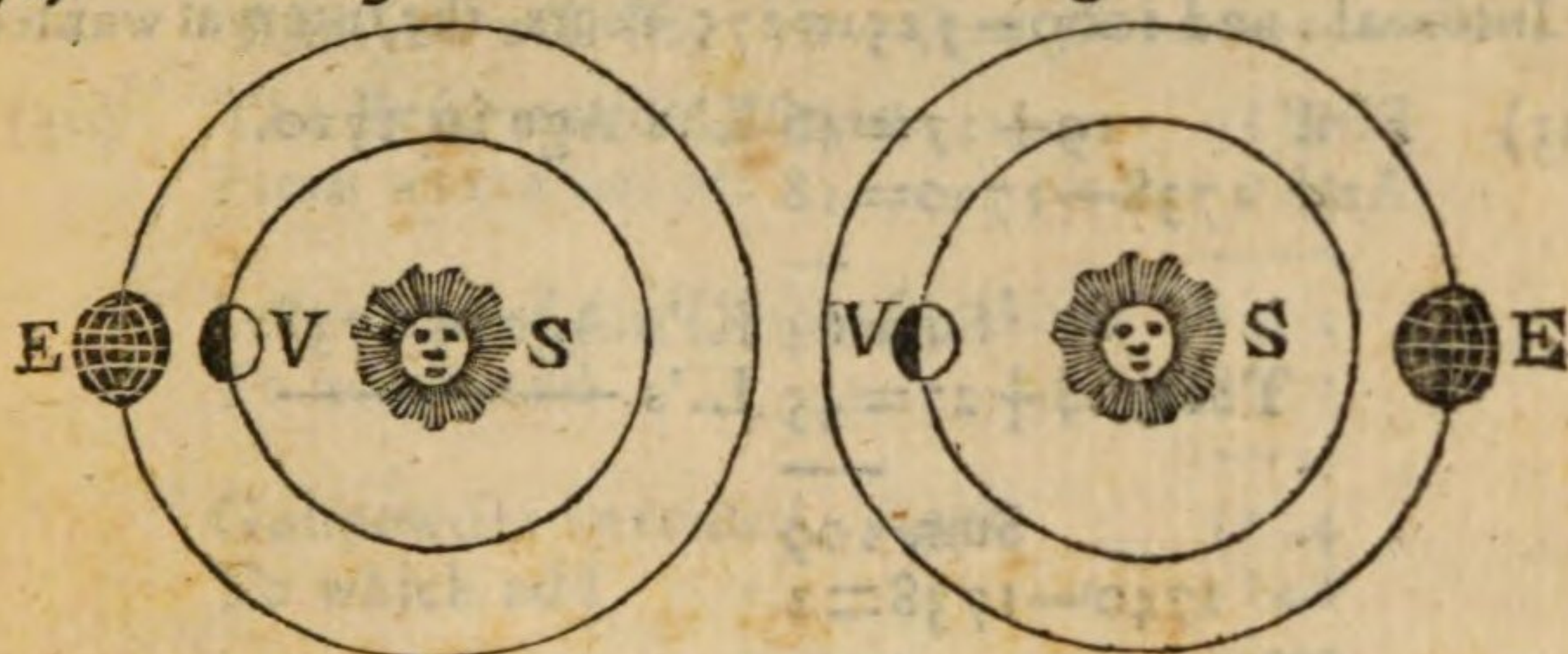
Then $\left. \begin{array}{r} 3960 \\ 4000 \\ 3648 \end{array} \right\} - 3265 = \left\{ \begin{array}{l} 695 \text{ from Homer to Cæsar.} \\ 735 \text{ ————— Christ.} \\ 383 \text{ ————— Alexander.} \end{array} \right.$

Answer 1813 Years, Sum of the Intervals.

(32)

Fig. 1.

Fig. 2.



Suppose S. the Sun ; V. Venus ; and E. the Earth ; then Fig. 1. will represent them when in Perigæo ; and Fig. 2, when they are in Apogæo.

∴ $180000000 - 59000000 = 22000000 = S. E. - S. V. = E. V.$

Fig. 1, or the Distance the Earth is from Venus, when in Perigæo.

And $81000000 + 59000000 = 140000000 = V. S. + S. E. = V. E.$ Fig. 2, or the Distance the Earth is from Venus, when in Apogæo.

Therefore $140000000 - 22000000 = 118000000$ Miles the Answer.

(33)
$$\left. \begin{array}{l} \text{First } 23 + 8 = 31 \text{ A.'s} \\ \text{And } 31 + 19 = 50 \text{ B.'s} \\ \text{Also } 50 - 14 = 36 \text{ C.'s} \end{array} \right\} \text{Age.}$$

Sum - - - - - 117
From which take - - - 22

Ans. 95 D.'s Age.

(34)

When Selah was born	Arphaxad was	—	35 Years old.
Eber	Selah	—	30
Peleg	Eber	—	34
Eber lived after the Birth of Peleg		—	430 Years.

From the Birth of Arphaxad to the Death of Eber was - - - - - } 529

Shem died after the Birth of Arphaxad, 500

Eber was the Survivor by - - - - - 29 Years.

Then

Then per Quest. $2 + 500 + 29 + 194 = 725$ Years, the whole Interval; and $1000 - 725 = 275$ Years, the Interval wanted.

(35) First $19 + 27 = 46$ K's Age in 1720.
And $1738 - 1720 = 18$

Sum 64 K's Age in 1738.
Then $18 + 27 = 45$ L's

Sum 109
 $1740 - 1738 = 2$

Sum $111 - 24 = 87$ Years the Age of M.
[in 1740.]

(36) First $318 + 207 = 525$ B.
Then $525 - 104 = 421$ C.
And $421 - 84 = 337$ D. } Flourished.
Then $337 + 112 = 449$ E.
Also $449 + 47 = 496$ F.

(37) Sam was born before Toby 28 Years.
Toby died at - - 12 ——— old.
After which Sam lived - 19
—
Sam's Age, - - 59 Years.
From which deduct $16 + 11 = 27$
—
Leaves Rachel's Age, - 32 Years.
To which add - $7 + 4 = 11$
—
Gives Joshua's Age - 43 Years.
—

Then $12 + 59 + 32 + 43 = 146$, the Sum of their Ages.

	(38)	Years.
B. born A. D. -	1108	Z. born A. D. - 1527
Lived before C. -	48	After which Y. was born 74
C. born -	1156	Y. born Anno - 1601
Lived before D. -	113	Before wh. X. was born 114
D. born Anno -	1269	X. born Anno - 1487

(39)

(39) I shall be $34 + 17 = 51$ Years of Age.
 And you will be $70 - 34 = 36$

(40) The Reformation, Anno 1517
 From which deduct - - 215

Invention of the Compass, Anno 1302
 To which add - - 42

Gunpowder Invented, Anno 1344
 To which add - - 148

America discovered, Anno - 1492
 To which add - - 77

Painting invented, Anno - 1569

(41) Restoration, Anno - 1660
 Grant made before, - - 33

First Grant made, Anno - 1627
 Duration, - - 210

First Grant ended, Anno - 1837
 Reversionary Grant's Continuance, +99

Its Expiration, Anno - 1936

(42) When B. is 41, A. will be $41 - 18 = 23$ Years old.
 But when A. is 72, B. will be $72 + 18 = 90$ Years old.

(43)

Temple built, Anno 3000 Christ born A. D. 4000
 Troy before, - 443 Rome built before, - 744

Troy built, Anno 2557 A.D. 3256
 London after, - 260 Carthage built before, - 113

London built, Anno 2817 A. D. 3143
 London built, - 2817

London older than Carthage by 326

(44)

(44)

Suppose E. the Earth, M. the Moon, and S. the Sun; then an Eclipse of the Sun will be represented by Fig. 3, and that of the Moon by Fig. 4.

Therefore $81000000 - 240000 = 80760000 = S M.$ Fig. 3, or the Distance these two luminaries are afunder, in an Eclipse of the Sun.

Likewise $81000000 + 240000 = 81240000 = S M.$ Fig. 4, or the Distance these two Luminaries are afunder, in an Eclipse of the Moon.

Fig. 3.

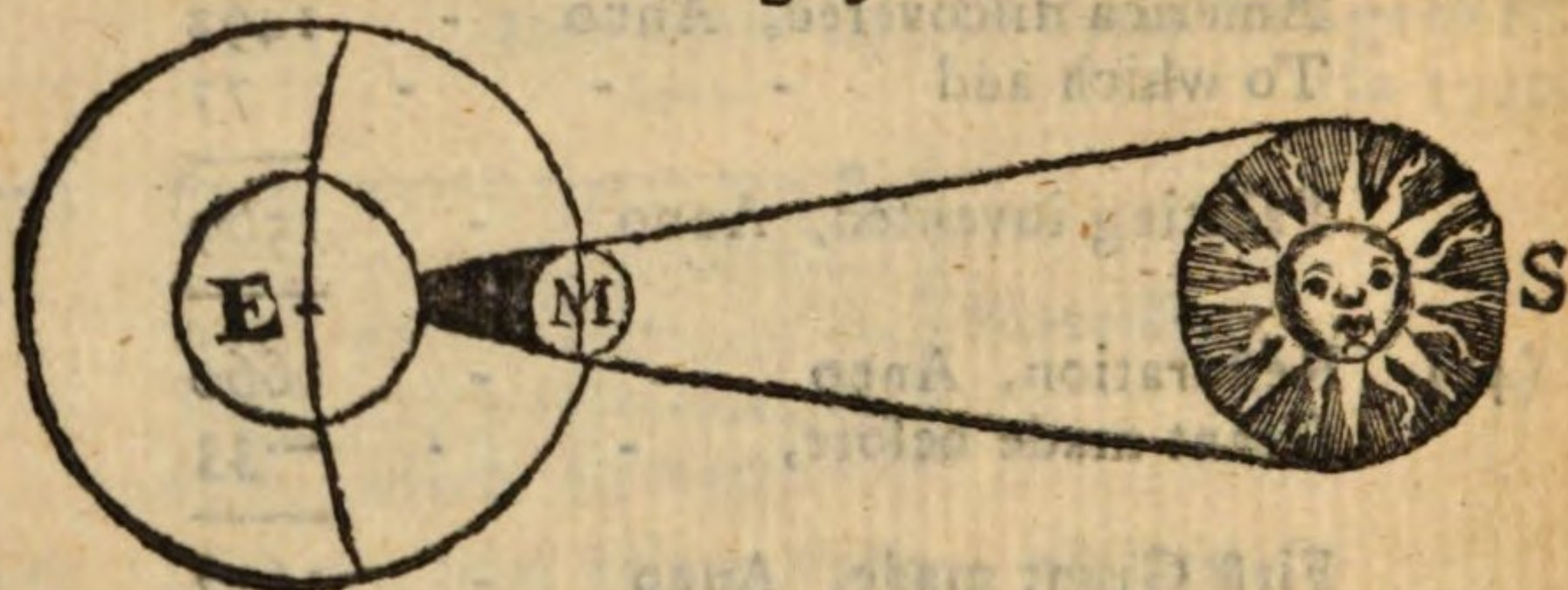
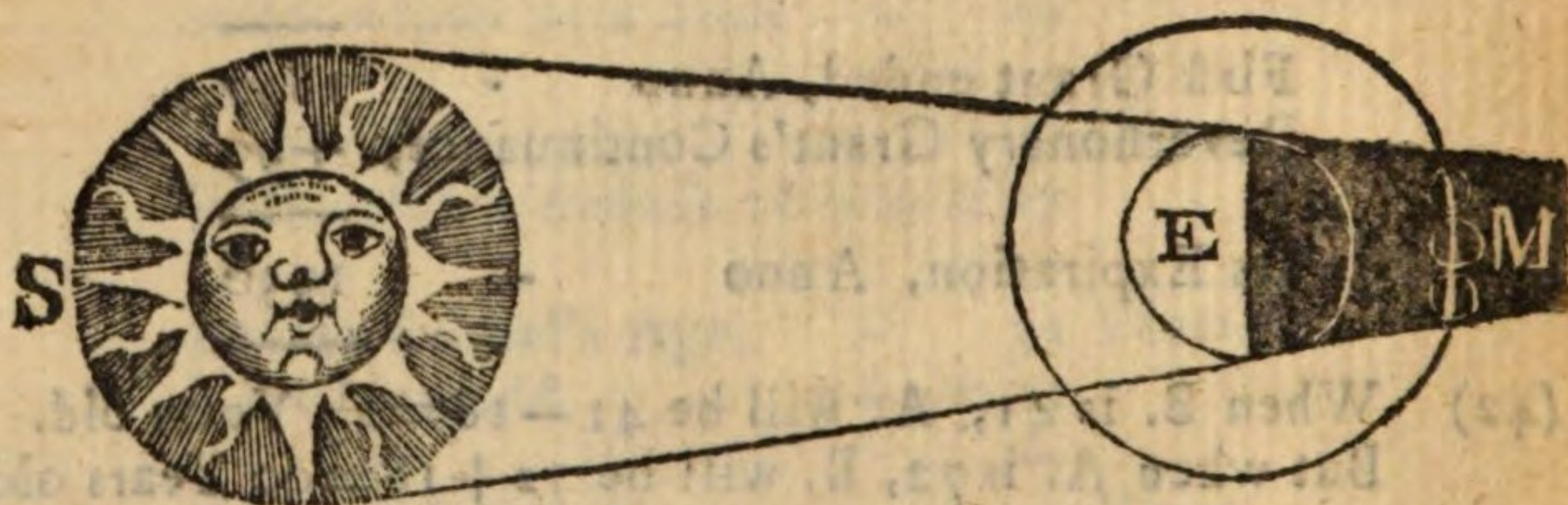


Fig. 4.



(45)	From the Creation to the Flood,	-	-	1656
	thence to the Building of Solomon's Tem.	-	-	1336
	to Mahomet,	-	-	1630
				—
	Together,	-	-	4622
	Mahomet after Christ,	-	-	622
				—
	Answ. Christ born, A. D.			4000
				—

(46)

Subtraction.

47

(46)

When Seth was born	Adam was	—	130 Years old.
Enos	Seth	—	105
Canaan	Enos	—	90
Mahaleel	Canaan	—	70
Jared	Mahaleel	—	65
Enoch	Jared	—	162
Methufelah	Enoch	—	65
Lamech	Methufelah	—	187
Noah	Lamech	—	182
When the Flood happened	Noah was		600 Years.

Year of the Flood, - - - 1656
 To Adam's Death $130 + 800 =$ - 930

After his Death, - - - 726 Years.

(47)

£.

£.

Miss Kitty's Fortune,	20000	And Miss Charlotte	15000
Left by the Father,	13200	By Father,	13200
Left by the Grandmo.	6800	By Grandmother,	£. 1800
	+ 1800		

Ans. Left them, £. 8600

(48)

Reformation was in Anno 1517
 To which add - - - 88

Powder Plot was discovered, 1605
 To which add - - - 43

King Charles murdered, Anno 1648

Then $1714 - 54 = 1660$ King Charles II. returned.
 And $1660 - 1648 = 12$ Years, the Answer.

(49)

First $180 - 47 = 133$ Years since B. died.
 Then $161 - 133 = 28$ Years, B.'s Age.
 And $47 + 43 = 90$ C.'s Age.

Sum, 181 — the Answer.

(50)

(50)

A. D.

First, $1772 - 50 = 1722$.
 And $1722 + 28 = 1750$ Jacob.
 Also, $1750 - 26 = 1724$ Timothy.
 Likewise, $1724 + 17 = 1741$ Sampson.

(51)

First, $1733 - 445 = 1288$ A. born.
 Then, $1362 - 1288 = 74$ A.'s
 And, $37 + 18 = 55$ B.'s } Age.
 Also, $256 - 197 = 59$ C.'s }
 Again, $1733 - 75 = 1658$ D. born.
 Then, $1658 - 1578 = 80$ D.'s Age.

(52)

Reformation, Anno - - 1517
 From which deduct - - 23

A. died, Anno - - 1494
 born - - 1441

53 A.'s Age

Then $1517 + 49 =$ Anno - 1566 B. died.
 And $1494 - 7 =$ - 1487 B. born.

Also, $36 - 7 + 9 =$ - - 79 B.'s }
 Then, $53 + 79 + 20 = 152$ the Answer. } 20 C.'s } Age.

(53)

Anno

A. D.

A. born 1438, lived 48 Years; then $1438 + 48 = 1486$ died.
 B. died 1502, bo. bef. 77 Yrs. — $1502 - 77 = 1425$ born.
 C. in 1577, was 22 Yrs. old, — $1577 - 22 = 1555$ — .
 And in $1577 + 54 = 1631$ died.
 D. died 1648, and in 1616 had lived $\frac{1}{2}$ his Time; then
 $1648 - 1616 = 32$ Half his Age double of which is 64.
 Then 1684, born bef. 64 Yrs. then $1648 - 64 = 1584$ died.
 E. in 1648 was 13 Years old, — $1648 - 13 = 1635$ born.
 And in $1635 + 13 + 14 + 31 + 7 = 1700$ died.
 F. in 1635 born after 27 Yrs. then $1635 + 27 = 1662$ born.
 G. in 1662 — 31 — $1662 + 31 = 1693$ — .

(54)

(54) Anno Anno
 $1727 - 42 = 1685$ A. died. } Aged 47 Years.
 $1685 - 47 = 1638$ — born. }
 $1638 + 13 = 1651$ B. born. } Aged 53 —.
 $1712 - 8 = 1704$ — died. }
 $1638 - 17 = 1621$ C. born. } Aged 91 —.
— — 1712 — died. }
 $1621 - 23 = 1598$ D. born. } Aged 64 —.
 $1598 + 64 = 1662$ — died. }
 $1704 + 11 = 1715$ E. born. } Aged 30. —.
 $1733 + 12 = 1745$ — died. }
 $1638 - 1598 = 40$, Half of which is 20.
 $1638 - 20 = 1618$ F. born. } Aged 113 Years.
 $1745 - 14 = 1731$ — died. }

The Sum of all their Ages is 398 Years.

Then $113 - 91 = 22$ Years F. Survivor.

M U L T I P L I C A T I O N .
I N T E G E R S .

(1) 14276084 4 <u> </u> Prod. <u>57104336</u>	(2) 20749509 9 <u> </u> <u>186745581</u>	(3) 1204674 12 <u> </u> <u>14456088</u>
(4) 4074746 16 <u> </u> Prod. <u>65195936</u>	(5) 147624 69 <u> </u> <u>10186056</u>	(6) 42768 748 <u> </u> <u>31990464</u>
(7) 10646 5278 <u> </u> Prod. <u>56189588</u>	(8) 14276 39674 <u> </u> <u>566386024</u>	(9) 3142708 467852 <u> </u> <u>1470322223216</u>

Multiplication.

(10)	27680709	(11)	2142760	(12)	21700
	40700609		4100		954000
Prod.	1126621713851781		8785316000		2070180000

The above Examples are performed as in Sect. 4.

CONTRACTIONS.

1. When the Multiplier consists of the same Figure in all the Places, that is, all Nines or all Sevens, &c. then for each Figure in the Multiplier annex a Cypher to the Multiplicand, and from that Number subtract the Multiplicand, and if the repeating Figure is 9, the Remainder is the Product; but if any other Figure, multiply it into the 9th Part of the Remainder; or, for the Figure 3, take the 3d Part of the Remainder, and for 6 multiply the 3d Part by 2.

EXAMPLES.

(1) Mul. 47627 by 9999. Thus, 476270000 47627 Prod. 476222373	(2) Mul. 27464 by 11111. Thus, 2746400000 27464 9)2746372536 Prod. 305152504
--	---

(3) Mul. 4674 by 2222. $ \begin{array}{r} 46740000 \\ 4674 \\ \hline 9)46735326 \\ \hline 5192814 \\ 2 \\ \hline \text{Prod. } 10385628 \end{array} $	(4) Mul. 47694 by 7777. $ \begin{array}{r} 476940000 \\ 47694 \\ \hline 9)476892306 \\ \hline 52988034 \\ 7 \\ \hline \text{Prod. } 370916238 \end{array} $
---	---

5. Mul.

Multiplication.

51

(5) Mul. 74769 by 3333.

Thus, 747690000
74769

3)747615231

Prod. 249205077

(6) Mul. 42763 by 6666.

Thus, 427630000
42763

3)427587237

142529079
2

Prod. 285058158

2. In many Cases the Work may be performed as in the following Examples.

(1) Mul. 4276
By 63

12828 × 2 •
25656

Prod. 262388

(2) Mul. 6946
486

41676 × 8 •
333408

Prod. 3375756

COMPOUND MULTIPLICATION.

M O N E Y.

C A S E I.

(1) £. s. d.
29 15 10

(2) £. s. d.
843 0 3

(3) £. s. d.
155 17 0

(2) £. s. d.
3 0 6
5

(3) s. d.
7 10
7

(4) s. d.
18 9½
8

£. 15 2 6

£. 2 14 10

£. 7 10 4

* Because 3 × 2 = 6, and 6 × 8 = 48, Figures of the Multipliers.

$$\begin{array}{r} \text{(5)} \quad \begin{array}{r} \text{£. s. d.} \\ 12 \quad 8 \\ 9 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 5} \quad 14 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(6)} \quad \begin{array}{r} \text{£. s. d.} \\ 2 \quad 6 \quad 4 \\ 10 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 23} \quad 3 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(7)} \quad \begin{array}{r} \text{£. s. d.} \\ 12 \quad 7 \\ 11 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 6} \quad 18 \quad 5 \\ \hline \end{array}$$

C A S E II.

$$\begin{array}{r} \text{(8)} \quad \begin{array}{r} \text{£. s. d.} \\ 1 \quad 17 \quad 6\frac{1}{4} \\ 12 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 17} \quad 10 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(10)} \quad \begin{array}{r} \text{d.} \\ 10\frac{1}{4} \\ 6 \times 3 = 18 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 0} \quad 15 \quad 4\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \text{(11)} \quad \begin{array}{r} \text{£. s. d.} \\ 2 \quad 9 \quad 6 \\ 9 \times 3 = 27 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 66} \quad 16 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(12)} \quad \begin{array}{r} \text{s. d.} \\ 4 \quad 11\frac{1}{2} \\ 6 \times 5 = 30 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \quad 8 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(13)} \quad \begin{array}{r} \text{s. d.} \\ 10 \quad 8 \\ 6 \times 6 = 36 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 19} \quad 4 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(14)} \quad \begin{array}{r} \text{d.} \\ 2\frac{1}{2} \\ 9 \times 5 = 45 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 0} \quad 9 \quad 4\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \text{(15)} \quad \begin{array}{r} \text{£. s. d.} \\ 1 \quad 7 \quad 0 \\ 10 \times 5 = 50 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 67} \quad 10 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(16)} \quad \begin{array}{r} \text{s. d.} \\ 2 \quad 7\frac{1}{2} \\ 8 \times 7 = 56 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \quad 7 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(17)} \quad \begin{array}{r} \text{£. s. d.} \\ 1 \quad 11 \quad 0 \\ 8 \times 8 = 64 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 99} \quad 4 \quad 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(18)} \quad \begin{array}{r} \text{s. d.} \\ 15 \quad 9 \\ 9 \times 8 = 72 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. 56} \quad 14 \quad 0 \\ \hline \end{array}$$

(19) $\begin{array}{r} d. \\ 11\frac{3}{4} \\ 10 \times 8 = 80 \end{array}$

$\begin{array}{r} \text{£. } 3 \ 18 \ 4 \end{array}$

(20) $\begin{array}{r} s. \\ 9 \\ 12 \times 7 = 84 \end{array}$

$\begin{array}{r} \text{£. } 37 \ 16 \end{array}$

(21) $\begin{array}{r} s. \ d. \\ 1 \ 10\frac{3}{4} \\ 12 \times 8 = 96 \end{array}$

$\begin{array}{r} \text{£. } 9 \ 2 \end{array}$

(22) $\begin{array}{r} s. \ d. \\ 18 \ 11\frac{1}{2} \\ 11 \times 9 = 99 \end{array}$

$\begin{array}{r} \text{£. } 93 \ 16 \ 10\frac{1}{2} \end{array}$

(23) $\begin{array}{r} s. \ d. \\ 11 \ 10 \\ 10 \times 10 = 100 \end{array}$

$\begin{array}{r} \text{£. } 59 \ 3 \ 4 \end{array}$

(24) $\begin{array}{r} s. \ d. \\ 2 \ 6 \\ 12 \times 10 = 120 \end{array}$

$\begin{array}{r} \text{£. } 15 \ 0 \ 0 \end{array}$

(25) $\begin{array}{r} s. \ d. \\ 1 \ 10 \\ 12 \times 11 = 132 \end{array}$

$\begin{array}{r} \text{£. } 12 \ 2 \ 0 \end{array}$

(26) $\begin{array}{r} s. \ d. \\ 1 \ 7\frac{3}{4} \\ 12 \times 12 = 144 \end{array}$

$\begin{array}{r} \text{£. } 11 \ 17 \ 0 \end{array}$

CASE III.

(28) $\begin{array}{r} s. \ d. \\ 19 \ 11\frac{1}{4} \\ 6 \times 3 + 1 = 19 \end{array}$

$\begin{array}{r} \text{£. } 18 \ 19 \ 7\frac{1}{4} = 1 + 18 = 19 \end{array}$

(29) $\begin{array}{r} s. \ d. \\ 1 \ 7\frac{1}{2} \\ 7 \times 4 + 1 = 29 \end{array}$

$\begin{array}{r} \text{£. } 2 \ 7 \ 1\frac{1}{2} = 29 \end{array}$

(30) $\begin{array}{r} \text{£. } s. \ d. \\ 2 \ 17 \ 6 \\ 6 \times 6 + 2 = 38 \end{array}$

$\begin{array}{r} \text{£. } 109 \ 5 \ 0 \end{array}$

(31) $\begin{array}{r} s. \ d. \\ 5 \ 10 \\ 9 \times 5 + 2 = 47 \end{array}$

$\begin{array}{r} \text{£. } 13 \ 14 \ 2 \end{array}$

$$(32) \quad \begin{array}{r} s. \quad d. \\ 10 \quad 4\frac{1}{2} \\ \hline 8 \times 7 + 2 = 58 \end{array}$$

$$\underline{\underline{\pounds. \quad 30 \quad 1 \quad 9}}$$

$$(33) \quad \begin{array}{r} \pounds. \quad s. \\ 5 \quad 17 \\ \hline 8 \times 8 + 3 = 67 \end{array}$$

$$\underline{\underline{\pounds. \quad 39 \quad 1 \quad 19}}$$

$$(34) \quad \begin{array}{r} s. \quad d. \\ 6 \quad 4\frac{1}{2} \\ \hline 9 \times 8 + 3 = 75 \end{array}$$

$$\underline{\underline{\pounds. \quad 23 \quad 18 \quad 1\frac{1}{2}}}$$

$$(35) \quad \begin{array}{r} s. \quad d. \\ 19 \quad 11\frac{3}{4} \\ \hline 9 \times 9 + 5 = 86 \end{array}$$

$$\underline{\underline{\pounds. \quad 85 \quad 18 \quad 2\frac{1}{2}}}$$

$$(36) \quad \begin{array}{r} s. \\ 3 \\ \hline 10 \times 10 + 6 = 106 \end{array}$$

$$\underline{\underline{\pounds. \quad 15 \quad 18}}$$

$$\text{or thus,} \quad \begin{array}{r} s. \\ 106 \\ 3 \\ \hline \end{array}$$

$$\underline{\underline{\pounds. \quad 15 \quad 18}}$$

C A S E IV.

$$(38) \quad \begin{array}{r} \pounds. \quad s. \quad d. \\ 3 \quad 0 \quad 6 \\ \hline 7 \times 5 + \frac{1}{2} = 35\frac{1}{2} \end{array}$$

$$\underline{\underline{\pounds. \quad 107 \quad 7 \quad 9}}$$

$$(39) \quad \begin{array}{r} \pounds. \quad s. \quad d. \\ 1 \quad 12 \quad 10 \\ \hline 9 \times 8 + 4\frac{1}{2} + \frac{1}{4} = 76\frac{3}{4} \end{array}$$

$$\underline{\underline{\pounds. \quad 125 \quad 19 \quad 11\frac{1}{2}}}$$

$$(40) \quad \begin{array}{r} \pounds. \quad s. \quad d. \\ 1 \quad 15 \quad 6\frac{1}{2} \\ \hline 4 \times 4 + 1 + \frac{1}{4} = 17\frac{1}{4} \end{array}$$

$$\underline{\underline{\pounds. \quad 31 \quad 10 \quad 4}}$$

$$(41) \quad \begin{array}{r} \pounds. \quad s. \quad d. \\ 4 \quad 6 \quad 7 \\ \hline 8 + \frac{1}{2} = 8\frac{1}{2} \end{array}$$

$$\underline{\underline{\pounds. \quad 36 \quad 15 \quad 11\frac{1}{2}}}$$

(42)

£. s. d.
26 17 6

$$10 \times 10 + \frac{1}{2} = 100\frac{1}{2}$$

£. 2700 18 9

(44)

s. d.

1 10 $\frac{3}{4}$

$$8 \times 4 \times 4 = 112$$

£. 720 0 0

s. d.

(45)

2 5

$$8 \times 7 \times 6 = 336$$

£. 40 12 0

d.

(46)

11 $\frac{3}{4}$

$$10 \times 7 \times 5 = 350$$

£. 17 2 8 $\frac{1}{2}$

WEIGHTS, MEASURES, &c.

Proceed here in the like Manner with the Product of each Denomination, as you did with the Sum of each Denomination in Addition.

lb. oz. dwt. grs.

(1) 14 10 0 21
4

59 4 3 12

T. C. qrs. lb.

(2) 17 17 0 24
2

35 14 1 20

C. qrs. lb. oz. drs.

(3) 14 0 21 0 14
7

99 1 7 6 2

lb. 3 3 6 grs.

(4) 10 6 4 1 17
9

94 11 1 1 13

Yds. qr. na.

(5) 127 0 3
12

1526 1 0

E.E. qr. na.

(6) 40 4 2
11

449 4 2

Lea. m. fu. p.

(7) 120 0 7 24
5

601 1 6 0

(8)

<i>Yds. f. in. b.c.</i>	<i>W.bds. ga. pts.</i>	<i>Tu. p. bds. g. qt.</i>
(8) 147 2 11 2 6	(9) 46 47 7 3	(10) 6 1 1 46 3 8
887 2 10 0	140 17 5	55 0 1 59 0

<i>Tier. g. qts.</i>	<i>B.bds. g. pts.</i>	<i>A.bds. gal. q. p.</i>
(11) 27 41 2 6	(12) 4 47 6 9	(13) 10 17 3 1 4
167 39 0	43 51 6	41 23 2 0

<i>B.b. ft. g. p.</i>	<i>A. r. p.</i>	<i>La. qr. bu. p.</i>
(14) 12 2 7 7 6	(15) 140 2 29 5	(16) 74 7 4 1 7
76 1 2 2	703 1 25	523 2 5 3

<i>D. b. m. sec.</i>
(17) 365 5 48 57 12
4382 21 47 24

QUESTIONS.

(1) First $54 \times 54 = 2916$
And $46 \times 19 = 874$

Ans. 2042

(2) £.
142
50

An. £. 7100

(3) $12 \times 12 \times 6 = 864 = \text{Six Dozen Dozen.}$
 $12 \times 6 = 72 = \text{Half a Dozen Dozen.}$

Answer, $\begin{cases} 792 \text{ Difference.} \\ 936 \text{ Sum.} \end{cases}$

(4)

(4)

$$\begin{array}{r} 52 \\ 7 \times 6 = 42 \\ \hline \end{array}$$

$$\begin{array}{r} 364 \\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 2184 \text{ Parishes.} \\ 246 \\ \hline \end{array}$$

$$\begin{array}{r} 13104 \\ 52416 \\ \hline \end{array}$$

$$\begin{array}{r} 537264 \text{ Houses.} \\ 10 \\ \hline \end{array}$$

$$\begin{array}{r} 5372640 \text{ Persons.} \\ \hline \end{array}$$

(5)

$$\text{First } 28 \times 2 = 56$$

$$\text{And } 8 \times 2 + 20 = 36$$

$$\text{Answ. Diff. } 20$$

$$\text{Again } 55 \times 2 = 110$$

$$\text{And } 5 \times 2 + 50 = 60$$

$$\text{Answ. } 50 \text{ Diff.}$$

(6)	Years.	£.	£.
The first	4 he cleared	364	per Ann. = 364 × 4 = 1456
The next	3	586	= 586 × 3 = 1758
And the last	3	873	= 873 × 3 = 2619

$$\text{His whole Gain, } \underline{\underline{£. 5833}}$$

Then $13000 - 5833 = 7167$ £. his original Stock.

To find the State of his Fortune at each Year's End, thus: He begun with 7167 £. cleared the first Year 364 £. which, added to 7167 £. = 7531 £. his Worth at the first Year's End; then $7531 + 364 = 7895$ £. at the second Year's End; and in this Manner proceed by a continual adding the preceding Year's Gain.

(7)	20 Remainder.	423 Quotient.
	423 Quotient.	× 462 Divisor.
	19 +	
	462 Divisor.	195426 Prod.
		+ 20 Rem.

$$\text{Ans. } \underline{\underline{195446 \text{ Dividend.}}}$$

(8)

Multiplication.

59

(13)

£.	s.	d.
100	0	0
1	7	0
0	10	6

In each Division was 101 17 6
× 6

In each Drawer was 611 5 0
× 12 0

Answ. £. 7335 0 0

(14) s. d.
19 11 spends daily.

$$9 \times 8 \times 5 + 5 = 365$$

£. 363 9 7 spends per Annum.
500 0 0 Income.

Answ. lays up, £. 136 10 5

(15) £. s. d.
1 12 6 spends daily.

$$9 \times 8 \times 5 + 5 = 365$$

593 2 6 spends.
294 12 6 lays up.

Answ. £. 887 15 0 per Annum.

(16)

Multiplication.

(16)

	<i>Yards.</i>
D.	73726 at one Circumvolution.
365 in the Y.	3
63 played.	<u>221178 in a Minute.</u>
Days 302 worked.	60
	<u>13270680 in an Hour.</u>
	10
	<u>132706800 in Day.</u>
	302 Days worked.
	<u>2654136</u>
	<u>3981204</u>
	<u>Answ. 40077453600 Yards.</u>

(17) First $20 \times 60 = 1200$ Feet, Piers stand on.
 And $21 \times 170 = 3570$ ——— Arches Span.

Width of the Danube,	4770
Thames,	<u>1200</u>

Answ. 3570 Feet the Difference.

(18) $2 \times 3 \times 4 \times 5 \times 6 = 720$, then $720 + 1 = 721$ the Answer,
 or any Term of the Arithmetical Progression, 301, 721,
 1141, &c. See page 358 and 359.

(19) 187 the lesser Number.
 $187 + 34 = 221$ the great Number.
 $221 \times 187 = 41327$ their Product.
 $41327 \times 41327 = 1707920929$ Square of their Products.
 $187 + 221 = 408$ their Sum, and $408 \times 408 = 166464$ Square
 of their Sum.
 $221 - 187 = 34$ their Difference, and $34 \times 34 = 1156$ Square
 of their Difference.
 And, lastly, $1707920929 + 166464 + 1156 = 1708088549$
 the Sum of those Squares.

(20) $109 \times 73 = 7957$ the greater Number.
 $28 \times 17 = 476$ Difference.

7481 lesser Number.

Then $7957 + 7481 = 15438$ their Sum,
 And $7957 \times 7481 = 59526317$ their Product.

Acres.

(21) A. had 757
 B. 2104
 C. 16410
 D. 12881
 E. 11008
 F. 9813
 H. 13800
 I. 8818
 Wanted, 416

(22) One comes up 6 Ways.
 6

Two comes up 36
 6

Three comes up 216
 6

Four comes up 1296 the Anf.

Sum, $76007 = \frac{1}{5}$ of the Whole.
 5

Answer, 380035 Acres.

(23) £. s. d.
 100 G. = 105 0 0
 Ten Marks = 6 13 4

W. had 98 6 8
 — 0 16 8

R. 97 10 0
 — 6 6 0

S. 91 4 0
 + 3 17 2

T. 95 1 2

£. s. d.
 98 6 8

- 97 10 0

- 91 4 0

- 95 1 2

Answ. £. 382 1 10

D I V I S I O N.

I N T E G E R S.

$$(1) \quad 4 \overline{)14076893} \\ \underline{3519223} - 1$$

$$(2) \quad 12 \overline{)30742165} \\ \underline{2561847} - 1$$

$$84 \left\{ \begin{array}{l} (3) \quad 7 \overline{)2410296} \\ (12) \quad 344328 \\ \underline{28694} \end{array} \right.$$

$$(4) \quad \begin{array}{ll} \text{Quot.} & \text{Rem.} \\ 576 \overline{)98420649} & (170869 - 105 \end{array}$$

$$(5) \quad 3029 \overline{)308763705} (101935 - 2590$$

$$\begin{array}{ll} \text{Quot.} & \text{Rem.} \\ (6) \quad 46058 \overline{)16221212499} & (352190 - 45479 \end{array}$$

$$(7) \quad 127345 \overline{)51799555} (406 - 97485$$

$$(8) \quad 3090807 \overline{)78855994985} (25513 - 235994$$

$$(9) \quad 37,00 \overline{)12764214,27} (344978 - 2827.$$

$$(10) \quad 827,000 \overline{)4074964,478} (4927 - 335478$$

$$(11) \quad 94,00 \overline{)247698514,00} (2635090 \frac{5}{9} \frac{4}{4}.$$

C O N T R A C T I O N S.

When the Divisor consists of the same Figure in all the Places, that is, all Nines, or all Sevens, &c. annex as many Cyphers to Unity or 1, as there are 9's or 7's, &c. in the given Divisor, for a new Divisor, and if the repeating Figure is 9, divide the Dividend by that Divisor, and multiply the integral Quotient by the Difference; do the same with the Product, and so proceed till you get 0 for an integral Quotient; then add all the Overplusses together, and divide that Sum by the given Divisor, the Overplus thence arising is that required, and the Sum of all the integral Quotients is the Quotient required; for any other Figure divide 9 Times the Dividend so, and the integral Quotient by the repeating Figure, this gives the true integral Quotient; and if the 9th Part of the first Overplus be added to the second, repeated as the given Figure, the Sum will be the true Overplus.

EXAMPLES.

E X A M P L E S.

(1) Divide 4677823 by 999.

$$\begin{array}{r} 1,000 \overline{) 4677} \quad 823 \\ \underline{4} \quad 677 \\ \underline{} \quad 4 \\ \hline 4681 \overline{) 1504} \end{array}$$

Then 1504 the Sum of the Overplusses, divided by 999 the Divisor gives 1, and 505 remains.

So $4681 + 1\frac{505}{999} = 4682\frac{505}{999}$, the Quotient required.

(2) Divide 2692464 by 1111.

First, 2692464

$$\begin{array}{r} \times 9 \\ \hline 2423 \overline{) 2176} \\ \underline{2423} \\ \hline 2423 \overline{) 4599} \end{array}$$

Then $4599 \div 9 = 511$, the Remainder
So $2423 + 511\frac{1}{11}$ is the Quotient required.

(3) Divide 4769042 by 7777.

First, 4769042

$$\begin{array}{r} \times 9 \\ \hline r \overline{) 1000} \overline{) 4292} \quad 1378 \\ \underline{4292} \\ \hline 7 \overline{) 4292} \quad 5670 \\ \underline{5670} \\ 631-1 \end{array}$$

Then $5670 \div 9 = 630$, and $630 + 1111 = 1741$ the Remainder; so the Quotient is $613\frac{1741}{7777}$.

C O M P O U N D D I V I S I O N.

M O N E Y.

C A S E I.

Here, in the 3d Example, I divide by 8, saying the 8's in 17 is 2 Times and 1 over, the 8's in 11 is 1 and 3 over, which is 3£. this 3£. I carry to the 11s. and it is 3£. 11s.

G 2

or

or 71s.; then I say the 8's in 71 is 8 Times 8 is 64, and 7s. over, which I carry to the 4 Pence, and it is 7s. 4d. or 88 Pence; then I say the 8's in 88 is 11 Times, and Nothing over, and so the Quotient is £. 21 8s. 11d.; in the same Manner proceed with all the Rest.

$$(1) \begin{array}{r} \text{£. s. d.} \\ \text{Qu. } 7 \quad 8 \quad 5\frac{1}{4} \\ \hline \end{array}$$

$$(2) \begin{array}{r} \text{£. s. d.} \\ 38 \quad 2 \quad 10\frac{1}{2} \\ \hline \end{array}$$

$$(3) \begin{array}{r} \text{£. s. d.} \\ 21 \quad 8 \quad 11 \\ \hline \end{array}$$

$$(4) \begin{array}{r} \text{£. s. d.} \\ \text{Quot. } 35 \quad 4 \quad 6 \\ \hline \end{array}$$

$$(5) \begin{array}{r} \text{£. s. d.} \\ 0 \quad 11 \quad 6\frac{1}{2} \\ \hline \end{array}$$

$$(7) \begin{array}{r} \text{£. s. d.} \\ 3)7 \quad 11 \quad 6 \\ \hline 2 \quad 10 \quad 6 \\ \hline \end{array}$$

$$(8) \begin{array}{r} \text{£. s. d.} \\ 10)3 \quad 17 \quad 1 \\ \hline \text{Quot. £. } 0 \quad 7 \quad 8\frac{1}{2} \\ \hline \end{array}$$

$$(9) \begin{array}{r} \text{£. s. d.} \\ 12)14 \quad 14 \quad 0 \\ \hline \text{£. } 1 \quad 4 \quad 6 \\ \hline \end{array}$$

C A S E II.

$$(2) \begin{array}{r} \text{£. s. d.} \\ 24 \left\{ \begin{array}{l} 4)3 \quad 13 \\ \hline 6)0 \quad 18 \quad 3 \\ \hline \end{array} \right. \\ \text{£. } 0 \quad 3 \quad 0\frac{1}{2} \\ \hline \end{array}$$

$$(3) \begin{array}{r} \text{£. s. d.} \\ 36 \left\{ \begin{array}{l} 6)64 \quad 19 \\ \hline 6)10 \quad 16 \quad 6 \\ \hline \end{array} \right. \\ \text{£. } 1 \quad 16 \quad 1 \\ \hline \end{array}$$

$$(4) \begin{array}{r} \text{£. s. d.} \\ 42 \left\{ \begin{array}{l} 6)190 \quad 4 \quad 6 \\ \hline 7)31 \quad 14 \quad 1 \\ \hline \end{array} \right. \\ \text{£. } 4 \quad 10 \quad 7 \\ \hline \end{array}$$

$$(5) \begin{array}{r} \text{£. s. d.} \\ 48 \left\{ \begin{array}{l} 6)37 \quad 14 \quad 8 \\ \hline 8)6 \quad 5 \quad 9\frac{1}{4} \\ \hline \end{array} \right. \\ \text{£. } 0 \quad 15 \quad 8\frac{1}{2} \\ \hline \end{array}$$

$$(6) \begin{array}{r} \text{£. s. d.} \\ 32 \left\{ \begin{array}{l} 4)78 \quad 16 \quad 8 \\ \hline 8)19 \quad 14 \quad 2 \\ \hline \end{array} \right. \\ \text{£. } 2 \quad 9 \quad 3\frac{1}{2} \\ \hline \end{array}$$

$$(7) \begin{array}{r} \text{£. s. d.} \\ 55 \left\{ \begin{array}{l} 5)4567 \quad 0 \quad 10 \\ \hline 11)913 \quad 8 \quad 2 \\ \hline \end{array} \right. \\ \text{£. } 83 \quad 0 \quad 8\frac{1}{4} \\ \hline \end{array}$$

$$(8) \begin{array}{r} \text{£. s.} \\ 63 \left\{ \begin{array}{l} 7)264 \quad 12 \\ \hline 9)37 \quad 16 \\ \hline \end{array} \right. \\ \text{£. } 4 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. s. d.} \\ 72 \left\{ \begin{array}{l} 8)18 \quad 18 \\ \hline 9)2 \quad 7 \quad 3 \\ \hline \end{array} \right. \\ \text{£. } 0 \quad 5 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r}
 (10) \quad \text{£. s. d.} \\
 81 \left\{ \begin{array}{l} 9) 121 \quad 12 \quad 6 \\ 9) 13 \quad 10 \quad 3\frac{1}{4} \end{array} \right. \\
 \hline
 1 \quad 10 \quad 0
 \end{array}$$

$$\begin{array}{r}
 (11) \quad \text{£. s. d.} \\
 120 \left\{ \begin{array}{l} 10) 174 \quad 1 \quad 8 \\ 12) 17 \quad 8 \quad 2 \end{array} \right. \\
 \hline
 1 \quad 9 \quad 0\frac{2}{12}
 \end{array}$$

C A S E III.

$$\begin{array}{r}
 (13) \quad \text{£. s. d.} \\
 26) 83 \quad 17) 3 \quad 4 \quad 6 \\
 78 \\
 \hline
 5 \\
 20
 \end{array}$$

$$\begin{array}{r}
 26) 117 (4s. \\
 104 \\
 \hline
 13 \\
 12
 \end{array}$$

$$\begin{array}{r}
 26) 156 (6d. \\
 156 \\
 \hline
 \dots
 \end{array}$$

Answ. £. 3 4 6

$$\begin{array}{r}
 (14) \quad \text{£. £. s. d.} \\
 145) 467 (3 \quad 4 \quad 4\frac{1}{4} \text{ Answer.} \\
 435 \\
 \hline
 \text{Rem. } 32\text{£.} \\
 20
 \end{array}$$

$$\begin{array}{r}
 145) 640 (4s. \\
 580 \\
 \hline
 \text{Rem. } 60d. \\
 12
 \end{array}$$

$$\begin{array}{r}
 145) 720 (4d. \\
 580 \\
 \hline
 \text{Rem. } 140d. \\
 4
 \end{array}$$

$$\begin{array}{r}
 145) 560 (\frac{3}{4} \\
 435 \\
 \hline
 \text{Rem. } 125 \text{ qrs.}
 \end{array}$$

$$\begin{array}{r}
 (15) \quad \text{£. s.} \\
 112 \left\{ \begin{array}{l} 4) 30 \quad 2 \\ 4) 7 \quad 10 \quad 6 \\ 7) 1 \quad 17 \quad 7\frac{1}{2} \end{array} \right. \\
 \hline
 \text{£. } 0 \quad 5 \quad 4\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 (16) \quad \text{£.} \\
 350 \left\{ \begin{array}{l} 5) 1009 \\ 7) 201 \quad 16 \\ 10) 28 \quad 16 \quad 6\frac{3}{4} \end{array} \right. \\
 \hline
 \text{£. } 2 \quad 17 \quad 7\frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 (17) \quad \text{£. s.} \\
 \left\{ \begin{array}{l} 4) 610 \quad 8 \\ 7) 152 \quad 12 \\ 8) 21 \quad 16 \end{array} \right. \\
 \hline
 \text{£. } 2 \quad 14 \quad 6
 \end{array}$$

$$\begin{array}{r}
 (18) \quad \text{£.} \\
 \left\{ \begin{array}{l} 8) 1426 \\ 8) 178 \quad 5 \\ 10) 22 \quad 7 \quad 7 \end{array} \right. \\
 \hline
 \text{£. } 2 \quad 4 \quad 6\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 (20) \quad \text{£. s. d.} \\
 \left\{ \begin{array}{l} 5) 12 \quad 12 \quad 11 \\ 5) 2 \quad 10 \quad 7 \end{array} \right. \\
 \hline
 0 \quad 10 \quad 1\frac{3}{4} \text{ per Qr.} \\
 4 \\
 \hline
 \text{£. } 2 \quad 0 \quad 5
 \end{array}$$

$$\begin{array}{r}
 (21) \quad \text{£. s. d.} \\
 \left\{ \begin{array}{l} 3) 1061 \quad 8 \quad 9\frac{1}{4} \\ 7) 353 \quad 16 \quad 3\frac{1}{4} \end{array} \right. \\
 \hline
 50 \quad 10 \quad 10\frac{3}{4} \\
 2 \\
 \hline
 \text{£. } 101 \quad 1 \quad 9\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 (22) \quad \text{£. s. d.} \\
 \left\{ \begin{array}{l} 5) 450 \quad 13 \quad 11\frac{1}{2} \\ 7) 90 \quad 3 \quad 9 \end{array} \right. \\
 \hline
 12 \quad 17 \quad 6\frac{1}{2} \\
 4 \\
 \hline
 \text{£. } 51 \quad 10 \quad 2
 \end{array}$$

WEIGHTS, MEASURES, &c.

lb. oz. dwt. grs.

$$(1) \quad 2) 8 \quad 1 \quad 5 \quad 8$$

$$\text{Quot. } 4 \quad 0 \quad 12 \quad 16$$

C. qr. lb. oz. drs.

$$3) 4) 17 \quad 2 \quad 27 \quad 14 \quad 15$$

$$\text{Quot. } 4 \quad 1 \quad 20 \quad 15 \quad 11\frac{3}{4}$$

Yds. qr. na.

$$(5) \quad 9) 214 \quad 3 \quad 2$$

$$\text{Quot. } 23 \quad 3 \quad 2$$

E.E. qr.

$$(6) \quad 8) 120 \quad 4$$

$$15 \quad 0 \quad 0$$

T. C. qrs. lb.

$$(2) \quad 3) 24 \quad 14 \quad 0 \quad 14$$

$$8 \quad 4 \quad 2 \quad 23\frac{1}{2}$$

lb. 3 3 6 grs.

$$(4) \quad 5) 4 \quad 11 \quad 4 \quad 2 \quad 12$$

$$0 \quad 11 \quad 7 \quad 1 \quad 2\frac{2}{5}$$

Lea. m. fu. p.

$$(7) \quad 9) 12 \quad 2 \quad 0 \quad 26$$

$$1 \quad 1 \quad 1 \quad 3$$

<i>Yds. f. in. b. c.</i>	<i>W. bds. ga.</i>	<i>Tu. p. bd. g. qt.</i>
(8) 10) 147 2 11 2	(9) 11) 24 57	(10) 8) 10 1 1 60 3
<u>14 2 4 2 $\frac{3}{10}$</u>	<u>2 16 5 $\frac{1}{11}$</u>	<u>1 0 1 13 1 $\frac{3}{8}$</u>

<i>Tier. g. pts.</i>	<i>A. bds. gal.</i>	<i>B. bds. g. qts.</i>
(11) 6) 16 20 7	(12) 5) 76 27	(13) 4) 12 49 2
<u>Quot. 2 31 5 $\frac{5}{6}$</u>	<u>15 15</u>	<u>3 12 1 $\frac{1}{2}$</u>

<i>B. b. fi. g.</i>	<i>A. r. p.</i>	<i>La. qr. bu. p.</i>
(14) 3) 61 2 6	(15) 12) 140 2 26	(16) 7) 60 6 7 2
<u>Quot. 20 2 2</u>	<u>11 2 35 $\frac{1}{2}$</u>	<u>8 6 5 2 $\frac{4}{7}$</u>

D. b. m. sec.

(17) 6) 146 23 24 56

Quot. 24 11 54 9 $\frac{1}{3}$

QUESTIONS.

(1) $\begin{array}{r} \text{£.} \\ 1,0000 \overline{) 22,0000} \end{array}$

Ans. 22 £. each.

(2) $\begin{array}{r} \text{M.} \\ 12 \overline{) 336} \end{array}$

Ans. 28 Miles per Day.

(3) First $4429 \div 43 = 103$, then $240 - 103 = 137$ the Ans.

(4) First $2262 \div 26 = 87$, and $2262 \div 87 = 26$.

Then $87 - 26 = 61$ the Number required.

(5) $5190048 \div 72084 = 72$ the Number required.

(6) First $419844 \div 3 = 139948$ the Remainder.

And $419844 \times 9494 = 3985998936$ Pro. of Div. and Quot.

Then $3985998936 + 139948 = 3986138884$ the Answer.

(7) First $360 - 144 = 216$ the greater Number.

And $216 - 144 = 72$ their Difference.

Also $216 \times 144 = 31104$ Product.

Likewise $216 \div 144 = 1 \frac{72}{144}$ or $1 \frac{1}{2}$ the larger Quote.

(8)

(8) First 3 Tons $\times 20 = 60$ Cwt. then $60 \div 15 = 4$ Cwt. per Man, the Answer.

$$(9) \text{ First } \begin{array}{r} 5 \overline{) 22525} \\ \underline{ 4505} \end{array} \text{ and } 16 \begin{array}{r} 4 \overline{) 9696} \\ \underline{ 2424} \end{array}$$

$$25\text{th Part} = \underline{901}$$

$$16\text{th Part} = \underline{606}$$

Then $1440 + 901 = 2341$, and $2341 - 606 = 1735$ the Answ.

(10) First $134 \times 71 = 9514$, consequently $9514 \div 57 = 166 \frac{2}{3}$ the Answer.

(11) Fourscore and thirteen Millions $= 93000000$.
Then $93000000 \div 30074 = 3091$, and the Remainder is 25811, from which deduct 21180 leaves 4631 excess. the Answer.

(12)
2,0)1640 Guineas.

+ 82

£. 1722

+ 4

£. 6888

- 1383

2)5505

+ 2752 10

£. 8257 10

2)12393 } o Sum of the
Mo. & Sift. }

£. 6196 10

£. s.
1722 0 College.

6888 0 Wife.

5505 0 Daughter.

8257 10 Eldest Son.

6196 10 Youngest Son.

988 10 Expences on his Burial.

£. 29557 10 died worth.

Then from £. 30000 take £. 29557 10 leaves £. 442 10s. the Answer.

(13)

£.	s.	
8)12	8	Worth of both.
1	7	Value of the Purse.

Answ. £. 11 1 Cash in the Purse.

(14)

	£.	s.	d.	
Brother owed at first,	74	18	2	
Paid in Part, - -	-41	14	8	
Remains on Balance,	2)33	3	6	£. s. d. 33 3 6 Bro.
Half of which is	+16	11	9	
Sister owed at first,	49	15	3	
Paid in Part, -	-13	12	10	
Remains on Balance,	36	2	5	36 2 5 Sist.
To which add the Brother's,	33	3	6	
Uncle William owed,	69	5	11	
Paid in Part, -	-24	7	3	
Rem. on B. unpaid by him,	44	18	8	44 18 8 Uncle.
All together owed, - -	£.	114	4 7	

Then from 150 £. take £. 114 4 7 leaves £. 35 15 5

(15	C. qrs. lb.		£.	s.	d.
Weight together,	9 3 16	Cost,	97	17	6
Difference,	-1 2 16	Diff.	8	13	3
	2)8 1 0 Rem.		2)89	4	3
Lesser Weight,	4 0 14	Cost, £.	44	12	1½
Difference,	+1 2 16	Diff.	8	13	3
Greoter Weight,	5 3 2	Cost, £.	53	5	4½

(16)

Division.

(16)		£.		£.
In Cash,	-	10000	Burial,	30
Bills, £.	54 10 6 × 8 = 436 4		Debts,	260
Left in all,	£. 10436 4		£. 290	
From which take	290 0 Rem.			

Daughter had,	9) 10146 4 0	
	£. 1127 7 1 $\frac{1}{4}$ $\frac{1}{3}$	
	7) 9018 16 10 $\frac{1}{2}$ $\frac{2}{3}$ Rem.	
Sons had each,	£. 1288 8 1 $\frac{1}{2}$ $\frac{2}{7}$	

(17) $14 \left\{ \begin{array}{l} 2) 2072 \\ 7) 1036 \end{array} \right.$

148 Elms in each Row, then $148 - 1 = 147$
 Vacancies, and $147 \times 25 = 3675$ Feet, which $\div 3 = 1225$
 Yards, the Length of the Grove required.

(18)	(19)	First	$129 + 178 = 307$	
$32 \left\{ \begin{array}{l} 4) 384 \\ 8) 96 \end{array} \right.$	Then	$1000 - 307 = 693$		
	And	$693 \div 3 = 231$ B's		} Crowns
	Also	$231 + 129 = 360$ A's		
	Likewise	$231 + 178 = 409$ C's		

Ans. in Rank 12 Men.

(20) First $37 - 28 = 9$, then $250 - 9 = 241$.
 $\therefore 3) 241$

gives $80\frac{1}{3}$ B.'s }
 And $80 + 37 = 117\frac{1}{3}$ A.'s } £.
 Also $80 - 28 = 52\frac{1}{3}$ C.'s }

(21) $207 + 213 + 189 + 234 + 222 + 250 = 1315$ the Sum,
 which being divided by 5, the Times each being men-
 tioned, viz. $5) 1315$

Quotient, 263
 To which add Four-score and 13 = 93

Sum, 356
 From which deduct 1

Leaves, Answer, 355

(22)

£.

First $10000 - 850 = 1500$ O.'s
 And $8500 - 6050 = 2450$ L.'s
 Then $6050 - 420 = 5630$, and $5630 \div 2 = 2815$ M.'s
 Consequently $2815 + 420 = 3235$ N.'s

} Subcrip-
tion.

(23) $46 \div 1610(35$ the lesser N^o. then $46 + 35 = 81$ their Sum, and $46 - 35 = 11$ their Difference; also $46 \div 35 = 1\frac{1}{5}$ their Quotient.

Then $46 \times 46 = 2116$ the Square of greater Number.

And $35 \times 35 = 1225$ ———— lesser.

3341 Sum of those Squares.

Again, $11 \times 11 \times 11 = 1331$ the Cube of their Difference.

(24)

First $7050 \div 94 = 75$ the lesser Number.

And $7050 \times 7050 = 49702500$ the Square of the greater.

Also $75 \times 75 = 5625$ Ditto of the lesser.

49696875 Difference of those Squares.

Again $7050 + 75 = 7125$ their Sum, and $7050 - 75 = 6975$ the Difference; then $7125 \times 6975 = 49696875$ the Product of their Sum, and Difference.

Consequently $49696875 \times 49696875 = 2469779384765625$ the Square of their Product, of their Sum and Difference.

(25) Suppose the Expence or Profit to be 2.

Then $2 \times 2 = 4$ Double the Expence.

And $2 - 2 = 1$ Half the Profit.

Answer, Difference 3, or as 4 to 1.

(26) First B. is to have 72
 And C. $72 + 112 = 184$ } more than A.

Sum, 256

Then

Division.

Then $1500 - 256 = 1244$.

$$\therefore \quad 3 \overline{) 1244}$$

gives $414\frac{2}{3}$ A.'s }
 And $414\frac{2}{3} + 72 = 486\frac{2}{3}$ B.'s } Share:
 Also $486\frac{2}{3} + 112 = 598\frac{2}{3}$ C.'s }

(27) $25000 + 33000 + 30000 + 28000 + 32000 = 148000$
 their Sum; then each being repeated four Times, that
 Sum must be divided by 4, viz.

$$4 \overline{) 148000}$$

Which gives 37000 £. Sum of their Fortune, then

From	Take	Rem.
£. 37000	{	$25000 = 12000$ £. the Youngest's Fortune.
		$33000 = 4000$ ——— Eldest's.
		$30000 = 7000$ ——— Second's.
		$28000 = 9000$ ——— Fourth's.
		$32000 = 5000$ Miss Charlotte's Fortune.

Proof 37000 £.

(28) $d. \quad s. \quad d.$
 First 120 at 2 for 1d. $= 120 \div 2 = 60$ or 5 0

And 120 at 3 ——— $= 120 \div 3 = 40$ — 3 4

8 4 Cost.

Then 240 at 5 for 2d. $= 240 \div 5 = 48$ } 8 0 Sold for.
 Two-pences, or ———

Difference 0 4 Lost.

(29) $£. \quad s. \quad d.$
 A. and B. had 13 10 0
 B. and C. 12 12 0
 A. and C. 11 16 6

Sum, £. 37 18 6

Which being divided by the Number of Players at each
 Time will give the Sum won, viz.

£.

viz. £. 37 18 6 ÷ 6 = 18 19 3 what was won.

Then from £. s. d. Take { £. s. d. Rem. { £. s. d. C.'s }
 18 19 3 { 13 10 0 } Rem. { 5 9 3 A.'s } Gain.
 { 12 12 0 } { 6 7 3 B.'s }
 { 11 16 6 }

(30) W. X. and Y. advanced £. s.
 W. X. and Z. - 350 10
 X. Y. and Z. - 344 10
 W. Y. and Z. - 400 0
 W. Y. and Z. - 378 4

They being mentioned each } 3) 1473 4 Sum.
 3 Times, - -

Sold for 450 Guineas, or £. 491 1 4 joint Property.
 472 10 0

Answer, Lost £. 18 11 4

(31) Worth at the End of 3½ Years, £. s. d.
 3179 11 8
 4) 100 £. = 25 0 0

¾ Year = 13 Quarters, 13) 3204 11 8
 — 246 10 1½

Worth at the End of 3 Years £. 2958 — 6½
 + 100 0 0

4) 3058 — 6½
 + 764 10 4½

Worth at the end of 2 Years, £. 2293 11 2
 + 100 0 0

4) 2393 11 2
 — 598 7 9½

Worth at 1 Year's End, £. 1795 3 4½
 + 100 0 0

H Carried over, 1895 3 4½

Brought over 4) 1395 3 4 $\frac{1}{2}$
— 473 15 10

Answer, He began with, £. 1421 7 6½

(32) £. £.

First, $50 \div 5 = 2$ each Man's Share, supposing they had attended equally.

And $2\text{ } \pounds . \div 7 = 5\text{ } s . 8\frac{4}{7}\text{ } d .$ each Man's daily Pay.

Then $5s. 8\frac{4}{7}d. \times 2 = 11s. 5\frac{1}{7}d.$ what C. and D. must each forfeit.

$\therefore 11s. 5\frac{1}{7}d. \times 2 = \pounds.1 \ 2s. 10\frac{2}{7}d.$ C and D.'s whole Defaults.

And £. 1 2s. 10 $\frac{2}{7}$ d. $\div 3 = 7$ s. 7 $\frac{3}{7}$ d. what A. B. and E. each received of C. and D.'s Default.

Also $5s. 8\frac{4}{7}d. \div 4 = 1s. 5\frac{1}{7}d.$ what A. B. C. and D. each received of E.'s Default.

	£.	s.	d.	
Therefore, 2£. + 7s. 7 $\frac{3}{4}$ d. + 1s. 5 $\frac{1}{4}$ d. =	2	9	0 $\frac{4}{7}$	A.
Ditto, - - - - -	2	9	0 $\frac{4}{7}$	B.
2£. + 1s. 5 $\frac{1}{4}$ d. less 11s. 5 $\frac{1}{4}$ d. =	1	10	0	C.
Ditto, - - - - -	1	10	0	D.
Also, 2£. + 7s. 7 $\frac{3}{4}$ d. less 5s. 8 $\frac{6}{7}$ d. =	2	1	10 $\frac{4}{7}$	E.

} Received.

(33) First $12 - 1 = 11$, and $11 \times 2 = 22$ she had before she met the last Boy; then $22 - 2 + 10 = 30$ she had before she met the second Boy; consequently $30 - 10 = 20$ what she had before the first returned her back 10; so that 20 being multiplied by 2 = 40, the Number of Apples she had at first.

S E C T. II. R E D U C T I O N.

M O N E Y.

	£.	s.	d.
(3)	80	15	11 $\frac{3}{4}$
	20		

1615
12

1939I
4

77567 Farthings.

(4) $\begin{array}{r} \text{qrs.} \\ 4 \overline{) 16921} \\ \hline 12 \overline{) 4230\frac{1}{4}} \\ \hline 2,0 \overline{) 35,2 \quad 6} \\ \hline \text{£.} \quad 17 \quad 12 \quad 6\frac{1}{4} \end{array}$

L.

$$(5) \begin{array}{r} \text{£. s. d.} \\ 110 \text{ } 0 \text{ } 6\frac{1}{2} \\ 20 \\ \hline \end{array}$$

2200

12

26406

2

52813 Half-pence.

$$(6) \quad 2)20553 \text{ Half-pence.}$$

12)10276 $\frac{1}{2}$

2,0)85,6—4

£. 42 16 4 $\frac{1}{2}$

$$(7) \begin{array}{r} \text{£. s. d.} \\ 107 \text{ } 10 \text{ } 8 \\ 20 \\ \hline \end{array}$$

2150

6 = Two-pences = 1s.

12904 Two-pences.

$$(8) \quad 6)5348 \text{ Two-pences.}$$

2,0)89,1 2 over = 4d.

£. 44 11 4

$$(9) \begin{array}{r} \text{£. s.} \\ 6 \text{ } 17 \\ 20 \\ \hline \end{array}$$

137

4 Three-pences = 1s.

548 Three-pences.

$$(10) \quad 4)2782 \text{ Three-pences.}$$

2,0)69,5—2 or 6d.

£. 34 15 6

$$(11) \begin{array}{r} \text{£. s. d.} \\ 10 \text{ } 10 \text{ } 8 \\ 20 \\ \hline \end{array}$$

210

3

632 Four-pences.

$$(12) \quad 3)3859 \text{ Four-pences.}$$

2,0)128,6 4

£. 64 6 4

$$\begin{array}{r}
 \text{(13)} \quad \begin{array}{r} \text{£.} \quad \text{s.} \\ 200 \quad 17 \\ 20 \\ \hline 4017 \\ 2 \\ \hline 8034 \end{array} \text{Six-pences.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(14)} \quad 2)705 \text{ Six-pences.} \\
 \hline
 2,0)35,2-6 \\
 \hline
 \text{£.} \quad 17 \quad 12 \quad 6 \\
 \hline
 \end{array}$$

Any Number of Pounds, &c may be reduced to Six-pences, Four-pences, Three-pences, or Two-pences, by multiplying the Pounds by as many of each Denomination that make one Pound; that is, to reduce Pounds, &c. to Six-pences, multiply by 40, if to Four-pences by 60, if to Three-pences by 80, and if to Two-pences by 120, observing to add in the odd Shillings and Pence, if any; accordingly, on the contrary, any Number of Six-pences, Four-pences, &c. may be reduced to Pounds, by dividing by as many as make one Pound, observing to value the Remainder (if any) right; thus, the two last Examples.

$$\begin{array}{r}
 \text{(13)} \quad \begin{array}{r} \text{£.} \quad \text{s.} \\ 200 \quad 17 \\ 40 \div 17 \text{ s.} = 34 \text{ Six-pences.} \\ \hline 8034 \end{array} \text{Six pences.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(14)} \quad \begin{array}{r} \text{Six-pences.} \quad \text{s.} \quad \text{d.} \\ 4,0)705 \text{ and } 25 \text{ over} = 12 \quad 6 \\ \hline \text{£.} \quad 17 \quad 12 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(15)} \quad \begin{array}{r} 21 \\ 7 \times 3 = 21 \\ \hline 147 \\ 3 \\ \hline 441 \text{ Shillings.} \\ 12 \\ \hline 5292 \text{ Pence.} \\ 4 \\ \hline 21168 \text{ Farthings.} \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(16)} \quad \begin{array}{r} 4)24192 \\ \hline 12)6048 \\ \hline 21 \left\{ \begin{array}{l} 3)504 \\ \hline 7)168 \\ \hline \end{array} \right. \\ \hline 24 \text{ Guinea.} \\ \hline \end{array}
 \end{array}$$

(17) 12 Moidores.

27

324

12

3888

4

15552 Farthings.

(18) 12)3240

27)270

10 Moidores.

(19) 30 £.

4

120 Crowns.

2

240 Half-Crowns.

30

7200 Pence.

(21) 25 Crowns.

5

125 Shillings.

3

375 Groats.

4

1500 Pence.

(20) 3,0)2016,0 Pence.

2)672 Half-Crowns.4)336 Crowns.

84 £.

(22) 4)25200 Pence.

3)6300 Groats.5)2100 Shillings.

420 Crowns.

(23) 25 £.

20

5)500 Shillings.

100 Crowns.

60

6000 Pence.

(24) 80 Crowns.

5

2,0)40,0 Shillings.

20 £.

	<i>d.</i>
(25) A Crown	=60
Half dit.	=30
Shilling	=12
One of each	102

<i>£.</i>	<i>s.</i>	<i>d.</i>
213	15	6
20		
4275		
12		

102) 51306 (503 of each.
510

306
306

...

<i>£.</i>	
(26) 120	
8	
2) 960	Half Crowns.
480	Crowns.
15	
3) 7200	Groats.
2400	Shillings.

	<i>s.</i>	<i>d.</i>
(27) A Crown	=5	0
Half dit.	=2	6
Shilling	=1	0
Groat	=0	4
Sum, 1 of each,	=8	10
	6 × 6 =	36

2 13 —
6

£. 15 18 —

(29) Guin.	
240	
7 × 3 = 21	
1680	
3	
5) 5040	Shillings.
1008	Crowns.
252	£.

	<i>£.</i>	<i>s.</i>
(30) 21 Guineas	=22	1
A Crown	=0	5
A Moidore	=1	7
In each Purse	23	13
7 × 3 = 21	165	11
		3
Answer, £.	496	13

Reduction.

79

(33)
First 17s. 6d. = 35 Six-pences.
And 1£. = 40 ditto.

Pist.

Then 470
 $7 \times 5 = 35$

—

3290

5

—

4,0)1645,0

Ans. £. 411 5s.

—

(34) [ces
First 10s. 6d. = 21 Six-pen-
Then 270 Half-Guin.

$7 \times 3 = 21$

—

1890

3

—

4,0)567,0 Six-pences.

£. 141 15s.

—

Or thus: 10s. 6d. $\times 10 \times 9 \times 3(270)$
will give the Answer.

(35)
First 6s. 4d. = 19 Four-pences.
Ducatoons.

Then 427

19

—

3)8113 Four-pences.

—

2,0)27,0 4s. 4d.

Answer, £. 135 4 4

—

(36) [pences.
First 5s. 3d. = 21 Three-
Qr. Guin.

Then 2740

$7 \times 3 = 21$

—

19180

3

—

4)57540 Three-pen-

—

2,0)1438,5

Ans. £. 719 5s.

—

(38)
First 3s. 2d. = 19 Two-pences.

£. s. d.

387 18 4

20

—

7758

6

—

19)46550

2450 Florins.

—

(39) [pences.
First 13s. 4d. = 40 Three-

£. s. d.

Then 496 18 4

20

—

9938

3

—

4,0)2981,5 Three-pences.

745 Mar. 5s.

—

(40)

(40)
First 4s. 3d. = 17 Three-pences.

£. s. d.
262 8 9

20

5248

4

17)20995

1235 Dollars, Answer.

(43) Moidores.

1240

9 × 3 = 27

11160

3

21 { 3)33480 Shillings.
7)11160

Answ. 1594 Guin. 6s.

(41) [pences.
First 18s. 6d. = 37 Six-

£.

Then 247

40

37)9880

267 Pist. and 6d. over.

(44)

First 5s. 6½d. = 133 Half-pence.

And 4s. 7d. = 110 ditto.

Cobs.

Then 476

110

133)52360

393 Duc. & 3s. 9½d.

WEIGHTS, MEASURES, &c.

(1) lb.

14

12

168 Ounces.

20

3360

6 × 4 = 24

20160

4

80640 Grains.

(2)

24 { 4)138240
6)34560

2,0)576,0

12)288

24 lb.

(3)

(2) *lb. oz. dwt. grs.*

19 10 17 22

12

238

20

4777

$6 \times 4 = 24$

28662

4

114670 Grains.

(5) *lb. oz. dwt. grs.*

4 6 0 22

12

54

20

1080

$6 \times 4 = 24$

6480

4

25942 grs. = 1 Ingot.

4

A. 103768

(4) *grs.*

24 { 4) 74342

{ 6) 18585 - 2

2,0) 309,7 - 3

14 grs.

12) 154 - 17 dwts.

Ans. *lb.* 12 10 17 14

(6) *lb. oz. dwt.*

6 11 14

12

83

20

1647

$6 \times 4 = 24$

10044

4

40176) 251056

6 Ingots.

(7)

One Dozen of Dishes, each Weight

Ditto ——— Plates, ———

One of each, - - -

lb. oz. dwt. grs.

2 1 15 0

1 3 15 22

3 5 10 22

12

Answer, *lb.* 41 6 11 0

(8)

(8)	oz. dwt. grs.	oz. dwt. grs.
Bowls, each	24 4 0	455 1 16
Tankards, -	11 14 0	20
Tea Pots, -	10 10 0	—
Lamps, -	20 17 21	9101
Pl. per Dozen,	127 11 0	$6 \times 4 = 24$
Spoons ditto,	36 17 23	—
	—	54606
One of each,	231 14 20	4
	20	—
	—	218440 grs.
	4634	—
	$6 \times 4 = 24$	
	—	
	27804	
	4	
	—	
	111236)218440(1 of each.	

Rem. 107204 grs. = 223 oz. 6 dwts. 20 grs.

Answer 1 of each Sort, and 223 oz. 6 dwts. 20 grs. over.

(9) lb. 3 3 9 grs.

4 10 4 1 12

12

—

58

8

468

3

1405

20

28112 Grains.

(10) grs.

2,0)5993,4

3)2996—14 grs.

8)998—29.

12)124—63

lb. 10 4 6 2 14

$$\begin{array}{r}
 (11) \quad 6 \text{ Tons.} \\
 \underline{20} \\
 120 \text{ Cwts.} \\
 \underline{4} \\
 480 \text{ Qrs.} \\
 7 \times 4 = 28 \\
 \underline{3360} \\
 \underline{4} \\
 13440 \text{ lb.}
 \end{array}$$

$$\begin{array}{r}
 (12) \quad \left\{ \begin{array}{l} 4) 26880 \text{ lb.} \\ \hline 7) 6720 \\ \hline 4) 960 \\ \hline 2,0) 24,0 \end{array} \right. \\
 \hline \\
 12 \text{ Tons.}
 \end{array}$$

$$\begin{array}{r}
 (13) \quad \begin{array}{cccc} \text{C.} & \text{qrs.} & \text{lb.} & \text{oz. drs.} \end{array} \\
 74 & 2 & 16 & 0 \quad 7 \\
 \underline{4} & & & \\
 298 & & & \\
 7 \times 4 = 28 & & & \\
 \hline
 2086 & & & \\
 \underline{4} & & & \\
 8360 & & & \\
 \underline{16} & & & \\
 133760 & & & \\
 \underline{16} & & & \\
 2140167 \text{ drs.}
 \end{array}$$

$$\begin{array}{r}
 (14) \quad \begin{array}{l} \text{oz.} \\ \left\{ \begin{array}{l} 4) 29768 \\ \hline 4) 7442 \end{array} \right\} 8 \text{ oz.} \\ \left\{ \begin{array}{l} 4) 1860 - 2 \\ \hline 7) 465 \\ \hline 4) 66 - 3 \end{array} \right\} 12 \text{ lb.} \end{array} \\
 \hline \\
 \text{C. } 16 \quad 2 \quad 12 \quad 8
 \end{array}$$

$$\begin{array}{r}
 (15) \quad \begin{array}{ccc} \text{lb.} & \text{oz.} & \text{drs.} \end{array} \\
 67 & 12 & 15 \\
 \underline{16} & & \\
 1084 & & \\
 \underline{16} & & \\
 17359 \text{ drs.}
 \end{array}$$

$$\begin{array}{r}
 (16) \quad \begin{array}{ccc} \text{C.} & \text{qrs.} & \text{lb.} \end{array} \\
 6 & 3 & 27 \\
 \underline{4} & & \\
 27 & & \\
 7 \times 4 = 28 & & \\
 \hline
 189 & & \\
 \underline{4} & & \\
 783 \text{ lbs. in 1 Hhd.} \\
 \underline{6}
 \end{array}$$

Answer 4693 lb.

(17)

84

Reduction.

(17) C. qrs.

11 2

4

46

 $7 \times 4 = 28$

322

4

lb. Hds.

1288)12880(10 Answ.

(19) C. qrs. lb.

12 3 12

4

51

 $7 \times 4 = 28$

357

4

12)1440

Anf. 120 Canisters.

(18)

C.

A Fother = $19\frac{1}{2}$ $\times 2$

39

507

2

1014(26 Fother.

234

...

(20) lb.

126 $\frac{1}{2}$

2

253

C. qrs.

8 3

4

35

 $7 \times 4 = 28$

245

4

980 lb.

2

253)1960(7 Parcels, and

Remains 189 half-lbs. = 3 qrs. 10 $\frac{1}{2}$ lb. over.(21) $6 + 8 + 12 + 16 = 42$ lb.

C. qrs. lb.

4 3 24

4

19

 $7 \times 4 = 28$

133

4

42 { 6)556

7) 92-4

31-1

10 lb.

Answ. 12 of each, and 10 lb. over.

(22)

lb.

24 great.

3

2)72

36 lb. common.

(23) lb.

120

2

3)240

80 lb. great.

(24) Yds. qrs.
 27 3
 4
 —
 111
 4
 —
 444 Nails.

(25) Nails.
 4)352
 —
 4)88
 —
 22 Yards.

(26) E. Eng. qrs. na.
 30 4 3
 5
 —
 154
 4
 —
 619 Nails.

(27)
 4)569
 —
 3)142—I
 —
 E. Fl. 47 1 1
 —

(28) Yards.
 24
 4
 —
 96
 4
 —
 384 Na. = 1 Piece.
 14
 —
 Anf. 5376 Nails.

(9) Nails.
 4)24768
 —
 4)6192
 —
 12)1548
 —
 Anf. 129 Pieces.

(30) E. Fl.
 20 each.
 3
 —
 60 qrs. in 1 Piece.
 12
 —
 5)720
 —
 144 Ells English.

(31) Yards.
 24 227
 3 4
 —
 72 { 8)908
 — { —
 9)113—4 } 44 Qrs.
 — { —
 12—5 }
 —
 Anf. 12 Ps. 11 Yards.
 I (32

(32) 24 E. Fl.

3

72 Qrs. in 1 Ps.

12

864 Qrs. in 12 Pieces.

4

5)3456 Qrs. in 4 Bales.

E. Eng. 691 1 Qr.

(34) Poles.

4,0)1280,0

8)320

Answ. 40 Miles.

(36) B. C.

3)2280960

12)760320

3)63360

176,0)2112,0(12 Miles.

• 352

• • •

(43) 60 Miles.

8

480 Furlongs.

40

19200 Poles.

(35) Miles.

16

1760 Yards in 1 Mile.

28160 Yards.

3
84480 Feet.

12

1013760 Inches.

3

3041280 Barley Corns.

(37) Miles.

276

1760

1656

4992

485760 Yards.

6 × 6 = 36.

2914560

6

17487360 Inches.

3

52462080 B. C.

(38) Feet.

First $18\frac{1}{2} = 37$ Half-Ft.
Miles.
$$\begin{array}{r} 197 \\ 1760 \end{array}$$

$$\begin{array}{r} 1182 \\ 3349 \end{array}$$

$$\begin{array}{r} 346720 \text{ Yards.} \\ 6 \end{array}$$

$$\begin{array}{r} 2080320 \text{ Half Ft.} \\ \text{which divided by } 37 = \\ 56224\frac{32}{37} \text{ Times, the An-} \\ \text{swer.} \end{array}$$

(39) M. fur.

$$\begin{array}{r} 69 \ 4 \\ 8 \end{array}$$

$$\begin{array}{r} 556 \\ 11 \times 20 = 220 \text{ Yards} \end{array} \quad [=1 \text{ Fur.}]$$

$$\begin{array}{r} 6116 \\ 20 \end{array}$$

$$\begin{array}{r} 122320 \text{ Yards in } 1^\circ \\ 6 \times 6 = 36 \end{array}$$

$$\begin{array}{r} 733920 \\ 6 \end{array}$$

$$\begin{array}{r} 4403520 \text{ Inches in } 1^\circ \\ 3 \end{array}$$

$$\begin{array}{r} 13210560 \text{ Bar. C. in } 1^\circ \\ 6 \times 60 = 360^\circ \end{array}$$

$$\begin{array}{r} 79263360 \\ 60 \end{array}$$

$$\text{Answ. } 4755801600 \text{ Barley-Corns.}$$

(40) A.

$$\begin{array}{r} 64 \\ 4 \end{array}$$

$$\begin{array}{r} 256 \text{ Roods.} \\ 4^\circ \end{array}$$

$$\begin{array}{r} 10240 \text{ Poles.} \end{array}$$

(41) P.

$$\begin{array}{r} 4,0)2176,0 \end{array}$$

$$\begin{array}{r} 4)544 \end{array}$$

$$\text{Anf. } 136 \text{ Acres.}$$

$$\begin{array}{r}
 (42) \quad A. \\
 774 \\
 \underline{4} \\
 3096 \\
 \underline{4,0} \\
 270 \left\{ \begin{array}{l} 3,0) 12384,0(\\ \underline{} \\ 9) 4128 \\ \underline{} \end{array} \right.
 \end{array}$$

Answer, 458 and 180 Poles
over.

$$\begin{array}{r}
 (44) \quad Tierce. \\
 12 \\
 \underline{42} \\
 504 \text{ Gal.} \\
 \underline{8} \\
 4032 \text{ Pts.}
 \end{array}$$

$$\begin{array}{r}
 (46) \quad Hhd. gal. qts. \\
 4 \quad 42 \quad 2 \\
 \underline{63} \\
 294 \\
 \underline{4} \\
 \text{Ans. } 1178 \text{ Qts.}
 \end{array}$$

$$\begin{array}{r}
 (48) \quad Gal. \\
 \text{A Pipe} = 126 \\
 \underline{4} \\
 12) 504 \\
 \underline{} \\
 \text{Ans. } 42 \text{ Doz.}
 \end{array}$$

$$\begin{array}{r}
 (43) \quad A. \quad r. \\
 \text{Rents } 200 \quad 0 \\
 \text{Tills } 96 \quad 2 \\
 \underline{} \\
 \text{Rem. } 103 \quad 2 \\
 \underline{4} \\
 414 \\
 \underline{40} \\
 \text{Ans. } 16560 \text{ Perches.}
 \end{array}$$

$$\begin{array}{r}
 (45) \quad Pts. \\
 8) 6048 \\
 \underline{} \\
 42 \left\{ \begin{array}{l} 6) 756 \\ \underline{} \\ 7) 126 \\ \underline{} \end{array} \right. \\
 18 \text{ Tierce.}
 \end{array}$$

$$\begin{array}{r}
 (47) \quad Pts. \\
 8) 5746 \\
 \underline{} \\
 63 \left\{ \begin{array}{l} 7) 718 - 2 \text{ Pts.} \\ \underline{} \\ 9) 102 - 4 \\ \underline{} \\ 11 - 3 \end{array} \right\} 25 \text{ Gal.} \\
 \text{Answ. } 11 \text{ hhds. } 25 \text{ gal. } 1. \text{ qt.}
 \end{array}$$

$$\begin{array}{r}
 (49) \quad \text{Tun} = 252 \text{ Gal.} \\
 \text{A Quart} = 4 \text{ Half-Pts.} \quad 4 \\
 \text{A Pint} = 2 \quad \underline{} \\
 \text{Half P.} = 1 \quad \underline{} \quad 1008 \text{ Qts.} \\
 \underline{} \\
 7 \\
 7) 4032 \\
 \underline{}
 \end{array}$$

Ans. 576 of each.
(50)

(50) Gal.
 A Pipe = 126
 A Punch = 84
 A Hogsh. = 63
 A Tierce = 42

Sum, $\frac{315}{\text{---}} \frac{1890}{\text{---}}$ (6 of each.

(51) A. Bar.
 12
 32

 384 Gal.
 8

 3072 Pints.

(52)
 8) 1704

 32) 213 (6 Bar. 21 Gal.

 Rem. 21 Gal.

(53) B. Bar. fir. gal.
 6 2 7
 4

 26
 9

 Answ. 241 Gal.

(54) A. hds. gal. pts.
 10 42 4
 48

 522
 8

 4180 Pints.

(55) Qts.
 4) 2017

 48 { 6) 504 — 1 Qt.
 { 8) 84

 Hhds. 10 24 1

(56) Hhds.
 12
 3

 2) 36 Half Barrels.

 Answ. 18 Barrels.

(57) Bar.
 18
 2

 3) 36 Half Barrels.

 Answ. 12 Hhds.

(58)	T. b. bds. gal.	(59)	qrs.
Gal.	4 1 1 49	24	
A Hhd = 54	2	8	
A Bar. = 36	—	—	
A Fir. = 9	9	192 Bush.	
—	2	4	
Sum, 99	—	—	
—	19	768 Pecks.	
—	54	2	
—	—	—	
99) 1075 (10 of each, and 85		1536	
Gal. over.		—	
Rem. 85 Gal.			

(60) Gal.
 2) 3360
 —
 4) 1680
 —
 8) 420
 —
 Answ. 52 qrs. 4 bu.

(62) Cha.
 40
 36
 —
 1440 Bush.
 4
 —

Answ. 5760 Pecks

(64) Cha. bu.
 47 30
 6 × 6 = 36 -
 —
 282
 6
 —
 3) 1722
 —

Answ. 574 Sacks.

(61) La. qrs. bu.
 42 4 7
 10
 —
 424 Qrs.
 8
 —
 3399 Bushels.

(63) Cha.
 36 { 6) 4762
 —
 6) 793—4 } 10 Pks.
 —
 132—1
 —

Cha. 132 10

(65) Sacks.
 12) 6450
 —

Answ. 537 Cha. 6 Sacks.

(66) *D. H.*

$$\begin{array}{r} 365 \quad 6 \\ 6 \times 4 = 24 \\ \hline 2190 \\ 4 \\ \hline 8766 \\ 60 \\ \hline \end{array}$$

Anfw. 525960 Minutes.

(67) *Sec.*

$$\begin{array}{r} 6,0) 207360,0 \\ \hline 6,0) 3456,0 \\ \hline 24 \left\{ \begin{array}{l} 4) 576 \\ \hline 6) 144 \\ \hline \end{array} \right. \\ \text{Anfw. } 24 \text{ Days.} \end{array}$$

(68) *D. h. m. sec.*

$$\begin{array}{r} 27 \quad 7 \quad 43 \quad 5 \\ 6 \times 4 = 24 \\ \hline 162 \\ 4 \\ \hline 655 \\ 60 \\ \hline 39343 \\ 60 \\ \hline \end{array}$$

Anf. 2360585 Seconds.

(69) *D. h. m. sec. '''*

$$\begin{array}{r} 365 \quad 5 \quad 48 \quad 57 \quad 39 \\ 6 \times 4 = 24 \\ \hline 2190 \\ 4 \\ \hline 8765 \\ 60 \\ \hline 525948 \\ 60 \\ \hline 31556937 \\ 60 \\ \hline \end{array}$$

Answer, 1893416259 Thirds.

(70)
$$\begin{array}{r} 6,0) 3155760,0 \\ \hline 6,0) 52596,0 \\ \hline 24 \left\{ \begin{array}{l} 4) 8766 \\ \hline 6) 2191-2 \\ \hline 365-1 \\ \hline \end{array} \right\} 6 \text{ Hours.} \end{array}$$

Anfw. 365 D. 6 H.

(71)

Reduction.

(71)

D. b.

365 6

 $6 \times 4 = 24$

2190

4

8766 Hours in 1 Year.

1772 Years since.

17532

61362

149022
$$24 \left\{ \begin{array}{l} 4) 15533352 \text{ Hours in 1772 Years.} \\ 6) 3883338 \end{array} \right.$$

Answ. 647223 Days since.

(27) Years.

London built 1108 before the birth of our Saviour:

1772 since

In all,

2880*Hours.*

Then 365 d. 6 h. = 8766 by last Ex.

2880

70128

70128

17532
$$24 \left\{ \begin{array}{l} 4) 25246080 \\ 6) 6311520 \end{array} \right.$$

Answer, 1051920 Days.

12. THE RULE OF THREE DIRECT.

(2) *lb. d. lb.*
 St. thus — If 2 : 18½ :: 24

$$\begin{array}{r} 4 \\ \hline 74 \\ 6 \times 4 = 24 \\ \hline 444 \\ 4 \\ \hline 2)1776 \\ \hline 4)888 \\ \hline 12)222 \\ \hline \end{array}$$

 Answ. 18s. 6d.

lb. s. d. lb.
 If 24 : 18 6 :: 2

$$\begin{array}{r} 12 \\ \hline 222 \\ 2 \\ \hline \end{array}$$

$$24 \left\{ \begin{array}{l} 4)444 \\ \hline 6)111 \end{array} \right.$$

 Proof, 18½ d.

Or whenever the first and second Terms consist of any Numbers that are in the Multiplication Table; then the most expeditious Way will be to multiply and divide as in Sect. 9. and 10.—Thus the above Example will be performed as follows.

lb. s. d. lb.
 If 2 : 1 6½ :: 24

$$\begin{array}{r} 6 \times 4 = 24 \\ \hline 9 \quad 3 \\ 4 \\ \hline 2)1 \quad 17 \quad 0 \\ \hline \end{array}$$

 Anf. £. 0 18 6

lb. s. d. lb.
 If 24 : 18 6 :: 2

$$\begin{array}{r} 2 \\ \hline 24 \left\{ \begin{array}{l} 4)1 \quad 17 \quad 0 \\ \hline 6)0 \quad 9 \quad 3 \\ \hline \end{array} \right. \end{array}$$

 Proof, 1s. 6½ d.

But in teaching, I would advise the Tutor to follow the general Rule, in which to make his Pupil quite perfect; after which the Pupil will, with more Ease, both to himself and Master, learn the Abbreviations.

lb. d. doz. lb.

(3) Stated thus—If $4 : 30 :: 6$ or 72
 Then $72 \times 30 = 2160$, this $\div 4 = 540$ Pence, which $\div$ by 12
 and 20 , will give $\text{£. } 2 \text{ } 5\text{s.}$ the Answer.

C. s. C.

(4) Stated thus—If $1 : 26 :: 40$

40

 $2,0)104,0$

 Answ. 52 £.

oz. d. C. lb.

(5) Sated thus—If $1 : 7\frac{1}{2} :: 1$ or 112
 Then by Reduction $7\frac{1}{2}d. = 15$ Half-pence, and $112lb. = 1792$
 oz. consequently $1792 \times 15 = 26880$ Half-pence, which
 divide by 2 , 12 , and 20 , will give 56 £. the Answer.

oz. s. d. oz.

(6) Stated thus—If $1 : 5 \text{ } 4 :: 36$
 Then by Reduction $5\text{s. } 4d. = 16$ Four-pences; these $\times$ by 36
 (the Third Term) $= 576$ Four-pences, which $\div$ by 3 ,
 and 20 , will give $\text{£. } 9 \text{ } 12\text{s.}$ the Answer.

Yd. s. d. Ps. Yds.

(7) Stated thus—If $1 : 17 \text{ } 6 :: 12$ each 30
 Then by Reduction $17\text{s. } 6d. = 35$ Six-pences, and 12×30
 $= 360$ Yards, consequently $360 \times 35 = 12600$ Six-pences,
 these $\div$ by $4,0$ will give 315 £. the Answer.

lb. s. d. C. lb.

(8) Stated thus—If $1 : 7 \text{ } 9 :: 1$ or 112
 Then by Reduction $7\text{s. } 9d. = 31$ Three-pences, these $\times$ by
 112 (the last Term) $= 3472$ Three-pences, which $\div$ by
 4 , and 20 , will give $\text{£. } 43 \text{ } 8\text{s.}$ the Answer.

(9) First Cost $\text{£. } 34 \text{ } 6$; gained 6 £. which, added toge-
 ther, $= \text{£. } 40 \text{ } 6$ —to be sold for—then

C. qrs. lb. £. s. lb.

Stated thus—If $2 \text{ } 1 \text{ } 14 : 40 \text{ } 6 :: 1$
 Then by Reduction $2C. \text{ } 1\text{qr. } 14lb. = 266lb.$ and $\text{£. } 40 \text{ } 6\text{s.} =$
 806s. which $\div$ by 266 will give $3\text{s. } 0\frac{1}{4}d. \frac{11}{266}$ the An-
 swer.

(10) First Cost 90 £. lost £. 7 10; Difference £. 82 10 fold for. Then,

Yds. £. s. Ell. qrs.

Stated thus—If 200 : 82 10 :: 1 or 5
Then by Reduction 200 Yards = 800 Qrs. and £. 82 10 = 1650s. which $\times 5$ (the last Term) = 8250s. these $\div$ by 800 (the first Term) will give 10s. 3 $\frac{3}{4}$ d. the Answer.

Doz. lb. s. lb.

(11) Stated thus—If 9 or 108 : 45 :: 4
Then $45 \times 4 = 180$ s. these $\div$ 108 will give 1s. 8d. the Answer.

lb. d. Hbs. C. qrs. lb.

(12) Stated thus—If 1 : 6 $\frac{1}{2}$:: 4 each 12 2 14
Then by Reduction 6 $\frac{1}{2}$ d. = 13 Half-pence, and 12 C. 2 qrs. 24 lb. = 1424 lb. in one Hoghead, which $\times$ by 4 the Number of Hhds. = 5696 lb. these $\times$ by 13 (the second Term) = 74048 Half-pence, which $\div$ 2, 12, and 20, will give £. 154 5s. 4d. the Answer.

(13)

	£.	s.
Cost, - - -	579	12
Freight, - - -	46	0
Loading, &c. - -	6	0
Custom, - - -	10	0
Cellar, - - -	4	0
Gain, - - -	360	0

Tuns. ——— Tuns.

Stated thus—If 46 : 1005 12 :: 26

Then by Reduction £. 1005 12s. = 20112s. these $\times$ by 26 (the third Term) = 522912s. which $\div$ by 46 (the first Term) will give 11367s. 7 $\frac{3}{4}$ d. $\frac{14}{46}$, or £. 568 7s. 7 $\frac{3}{4}$ d. $\frac{14}{46}$. the Answer.

C. lb. s. d. C. qrs. lb.

(14) Stated thus—If 1 or 112 : 32 6 :: 12 2 14
Then by Reduction 32s. 6d. = 390d. and 12 C. 2 qrs. 14 lb. = 1414 lb. which $\times$ by 390d. = 551460d. these $\div$ by 112 (the first Term) will give 4923 $\frac{3}{4}$ d. or £. 20 10s. 3 $\frac{3}{4}$ d. the Answer.

(15)

- (15) First Cost £. 89 16s. 4d; Loss 12 £.; Difference £. 77 16s. 4d.— then

C. lb. £. s. d. lb.

Stated thus—If 1 or 112 : 77 16 4 :: 1

Then by Reduction £. 77 16s. 4d. = 18676d. which ÷ by 112 will give 166 $\frac{3}{4}$ d. or 13s. 10 $\frac{3}{4}$ d. the Answer.

- (16) Here, as he spends as much in 4 Months as he gains in 3, consequently he will spend in 8 Months what he gains in 6— then

M.	£.	s.	d.	M.	£.	s.	d.
8	185	5	6	12	185	5	6
			12				2
8)	2223	6	0	Gains	370	11	0 per An.
				Spends	277	18	3
Spends	£.	277	18	3			
				Answ.	92	12	9

- (17) First 4s. 6d. × by 12 × 6 × 6 + 8 (36 Doz. 8 Prs.) will give 99 £. the Value of the 36 Dozen 8 Pair of Stockings;— then

s. d. P. £.

Stated thus—If 1 4 : 1 :: 99

Then by Reduction 1s. 4d. = 4 Four-pences, and 99 £. = 5940 Four-pences, which ÷ by 4 (the first Term) will give 1485 Pair; these ÷ 12, will give 123 Dozen and 9 Pair, the Answer.

- (18) First 18s. 9d. ÷ by 5 = 3s. 9d. sold for per Yard, and 6s. 6d. ÷ 2 = 3s. 3d. cost per Yard; then from 3s. 9d. take 3s. 3d. remains 6d. gained per Yard; and he gained as much as 180 Yards cost; thus, cost per Yard 3s. 3d. or 39d.; consequently 39 × 180 = 7020d. his whole Gain — then

d. Yd. d.

Stated thus—If 6 : 1 :: 7020

6)7020

Answer, 1170 Yards.

(12) *s. d. Ell. qrs. £. s.*

Stated thus—If 6 6 : 1 or 5 :: 28 8

Then by Reduction 6*s.* 6*d.* = 13 Six-pences, and 28*£.* 8*s.* = 936 Six-pences, which multiplied by 5 (the second Term) = 4680 *qrs.* these ÷ by 13 (the first Term) will give 360*qrs.* bought; this ÷ by 60, (the Quarters contained in 20 Ells or 1 Piece) gives 6 Pieces, the Answ.

(20) *s. d. oz. dwts. £. s. d.*

Stated thus—If 5 10 : 1 or 20 :: 102 16 6

Then by Reduction 5*s.* 10*d.* = 70*d.* and £. 102 16 6 = 24678*d.* which × by 20, (the second Term) = 493560 *dwts.* these ÷ by 70 (the first Term) gives 7050⁶/₇ *dwts.* the Quantity of Silver bought, which, ÷ by 994 *dwts.* (4 *lb.* 1 *oz.* 14 *dwts.*) the Weight of one Ingot, will give 7 Ingots, and 92*dwts.* or 4*oz.* 12*dwts.*⁶/₇ over, the Answer.

(21) *d. oz. £. s.*

Stated thus—If 8³/₄ : 1 :: 426 16

Then by Reduction 8³/₄ *d.* = 35*qrs.* and £. 426 16*s.* = 409728 *qrs.* which ÷ by 35, gives 11706¹⁸/₃₅ *oz.* these ÷ by 16, 28, and 4, will give 6 Par. and 2 *qrs.* 3 *lb.* 10¹⁸/₃₅ *oz.* over, the Answer.

(22) Here, first 100 Guineas = 105£. lays by; then from £. 488 5*s.* his yearly Income, take 105£. remains £. 383 5*s.* what he spends per Annum—then

D. £. s. D.

Stated thus—If 365 : 383 5 :: 1
20

365)7665 (21 Shillings per Day,
[the Answer.

£. s. s. d. £. s.

(13) Stated thus—If 1 or 20 : 3 9 :: 564 12

Then by Reduction 3*s.* 9*d.* = 45*d.* and 564£. 12*s.* = 11292*s.* which × 45*d.* (the second Term) = 508140*d.* this, ÷ by 20, (the first Term) = 25407*d.* these, ÷ by 12 and 20, will give £. 105 17*s.* 3*d.* Taxes, which, subtracted from his Income, viz. £. 564 12*s.* leaves £. 458 14*s.* 9*d.* his neat Income, the Answer.

The Rule of Three Direct.

(24) *s. d. oz. dwts. £.*

Stated thus—If 5 9 : 1 or 20 :: 200

Then by Reduction 5*s.* 9*d.* = 69*d.* and 200 £. = 48000 *d.*
 which $\times$ by 20, (the second Term) = 960000 *dwt.* this,
 $\div$ by 69 (the first Term) gives 13913 *dwt.* $\frac{3}{9}$, these
dwt. $\div$ by 692, (2 *lb.* 10 *oz.* 12 *dwt.*) the Penny-
 weights in an Ingot, will give 20 Ingots, and 73 *dwt.*
 or 3 *oz.* 13 *dwt.* $\frac{3}{9}$ over, the Answer.

(25) First $6 \times 6 \times 10 \times 30 = 10800$ Yards in the six Packs;
 then*Yds. £. s. d. Yds.*

Stated thus—If 3 : 2 4 3 :: 10800

Then by Reduct. £. 2 4*s.* 4*d.* = 531 Pence, these $\times$ by 10800
 = 5734800 *d.* which $\div$ by 3, (the first Term) = 1911600
 Pence, or 7965 £. cost him, and 3) 2 £. 4*s.* 3*d.* (14*s.*
 9*d.* cost per Yard.

(26) *W. £. s. W.*

Stated thus—If 16 : 14 16 :: 52

By Reduction £. 14 16*s.* = 296*s.* which $\times$ 52 = 15392*s.* these
 $\div$ by 16, gives 962*s.* or £. 48. 2*s.* the Answer.

(27)

£. s.

Oxen, each,	-	-	10	0
Cows,	-	-	7	0
Calves,	-	-	1	10
Sheep,	-	-	0	19

One of each cost,	-	£. 19	9
-------------------	---	-------	---

£. s. of ea. £. s.

Stated thus—If 19 9 : 1 :: 116 14

20

20

389

) 2334 (6 of each.

(28)

*£. £. s. d. £.*Stated thus—If 560 : 374 10 6 $\frac{1}{2}$:: 1

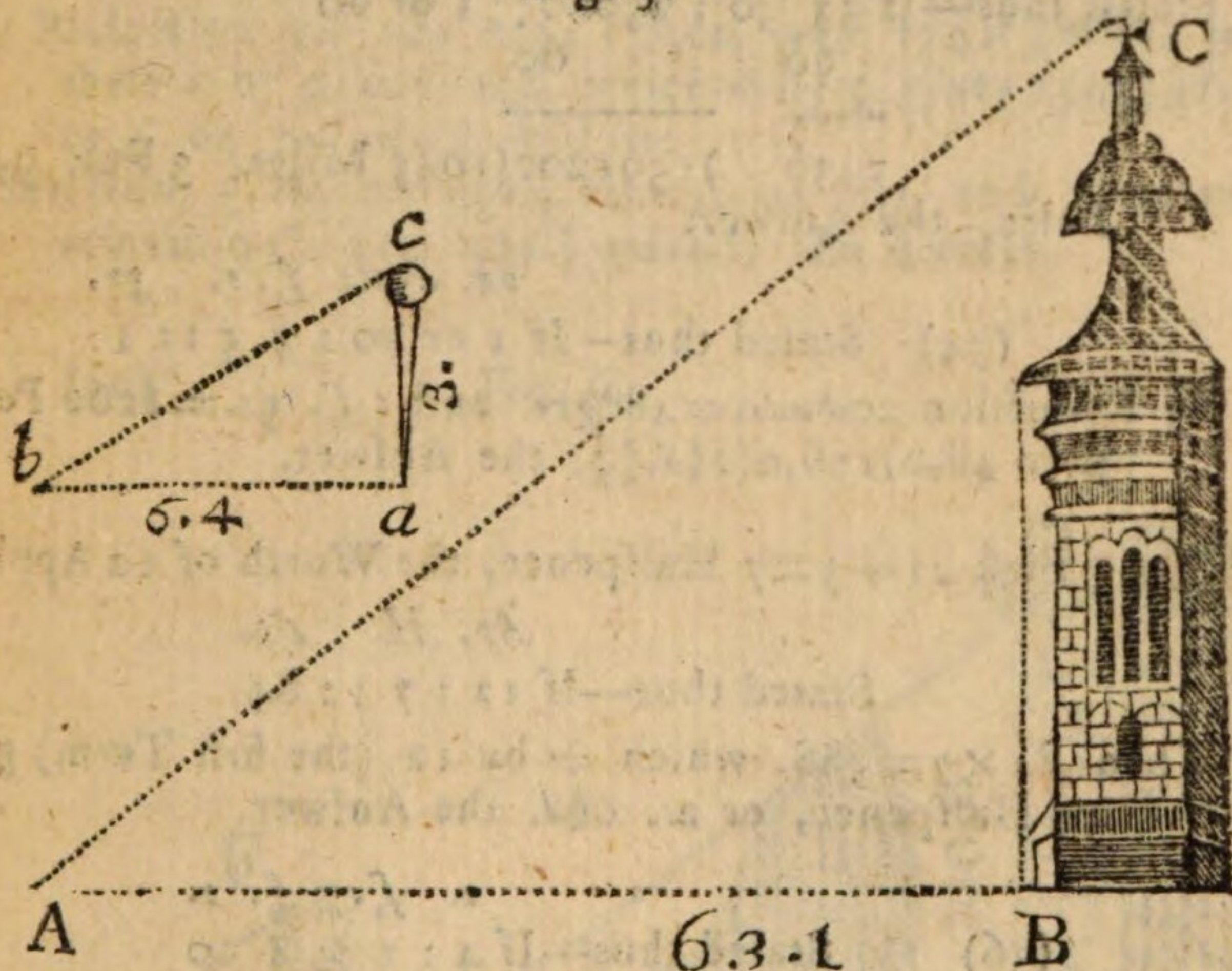
By Reduction £. 374 10*s.* 6 $\frac{1}{2}$ *d.* = 359546 Farthings, which $\div$
 by 560 gives 642 $\frac{26}{80}$ *grs.* which, reduced to Shillings,
&c. will be 13*s.* 4 $\frac{1}{2}$ *d.* $\frac{26}{80}$, the Answer.

(29)

First the Cost, Custom, Fees, Freight, Factorage, and Profit, added into one Sum = £. 14 os. 2d. — then

C. qrs. lb. oz. £. s. d. C. lb.
 (29) Stated thus — If 18 3 17 7 : 14 0 2 :: 1 or 112
 By Reduction 18 C. 3 qrs. 17 lb. 7 oz. = 33879 oz. and 112 lb.
 = 17920 oz. Also 14 £. 2d. = 3362 Pence; then 3362 p.
 $\times$ 17920 oz = 6024704 Pence, which $\div$ by 33879 (the
 first Term) gives $177\frac{3}{4}d. \frac{10847}{33879}$, these d. $\div$ by 12, give
 14s. $9\frac{3}{4}d. \frac{10847}{33879}$, the Answer.

Fig. 5.



(30) Here the above Figures $ac = 3$ Feet, the Length of the Staff, $ab = 6$ Feet 4 Inches, or 76 Inches, Distance of its Shadow. Also $BC =$ the Height of the Tower, and $AB = 63$ Yards, 1 Foot, or 2280 Inches Distance of its Shadow — then

In. Ft. In.
 Stated thus — If 76 : 3 :: 2280
 Then $2280 \times 3 = 6840$ Feet, which $\div$ by 79 = 90 Feet, the
 Answer. For as $ac : ab :: BC : AB$.

D. H. M. H.
 (31) Stated thus — If 6 4 : 285 :: 1

$\frac{12}{76} 285$ (3 Miles 6 Fur. the Answ.

*The Rule of Three Direct.**Pipe G. £. s.*

(32) Stated thus—If 1 or 126 : 44 2 :: 1 Pt.

By Reduction 126 Gal. = 1008 Pts. and 44 £. 2s. = 10584 Pence, which $\div$ by 1008 (the first Term) gives $10\frac{1}{2}d.$ per Pint, the Answer.

(33) First $360 \times 69\frac{1}{2} = 25020$ Miles, the Circumference of the Globe—then*H. M. Miles. H. M.*

Stated thus—If $\begin{array}{c} 23 \ 56 \\ 60 \end{array} : 25020 :: 1 \text{ or } 60$

$\begin{array}{r} 1436 \end{array}) 1501200 (1045 \text{ Miles, } 3 \text{ Fur. } 9\frac{356}{1436}$
Poles, the Answer.

oz. dwts. £. s. gr.

(34) Stated thus—If 1 or 20 : 5 5 :: 1

By Reduction 20 dwts. = 480 grs. and 5 £. 5s. = 1260 Pence, then $48,000 \div 1260 = 2\frac{1}{2}d. \frac{24}{8}$, the Answer.

(35) First $21 \div 3 = 7$ Halfpence, the Worth of 12 Apples.*Ap. H. Ap.*

Stated thus—If 12 : 7 :: 84

Then $84 \times 7 = 588$, which $\div$ by 12 (the first Term) gives 49 Halfpence, or 2s. $0\frac{1}{2}d.$ the Answer.

s. £. £. s.

(36) Stated thus—If 4 : 1 :: 8 10

By Reduction 8 £. 10s. = 170s. which $\div$ by 4 gives 42 £. 10s. the Answer.

(37) First $184 \times 7 \times 4 + \frac{1}{2}(28\frac{1}{2}) = 5244$ Yards of the former.*Yds. Yds. Yds.*

Stated thus—If 19 : 14 :: 5244

 $\begin{array}{r} 14 \end{array}$

$\begin{array}{r} 19 \end{array}) 73416 (3864 \text{ Yds. the Answer.}$

(38) First $406 \times 117 = 47502$ Pieces; these $\times$ by 44d. (3s. 8d.) or the Value of one Piece, will give 2090088 Pence, Value of all the Pieces—then

Stated

The Rule of Three Direct.

101

d. Reas. d.

Stated thus—If $3 : 20 :: 2090088$

20

3)41801760(13933920 Reas,

the Answer.

(39) First $274 \times 4 = 1096s.$ or £. 54 16s. cost him; then—

Ell E. qrs. s. d. Ell F.

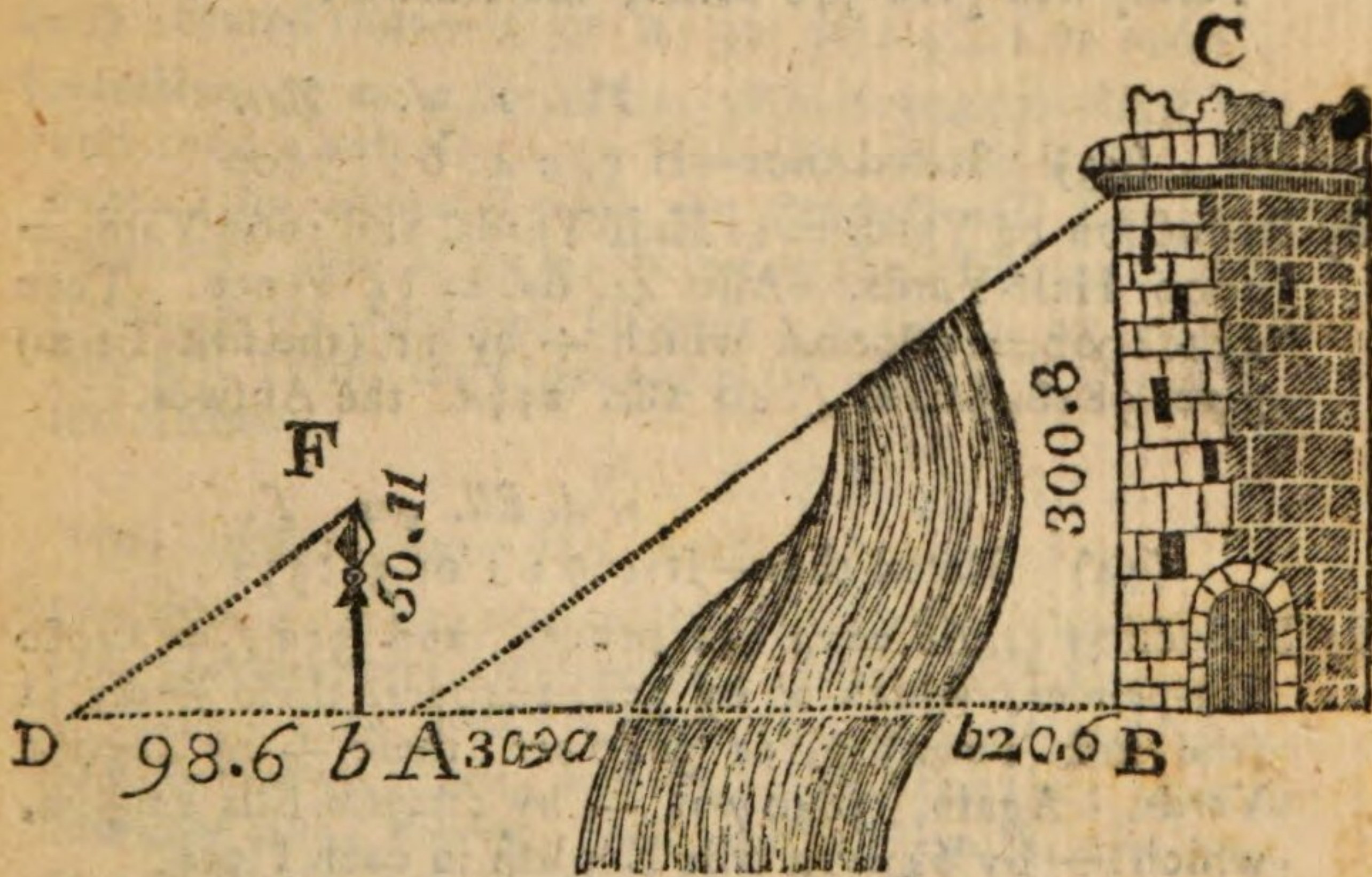
Stated thus—If $1 \text{ or } 5 : 7 \text{ } 10 :: 274$

By Reduction $7s. \text{ } 10d. = 94$ Pence, and $274 \times 3 = 822 \text{ qrs.}$
these $\times$ by $94 = 77268$, which $\div$ by 5 , gives $15453\frac{1}{2}d.\frac{2}{5}$,
or £. 64 7s. $9\frac{1}{2}d.\frac{2}{5}$ sold for.

Then from £. 64 7s. $9\frac{1}{2}d.\frac{2}{5}$ take £. 54 16s. and there will
remain 9 £. 11s. $9\frac{1}{2}d.\frac{2}{5}$ gained, the Answer.

(40)

Fig. 6.



Here $Fb = 50$ Feet, 11 Inches, or 611 Inches, Height of the May Pole, and $Db = 98$ Feet, 6 Inches, or 1182 Inches, Length of its Shadow. Also $BC = 300$ Feet, 8 Inches, or 3608 Inches, and AB the Length of its Shadow—then

The Rule of Three Direct.

In. In. In. In.

Stated thus—If 611 : 1182 :: 3608 : 6979

Ft. In.

Then $6979 \div 12 = 581 \frac{7}{12} = A. B.$ And 30 F. 9 In. + 20 F. 6 In. = $51 \frac{3}{4} = A a + B b.$ $530 \frac{4}{11} = a b, \text{ the}$

Breadth of the Stream required.

M. s. M.

(41) Stated thus—If 2 : 15 :: 7

Then $15 \times 7 = 105s.$ which $\div$ by 2 = 52s. 6d. or £. 2 12s. 6d. the Answer.

H. M. D. H.

(42) Stated thus—If 4 : 12 :: 9 8

By Reduction 9 D. 8 H. = 116 Hours, which $\times$ by 12 (the second Term) = 1392 Hours; these $\div$ by 4 (the first Term) will give 348 Miles, the Answer.

Yds s. d. Yds.

(43) Stated thus—If $5\frac{1}{2} : 4 : 6 :: 1000$ By Reduction $5\frac{1}{2}$ Yards = 11 Half-Yards, and 1000 Yards = 2000 Half-Yards. Also 4s. 6d. = 54 Pence. Then $54 \times 2000 = 108000d.$ which $\div$ by 11 (the first Term) gives $9818\frac{2}{11}d.$ or £. 40 18s. $2\frac{2}{11}d.$ the Answer.

s. d. Ell. qrs. £.

(44) Stated thus—If 5 6 : 1 or 3 :: 352

By Reduction 5s. 6d. = 11 Six-pences, and 352 £. = 14080 Six-pences, which $\times$ by 3 = 42240 qrs. these $\div$ by 11 (the first Term) gives 3840 qrs. these $\div$ by 4 = 960 Yards. Again, 3840 qrs. $\div$ by 5 = 768 Ells English, which $\div$ by 64 = 12 Ells English in each Piece.

£. £. s. £. s.

(45) Stated thus—If 100 : 4 15 :: 40 10

By Reduction 100 £ = 2000s. and £. 4 15s. = 95s. Also £. 40 10s. = 810s. which $\times$ by 95 = 7695s. these $\div$ by 2,00 will give 38s. $5\frac{1}{2}d. \frac{16}{20}$, or £. 1 18s. $5\frac{1}{4}d. \frac{16}{20}$, the Answer.

(46)

(46) First $24 \times 20 = 480$ Yards, contained in the 24 Pieces.

Yds. Yds. Yds.

Stated thus—If $12 : 8 :: 480$

Here $480 \times 8 = 3840$ Yards, which $\div$ by 12, gives 320 Yds. the Answer.

A. £. s. d. A.

(47) Stated thus—If $1 : 1 \ 19 \ 6 :: 240$

By Reduction £. 1 19s. 6d. = 79 Sixpences, which $\times$ by 4 $\times 60$ (240) the Product is 18960 Six-pences; these $\div$ by 40, will give 474 £. per Annum; this $\div$ by 4 = £. 118 10s. per Quarter, the Answer.

£. s. d. £.

(48) Stated thus—If $1 : 10 \ 6 :: 1000$

By Reduction 10s. 6d. = 11 Six-pences, which $\times$ by 1000, the Product is 11000 Six-pences, or 525 £. the Answer.

M. fur. p. ft. Min. Mile Yds.

(49) Stated thus—If $37 \ 2 \ 37 \ 5\frac{1}{2} : 4 :: 1$ or 1760

By Reduction $37 \ m. \ 2 \ fur. \ 37 \ p. \ 5\frac{1}{2} \ ft. = 394592$ Half-feet, and 1760 Yards. = 10560 Half-feet, which $\times$ by 4 Minutes (the second Term) the Product will be 42240 Minutes; this $\times$ by 60, to bring the Minutes into Seconds, gives 2534400 Seconds; these $\div$ by 394592 (the first Term) the Quotient is $6'' \ 25 \frac{146080}{394592}'''$ past 12, the Answer.

£. s. d. E. B. £. s. d.

(50) Stated thus—If $1 \ 7 \ 10 : 4 :: 118 \ 17 \ 7\frac{1}{2}$

By Reduction £. 1 7s. 10d. = 668 Half-pence, and £. 118 17s. $7\frac{1}{2}d. = 57063$ Half-pence. Also 4 Ells English = 80 Nails; then $57063 \times 80 = 4565040$ Nails, which $\div$ by 668 = 6833 Nails, and $\frac{596}{668}$ over, bought; then to find how many Pieces, each 33 Ells Flemish, 1qr. 2 na. or 402 Nails, are contained in the Quantity bought; thus, $402 \overline{) 6833} (16$ Pieces, and 33 Ells. 1qr. 1 na. $\frac{596}{668}$ over, the Answer.

(51) First, from 100 £. take 60 £. Remains 40 £. Serge cost.

Yds. Y. Yds.

Stated thus—If $2 : 3 :: 236$

Here

Here $236 \times 3 = 708$; this $\div$ by $2 = 354$ Yards of Shalloon, which cost $60 \text{ } \pounds$. consequently $60 \text{ } \pounds$. $\div$ by 354 , will give $3\text{s. } 4\text{d. } \frac{2}{3} \frac{5}{4}$, what the Shalloon cost per Yard; and $40 \text{ } \pounds$. $\div$ by 236 , will give $3\text{s. } 4\text{d. } \frac{1}{2} \frac{6}{3} \frac{8}{4}$, what the Serge cost per Yard.

s. lb. £. s. d.

(52) Stated thus—If $14 : 8 :: 4 \text{ } 19 \text{ } 1\frac{1}{4}$

By Reduction $14\text{s.} = 672$ Farthings, and $\text{£. } 4 \text{ } 19\text{s. } 1\frac{3}{4}\text{d.} = 4757$ Farthings, which $\times$ by 8 , (the second Term) the Product will be 38056 ; this, $\div$ by 672 (the first Term) will give $56 \text{ lb. } 10 \text{ oz. } 1 \text{ dr. } \frac{3}{6} \frac{5}{7} \frac{2}{2}$, the Answer.

C. lb. s. C. qrs. lb.

(53) Stated thus—If $1 \text{ or } 112 : 7 :: 20 \text{ } 2 \text{ } 16$.

By Reduction $20 \text{ C. } 2 \text{ qrs. } 16 \text{ lb.} = 2312 \text{ lb.}$ these $\times 7\text{s.} = 16184\text{s.}$ which $\div$ by 112 will give $144\text{s. } 6\text{d.}$ or $\text{£. } 7 \text{ } 4\text{s. } 6\text{d.}$ the Answer.

(54) A Pipe $= 126$ Gallons, from which take 12 Gallons. Remains 114 Gallons, or 456 Quarts, at 18d. per Qt. Then

Qt. d. Qts.

Stated thus—If $1 : 18 :: 456$

Here $456 \times 18 = 8208$ Pence, which, $\div$ by 12 and 20 , gives $\text{£. } 34 \text{ } 4\text{s.}$ sold for, from which take $\text{£. } 25 \text{ } 4\text{s.}$ Remains $9 \text{ } \pounds$. gained, the Answer.

C. qrs. lb. £. s. d. lb.

(55) Stated thus—If $20 \text{ } 2 \text{ } 16 : 7 \text{ } 4 \text{ } 6 :: 1$

Here by Reduction $20 \text{ C. } 2 \text{ qrs. } 16 \text{ lb.} = 2312 \text{ lb.}$ and $\text{£. } 7 \text{ } 4\text{s. } 6\text{d.} = 6936$ Farthings, which $\div$ by 2312 (the first Term) gives $\frac{3}{4}$, the Answer.

(56) First $9 \times 4 = 36$ square Inches in a Brick, and $20 \times 20 = 400$ Feet square, which $\times$ by $144 = 57600$ square Inches, the Content of the Room.—Then

Inch. B. Inch.

Stated thus—If $56 : 1 :: 5760$

$36)57600(1600$ Bricks, the Answer.

(57) First $22 \times 4 = 88$ Miles I have gone before you set out,
and $32 - 22 = 10$ — you gain of me per Day; then

Stated thus—If $10 : 1 :: 88$

$10)88(8\frac{8}{10}$ Days, before you overtake me.

Then $8\frac{8}{10} \times 4 \times 8(32) = 281\frac{2}{5}$ Miles you travelled before I was
overtaken, consequently $350 - 281\frac{2}{5} = 68\frac{3}{5}$ Miles on this
Side Edinburgh, the Answer.

(58) First, from 20 take 11. Remains 9 Days to be sunk.

Days. Yea. Days.

Then stated thus—If $3 : 400 :: 9$

Here $400 \times 9 = 3600$; this $\div$ by $3 = 1200$ Years after Anno
1700; then 1200 added to 1700 = 2900, the Year of
Christ required.

(59) $d. lb. d.$

First, Beef cost per Day, $2\frac{1}{4} \times 5 = 11\frac{1}{4}$, and per Week, $11\frac{1}{4}d.$
 $\times 7 = 6s. 6\frac{3}{4}d. = 315 qrs.$

Biscuit cost per Day, $1\frac{1}{2}d. \times 3lb. = 4\frac{1}{2}d.$ and per Week, $4\frac{1}{2}d.$
 $\times 7 = 2s. 7\frac{1}{2}d. = 126 qrs.$

Meat for the Ship's Company cost per Day $\pounds. 12 12s.$ and
per Week $\pounds. 12 12s. \times 7 = \pounds. 88 4s.$ or 84672 Farthings.

$Qrs. Qrs. Qrs.$

Stated thus—If $315 : 126 :: 84672$
 126

$315)10668672(33868\frac{252}{315}$ Farthings, or
 $\pounds. 35 5s. 7d. \frac{252}{315}$, the Answer.

$H. M. H.$

(60) Stated thus—If $8 : 135 :: 1$

Here $135 \div 8 = 16$ Miles, 7 Furlongs, what both rode per
Hour.

$M. fur. p. M. f. M. fur. p.$

Then, 16 7 0 less 2 4 = 14 3 0

And 14 3 0 $\div$ by 2 = 7 1 20 B. rode per Hour.

Also, 7 1 20 $\div$ by 2 4 = 9 5 20 A. ditto.

$M. fur. p.$

$M. fur.$

For $\left\{ \begin{array}{l} 7 \ 1 \ 20 \\ 9 \ 5 \ 20 \end{array} \right\} \times \text{by } 8 = \left\{ \begin{array}{l} 57 \ 4 \text{ B.} \\ 77 \ 4 \text{ A.} \end{array} \right\} \text{ travelled.}$

Proof, 135 Miles.

(61)

- (61) Suppose 130 to be the whole Work; then A will perform $\frac{1}{3}$ th Part, and B. $\frac{1}{6}$ th, which will be $13 + 10 = 23$ d Part of the Work performed by them both together in one Day.

W. Day. W.

Then Stated thus—If $23 : 1 :: 130$

23)130(5 Days, 7 Hours, $49\frac{1}{2}\frac{3}{4}$ Minutes, the Answer.

- (62) Here, suppose 198 to be the whole Work, of which B and C will perform $\frac{1}{8}$, viz. 11, and with the Help of A they will perform $\frac{1}{18}$, viz. the 18th Part in one Day; then $18 - 11 = 7$ th Part performed by A alone.

W. Day. W.

Then stated thus—If $7 : 1 :: 198$

7)198(28 Days, 3 H. $25\frac{1}{2}$ M. the Answ.

Here the Reason of my supposing 130 to be the whole Work in the 61st Question, is because $\frac{1}{6}$ and $\frac{1}{3}$ reduced to a Common Denominator becomes $\frac{1}{3}$ and $\frac{2}{3}$ by (Case V. p. 197.) whose sum is $\frac{3}{3}$ by (Sect. 39. p. 202.) Likewise in the last Question I supposed 198, for $\frac{1}{18}$ and $\frac{1}{8}$ reduced to a Common Denominator becomes $\frac{1}{18}$ and $\frac{1}{8}$, whose difference is $\frac{7}{144}$ per Sect. 40. page 203.

£. £. £.

(63) Stated thus—If $17 : 3 :: 140$

£. s. d.

Here $140 \times 3 = 420$; this $\div$ by 17 = 24 14 $\frac{1}{4}$ $\frac{1}{7}$ taken off.

Then from £. 140 take £. 24 14s. $14\frac{1}{4}d. \frac{1}{7}$. Remains £. 115 5s. $10\frac{1}{2}d. \frac{6}{7}$, the Assessment, which $\div$ by 5, gives £. 23 1s. $2d. \frac{3}{7}$ the Taxes; this, taken from £. 140, leaves £. 116 18s. $9\frac{3}{4}d. \frac{9}{7}$, the Answer.

- (64) First 1000 $\div$ by $\begin{cases} 5 = 200 \text{ gained by Land Trade.} \\ 8 = 125 \text{ ————— Sea ditto.} \end{cases}$

Sum, 325 Gain per Year.

Then

Then $1000 \div 2\frac{1}{2} = 400\text{£}$. lost in 1 Year.

And $400\text{£} - 325 = 75\text{£}$. run out per Year.

Y. Y. £.

Stated thus—If $75 : 1 :: 1000$

$75)1000(13\frac{1}{3}$ Years, the Answer.

(65) Here, from 10 in the Morning to 6 in the Evening is 8 Hours—I set out before you, then $2 \times 8 = 16$ Miles I travelled before you set out, and $170 - 16 = 154$ Miles. Also, $2 + 3 = 5$ Miles both travelled per hour.

M. H. M.

Stated thus—If $5 : 1 :: 154$

$5)154(30\frac{4}{5}$ Hours they will meet.

Then $30\frac{4}{5} \times 2 = 61\frac{3}{5}$, which added to $16 = 77\frac{3}{5}$ Miles, or $77\text{ m. } 4\text{ fur. } 32\text{ p.}$ from Exeter; and $30\frac{4}{5} \times 3 = 92\frac{2}{5}$ M. or $92\text{ m. } 3\text{ fur. } 8\text{ p.}$ from London.

M. fu. p.

Answer, $\left\{ \begin{array}{l} 77 \quad 4 \quad 32 \\ 92 \quad 3 \quad 8 \end{array} \right\}$ distant from $\left\{ \begin{array}{l} \text{Exeter.} \\ \text{London.} \end{array} \right.$

Proof, 170 Miles.

(66) First, from 13° take 1° , remains 12° , Moon gains of the Sun per Day.

And $30^\circ \times 3 = 90^\circ$, from the first of Aries to the first of Cancer.

Also, $90^\circ + 3 = 93^\circ$ Sun before the Moon.

D.

Then stated thus—If $12^\circ : 1 :: 93^\circ$

$12)93(7\frac{3}{4}$ Days, in which Time,

the Sun will be overtaken by the Moon.

$\therefore 7\frac{3}{4} + 3 = 10\frac{3}{4}$ Degrees of Cancer, the Answer.

(67) First, from 21 take 15. Remains 6 Rods, the Dogs gained in running 21 Rods, and fourscore $= 80$, $+ 16 = 96$ Rods the Hare started before the Dogs.—Then

R. R. R.

Stated thus—If $6 : 21 :: 96$

Here $96 \times 21 = 2016$; this $\div$ by 6, gives 336 Rods the Dogs ran, and $336 - 96 = 240$ Rods the Hare ran.

(68)

The Rule of Three Direct.(68) First, $5\frac{2}{3}$ Miles = 29920 Feet.*Feet. Sec. Feet.*

Stated thus—If 1150 : 1 :: 29920

115,0)2992,0(26 Sec. $1\frac{5}{115}$ Thirds, the Answer.

(69) First, 1 Min. 3 Seconds = 63 Seconds.

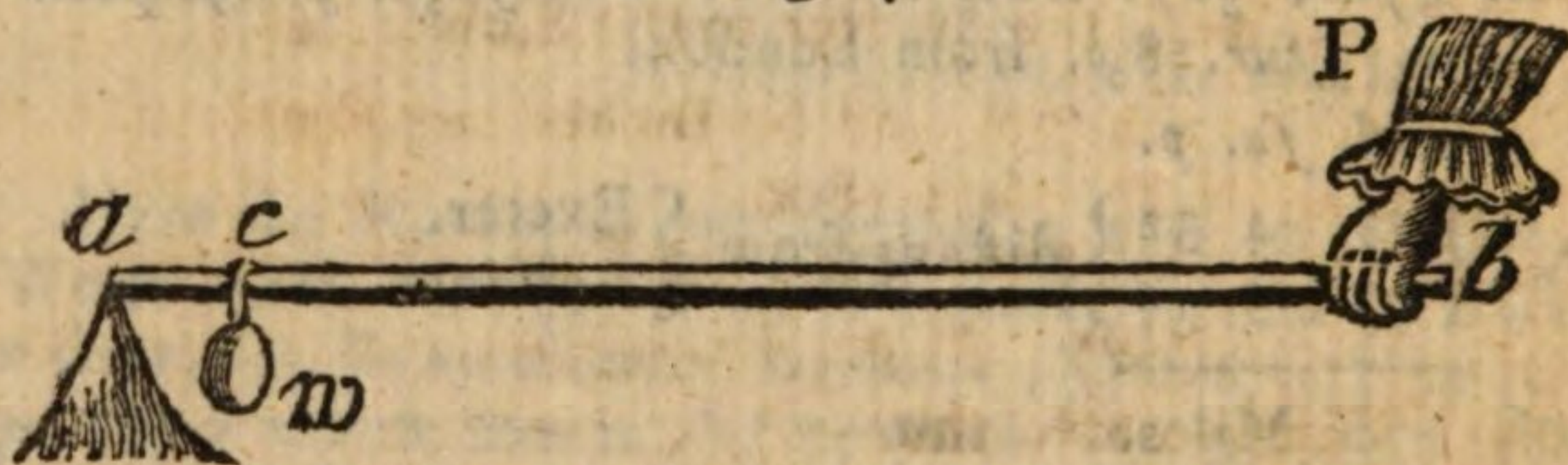
Feet.

Stated thus—If 1 : 1150 :: 63

63

72450 Feet, or 13 Miles, 5 Furlongs, 30 Poles, 5 Yards, the Answer.

Fig. 7.

(70) Here $ab = 100$ Inches, and $ac = 7\frac{1}{2}$, P the Power, or 168 *lb.* and w . the Weight to be moved.Then from 100 take $7\frac{1}{2}$, remains $92\frac{1}{2} = ab - ac = cb$.*In. lb. In.*Stated thus—If $7\frac{1}{2} : 168 :: 92\frac{1}{2}$ Here $7\frac{1}{2}$ Inches = 15 Half Inches, and $92\frac{1}{2} = 185$ Half Inches, which $\times 168 = 31080$ *lb.* these $\div$ by 15 = 2072 *lb.* = w . the Answer.For A's $ac : P :: cb : w$.(71) First $9 \times 2 = 18$ Inches the Diameter of the Crank.*Ft. In. Ft.*

Stated thus—If 6 : 18 :: 9

9

6)162(27 Inches the Answ.

(72)

(72) First $62\frac{1}{2} \times 3 = 187\frac{1}{2} \text{ lb.}$ Wt. of 3 solid Feet of Water;

Feet. lb. Feet.

Therefore, stated thus—If $6 : 187\frac{1}{2} :: 9$

9

6)1687 $\frac{1}{2}$ (281 $\frac{1}{4}$ lb. the Answ.

(73) Stated thus—As $25 : 1000 :: 1 : 40$, so the less is moved with a Force 40 Times greater than the other.

(74) Stated thus—As $60 : 8 :: 100 :$

6,0)80,0 (13 $\frac{1}{3}$. Answ. as $13\frac{1}{2}$ to 1.

(75) Stated thus—As $8 : 48 :: 1 : 6$

Answer, Lesser to the Greater as 1 to 6

(76) First 2 Hours $\times 60 = 120$ Minutes.

Then—stated thus—As $40 : 120 :: 1 : 3$.

Answer, the Swifter to the Slower, as 1 to 3.

(77) First $30 \times 12 = 360$, and 60 Inches = 5 Feet.

Then as $1 : 5 :: 360 : 1800$

Consequently $1800 \div 5 = 1795$ Feet, the Answer.

(78) First $50 \div 5 = 10$

Then—stated thus—As $5 : 10 :: 1 : 2$

So that the first Body hath been in Motion double the Time of the latter.

THE RULE OF THREE INVERSE.

In. B. In. L. In. B.

(2) Stated thus—If $12 : 12 :: 4$

Here $12 \times 12 = 144$; this $\div 4 = 36$ Inches long, the Answer.

£. M. £.

(3) Stated thus—If $500 : 6 :: 220$

Here $500 \times 6 = 3000$, this $\div$ by 220, gives 13 Months, $19\frac{3}{4}$ Days, the Answer.

M. oz. M.

(4) Stated thus—If $3 : 14 :: 8$

Here $3 \times 14 = 42$, this $\div$ by 8 = 5 oz. 4 dwt. the Answer.

s. d. oz. s.

(5) Stated thus—If $4 6 : 12 :: 3$

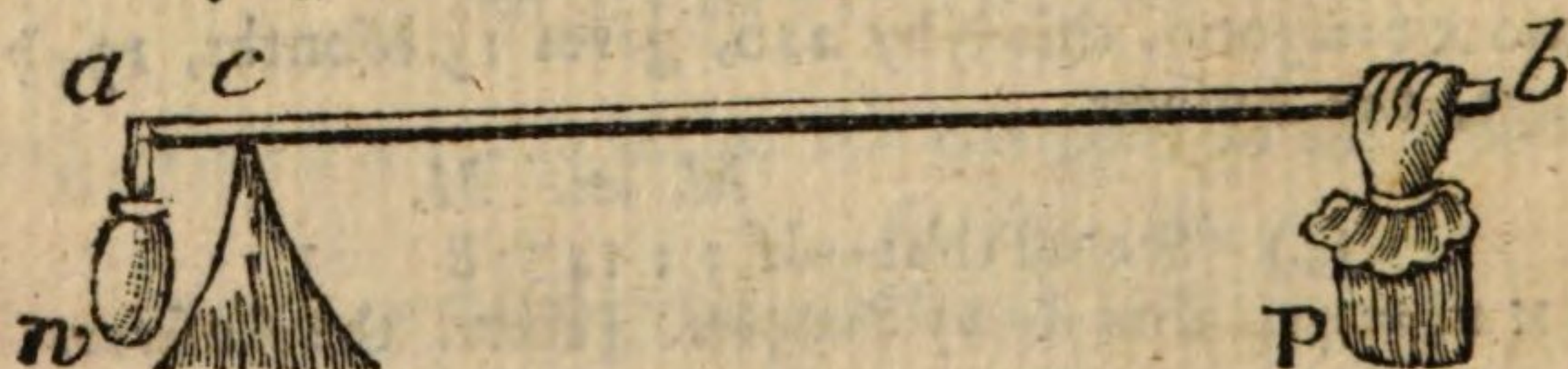
Here 4s. 6d. = 9 Six-pences, and 3s. = 6; then $12 \times 9 = 108$; this $\div$ by 6, the second Term, gives 18 oz. the Answ.

L

(6)

*The Rule of Three Inverse.**Qrs. Yds. Qrs.*(6) Stated thus—If $5 : 275 :: 3$ Here $275 \times 5 = 1375$, this $\div$ by 3, gives 458 yds. 1 qr. $1\frac{1}{3}$ na. the Answer.*£. Yrs. £.*(7) Stated thus—If $80 : 15 :: 600$ Here $15 \times 80 = 1200$, this $\div$ by 600, gives 2 Years, the Answer.*M. W. D. M.*(8) Stated thus—If $10 : 43\ 5 :: 6$ Here $43\ w. 5\ d. = 306$ Days, which $\times$ by 10 = 3060, this $\div$ by 6, gives 510 Days, or 72 Weeks, 6 Days, the Answer.*Ft. Ft. Ft.*(9) Stated thus—If $24 : 30 :: 2\frac{1}{4}$ Here $24\ ft. = 96\ qrs.$ and $2\frac{1}{4} = 9\ qrs.$ then 96×30 (the second Term) = 2880 Feet; these $\div$ by 9 (the last Term) gives 320 Feet, or $106\frac{2}{3}$ Yards, the Answer.Note, Paper is 27 Inches, or $2\frac{1}{4}$ Feet wide.*Mo. Men. Mo.*(10) Stated thus—If $3 : 1500 :: 5$ Here $1500 \times 3 = 4500$, this $\div$ by 5, gives 900 Men, to continue in; then from 1500 take 900, remains 600 Men to depart.*H. Cock. M.*(11) Stated thus—If $6 : 1 :: 15$ Here 6 Hours $\times 60 = 360$ Minutes, which $\div$ by 15, (the last Term) gives 24 Cocks, the Answer.

(12) Fig. 8.

Here in the above Figure $ab = 100$ Inches, $ac = 7\frac{1}{2}$, P the Power, or $1\frac{1}{2}$ Cwt. and w the Weight.Then $100 - 7\frac{1}{2} = 92\frac{1}{2}$ Inches = $ab - ac$.Stated thus—If $92\frac{1}{2} : 1\frac{1}{2}\ C. \text{ or } 168\ lb. :: 7\frac{1}{2}$

Here

The Rule of Three Contracted.

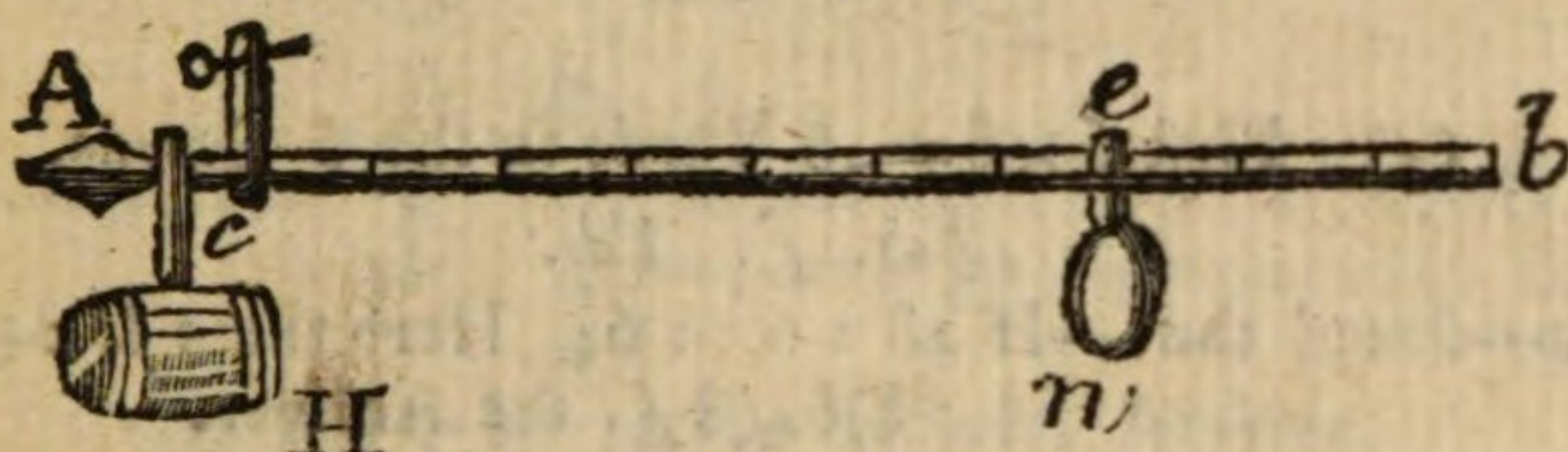
III

Here $92\frac{1}{2} = 185$ Half-inches, and $7\frac{1}{2} = 15$; then $185 \times 168 = 31080$; this $\div$ by 15, (the last Term) gives 2072 *lb.* or $18\frac{1}{2}$ C. the Answer.

For as $bc : w :: ac : P$.

(13)

Fig. 9.



In the above Fig. $ce = 70$ Inches, $ac = 2$, H the Hoghead, and w the Weight.

In. C. In.

Stated thus—If $2 : 9\frac{1}{2} :: 70$

Here $9\frac{1}{2}$ C. = 1064 *lb.* which $\times$ by 2 = 2128 *lb.* these $\div$ by 70, (the last Term) gives 30 *lb.* 6 *oz.* $6\frac{2}{3}$ *drs.* the Weight required.

For as $ac : w :: ec : H$

(14) Stated thus—As 100000 : 20 :: 30 : 66666 Feet, 8 Inches per Second, the Answer.

lb. Feet. lb.

(15) Stated thus—As 200 : 100 :: 8 : 2500 Feet per Second.

CONTRACTIONS in the RULE of THREE.

(2) First $3 \times 3 = 9$ C. in the three Chests.

C. £. s. C.

Stated thus—If $9 : 11 \frac{13}{8} :: 72$

Here the 1 and 3 Terms [$\div$ by 9.

Answer, £. 93 4

C. £. C.

(3) Stated thus—If $26 : 78 :: 156$

Here 1 and 2 $\div$ by 26.

£. 468 the Answer.

L 2

(4)

The Rule of Three Contracted.

(4) Stated thus—If $\begin{array}{ccc} lb. & s. & lb. \\ 3 & 18 & 112 \\ 1 & 6 & 6 \end{array}$ Here the 1 and 2 $\div$ by 3.

Answer, $672 s. = \text{£}. 33 \ 12 s.$

(5) First $6 \times 8 = 48$ Yards in the 6 Gowns.

$Yds. \ \text{£}. \ Yds.$

Then—stated thus—If $48 : 6 :: 64$ Here the 1 & 2 $\div$ by 6.
 $8)64(8 \text{ £}. \text{ the Answer.}$

(6) First $1\frac{1}{2} C. = 168 lb.$

$lb. \ s. \ lb.$

Then—stated thus—If $24 : 12 :: 168$ Here 1 and 2 $\div$ 12.
 $2)168(84s. \text{ or } 4 \text{ £}. 4s. \text{ the Answer.}$

(7) First $24 \times 3 = 72$ Yards in the 3 Pieces.

$Yds. \ \text{£}. \ s. \ Yds.$

Then—stated thus—If $72 : 64 \ 14 :: 8$ Here 1 and 3 $\div$ by 8.
 $9)64 \ 14(7 \text{ £}. 3s. 9\frac{1}{4} d. \frac{1}{3} \text{ Answer.}$

(8) First $6 \times 3 = 18 lb.$ in the 6 Parcels.

$lb. \ \text{£}. \ s. \ lb.$

Stated thus—If $18 : 12 \ 19 :: 6$ Here 1 and 3 Terms $\div$ by 6.
 $3)12 \ 19(4 \text{ £}. 6s. 4d. \text{ the Answer.}$

$In. \ In. \ In.$

(9) Stated thus—If $12 : 12 :: 3$ Here the 1 and 3 $\div$ by 3.

$4 \ 4 \ 1$

Answer, 48 Inches, or 4 Feet,

$M. \ C. \ M.$

(10) Stated thus—If $512 : 225 :: 64$ Here 1 and 3 $\div$ by 64.

$8 \ 8 \ 1$

Answer, 1800 Cwt.

THE RULE OF FIVE:

O R,

THE DOUBLE RULE OF THREE.

P. D. B.

- (2) Placed thus— $6 : 12 : 2$, then $6 \times 12 = 72$, the Divisor.
 $36 : 4 : -$, and $36 \times 4 \times 2 = 288$, the Dividend.

So $288 \div \text{by } 72 = 4$ Bushels, Answer.

P. w. £. Then $21 \times 6 = 126$ the Divisor.

- (3) $6 : 21 : 120$, And $46 \times 14 \times 120 = 77280$ £. Dividend.
 $14 \quad 6 \text{ ———}$. So $77280 \div 126 = \text{£.} 613 \text{ } 6s. \text{ } 8d.$ the Answer.

- (4) Here $9\text{£. } 5s. = 8880$ Farthings, and $\text{£.} 70 \text{ } 10s. \text{ } 3\frac{3}{4}d. = 67695$ Farthings.

C. C. qrs. Then $8880 \times 125 = 1110000$ the Divisor.
 $40 : 100 : 8880$ And $67695 \times 40 \times 100 = 270780000$ Divid.
 $- : 125 \quad 67695$ So $270780000 \div 1110000 = 243 \text{ } C. \text{ } 3 \text{ } qrs. \text{ } 21lb. \text{ } 15oz. \text{ } 2\frac{1}{11}\frac{8}{11} \text{ drs.}$

- (5) First, from $125\text{£. } 8s.$ take 120£. Remains $5\text{£. } 8s. = 108s.$ Interest.

£. M. s. Then $120 \times 9 = 1080$ the Divisor.
 $120 : 9 : 108$ And $108 \times 12 \times 100 = 129600s.$ Dividend.
 $100 \quad 12 : -$ Therefore $129600 \div 1080 = 120s.$ or 6£. the Answer.

- (6) First, $100\text{£.} = 24000$ Pence, and $\text{£.} 259 \text{ } 13s. \text{ } 5d. = 62321d.$ then

D. W. £. Here $52 \times 24000 = 1248000$ the Divisor.
 $24000 : 52 : 5$ And $62321 \times 5 \times 20 = 6232100$ £. therefore
 $62321 : 20 : -$ $6232100 \div 1248000 = \text{£.} 4 \text{ } 19s. \text{ } 10\frac{1}{4}d. \text{ } 1\frac{1}{2}\frac{5}{4}\frac{2}{3}$
the Answer.

- (7) First, 8 Months = 32 Weeks, then

Men. W. oz. Here $32 \times 14 = 448$ the Divisor.
 $1400 : 20 : 14$ And $8 \times 20 \times 1400 = 224000$ the Dividend.
 $- : 32 : 8$ Therefore $224000 \div 448 = 500$ Men, Answ.

(8) First, 10 Guineas = 210 Shillings, and 5*£*. = 100*s*. then
M. s. D. Here $100 \times 12 = 1200$ the Divisor.

8 : 5 : 100 And $210 \times 8 \times 5 = 8400$ the Dividend.

: 12 : 210 Therefore $84,00 \div 12,00 = 7$ Men, the Answer.

(9) First, 24 Weeks, 4 Days = 172 Days; then

M. D. £. Here $140 \times 12 = 1680$ the Divisor.

5 : 172 : 140 And $172 \times 5 \times 200 = 172000$ the Dividend.

2 : — : 200 Therefore $17200,0 \div 168,0 = 102\frac{64}{168}$ Days,
 or 3 *Mo.* 2 *W.* $4\frac{64}{168}$ *D.* the Answer.

(10) First, 3*£*. 10*s*. = 70*s*. and 38*£*. 10*s*. = 770*s*. Also 1
 Year = 4 *qrs.* and $1\frac{1}{4} = 5$ *qrs.*

£. qrs. s. Here $70 \times 5 = 350$ the Divisor.

100 : 4 : 70 And $770 \times 4 \times 100 = 308000$ the Dividend.

— : 5 : 770 Therefore $30800,0 \div 35,0 = 880$ *£.* Answer.

(11) First, 36*s.* $\div$ by 12 (the Sacks in a Chaldron) = 3*s.*
 what the Coals cost per Week, and 41 *£.* = 820*s*. then

P. w. s. Here $3 \times 6 = 18$ the Divisor.

7 : 1 : 3 And $820 \times 7 = 5740$ the Dividend.

: 6 : 820 Therefore $5740 \div 18 = 318\frac{8}{9}$ Poor, the Answer.

THE RULE OF THREE REPEATED.

(2) First $18 \times 2 = 36$ Inches, Length of the Lever; then

In. Ft. In. Ft.

Stated thus—As 40 : 104 : 36 : $93\frac{3}{5}$

Feet. Hou. Feet.

Again,—As $93\frac{3}{5} : 13 :: 73 : 10$ Hours, 8 Min. 20 Sec. the
 Answer.

In. lb. In. lb.

(3) Stated thus—As 1 : $1\frac{1}{2} :: 12 : 18$ wt. 12 In. } from the
 Again, As 1 : $1\frac{1}{2} :: 28 : 42$ wt. 28 — } Shoulders

Consequently $42 - 18 = 24$ *lb.* the Answer.

(4) First 30 Inches = $2\frac{1}{2}$ Feet, 24 = 2 Feet, and 4 C. 70 *lb.*
 = 518 *lb.* then $11 - 2\frac{1}{2} = 8\frac{1}{2}$ Feet; also $11 - 2 = 9$.

Feet. lb. Feet. lb.

[mer Case.

Stated thus—As 11 : 518 :: $8\frac{1}{2} : 400\frac{3}{4}$ Force in the for-

And Contra, As $8\frac{1}{2} : 400\frac{3}{4} :: 2\frac{1}{2} : 117\frac{8}{11}$ Pressure.

Again, As 11 : 518 :: 9 : $423\frac{9}{11}$ Force in lat. Ca.

Also, As 9 : $423\frac{9}{11} :: 2 : 94\frac{2}{11}$ Pressure.

(5)

H. D. H. D.

(5) Stated thus—As 12 : 7 :: 16 : $5\frac{1}{4}$ of 16 Hours each.

M. D. M. D. H. M.

Then, As 152 : $5\frac{1}{4}$:: 576 : 19 14 $18\frac{1}{2}$ the Answer.

M. gall. M. gall.

(6) Stated thus—As 9 : 14 :: 31 : $48\frac{2}{9}$ fills in 31 Minutes.

Then $48\frac{2}{9} - 40 = 8\frac{2}{9}$ Gallons in the Tub at the End of 31 Minutes.

And from 2 to 5 = 3 Hours, or 180 Minutes.

M. gal. M. sec.

Again, as 31 $8\frac{2}{9}$:: 180 : $47\frac{2}{3}$ fills in 3 Hours.

And $147 - 47\frac{2}{3} = 99\frac{1}{3}$ Gallons wants of being full.

gal. M. gal. M. sec.

Also, as 14 : 9 :: $99\frac{1}{3}$: 63 $48\frac{2}{3}$ the Tub will be full.

Which added to 5 o'Clock, will give 3 Min. $48\frac{2}{3}$ Sec. after 6, the Tub will be full.

(7) First $1\frac{1}{2}d. = 3$ Halfpence, and 6s. $7\frac{1}{2}d. = 159$.

£. £.

Stated thus—As 3 : 1 :: 159 : $53 = \frac{4}{5}$ of the Rent.

Then $53£. \div 4 = 13£. 5s. = \frac{1}{5}$ of the Rent, or the Answer!

For 53£. added to 13£. 5s. gives 66£. 5s. the whole Rent.

£. s. £. s. s.

Again, if 1 : 4 :: 66 5, or 265 : £. 13 5s. the Answer.

Or, instead of the last Stating, thus; from £. 66 5s. the whole Rent, take $\frac{4}{5}$, viz. 53£. Remains £. 13 5s. the King's Tax, as before.

(8) First, $45 + 13 = 58$ Persons, and 17 Guineas = 357 Shillings.

P. £. P.

Stated thus—If 45 : 20 :: 58 ; $25\frac{7}{9}£.$ in the same Time —

And in twice that Time the 58 Persons will spend $25\frac{7}{9} \times 2 = 51\frac{4}{9}£.$'s Worth, at 17 Guineas per Head.

£. G. s. £. s. d.

Then—If $51\frac{4}{9}$: 17 or 357 :: 63 : $436\frac{2}{3}, 4\frac{1}{6} = £. 21\ 16s.$

$5\frac{3}{4}d. 4\frac{1}{6}$ per Hhd. which $\times$ by 2 = £. 43 12s. $5\frac{3}{4}d. 3\frac{6}{8}$ per Pipe, the Answer.

(9) First $8 - 5 = 3$ Mile B. } gained of A, in one Day,
And $10 - 5 = 5$ — C. }

Then

M. D. M. D.

Then stated thus—If $3 : 1 :: 73 : 24\frac{1}{3}$ when A and B meet.Again, $5 : 1 :: 73 : 14\frac{3}{5}$ ——— A and C ———.

So that B nor C can never meet with A but at the End of these Periods, when A and C will have travelled 219 Miles.

D. D. D. M.

∴ Then, if $14\frac{3}{5} : 219 :: 24\frac{1}{3} : 365$

$$\left. \begin{matrix} 219 \\ 365 \end{matrix} \right\} \times \text{by } \left\{ \begin{matrix} 24\frac{1}{3} \\ 14\frac{3}{5} \end{matrix} \right\} = 5329 \text{ Days, the 73d Time of their}$$

general Meeting ∴ $5329 \div 73 = 73$ Days, their first general Meeting.

M. D. M.

For as $73 : \left\{ \begin{matrix} 24\frac{1}{3} \\ 14\frac{3}{5} \end{matrix} \right\} :: \left\{ \begin{matrix} 219 \\ 365 \end{matrix} \right\} : 73 \text{ Days.}$ (10) First, for every Day he worked he received 12 Pence.
And for every Day he played he paid - 8 —

Sum 20

Likewise, as his idle Days came to the same Money as those he worked, therefore the Proportion will be as follows.

D. D. D.

As $20 : 390 : 8 : 156$ Days he worked.And $20 : 390 : 12 : 234$ ——— played.

For 156, at 12d. per Day, comes to the same Money as 234 at 8d. per Day, viz. £. 7 16s. the Proof.

(11) First, fourscore and eleven, or 91 Guineas = 5733 Four-pences.

And from December 11th to May 20th = 150 Days.

Also, 100 Marks = 4000 Four-pences.

And from September 3d to Christmas = 113 Days.

Then stated thus—If $5733 : 150 :: 4000 : 214\frac{3}{4}\frac{9}{5}$, or rather 215 Days, from which take 113 = 102 Days, and 40£. = 2400 Four-pences.Again, Recep. If $4000 : 102 :: 2400 : 170$ Days, the Answer nearly.(12) First £. 41 8d. = 500 Pence, and $500 \div 20 = 25$ Day's Wages; then $40 - 25 = 15$ Days more.

For every Day he worked he had - 20 Pence.

And for every Day he played, - 10

Sum 30 Pence.

Then

d. D. d. Days.

Then—If $30 : 15 :: \begin{cases} 10 : 5 \text{ worked.} \\ 20 : 10 \text{ idle.} \end{cases}$

$\therefore$ He was idle 10 Days, and worked $(5+25)=30$ Days.

PRACTICE.

CASE I.

$$(1) \begin{array}{r|l} \frac{1}{4} & \frac{1}{4} 2107 \text{ at } \frac{3}{4}d. \\ \hline 12 & 526\frac{3}{4} \\ \hline 2,0 & 4,3 \ 10 \\ \hline & \text{£.} 2 \ 3 \ 10\frac{3}{4} \end{array}$$

$$(2) \begin{array}{r|l} \frac{1}{2} & \frac{1}{2} 1470 \text{ at } \frac{1}{2}d. \\ \hline 12 & 735 \\ \hline 2,0 & 6,1 \ 3 \\ \hline & \text{£.} 3 \ 1 \ 3 \end{array}$$

$$(3) \begin{array}{r|l} \frac{1}{2} & \frac{1}{2} 1276 \text{ at } \frac{3}{4}d. \\ \hline \frac{1}{4} & \frac{1}{2} 638 \\ & 319 \\ \hline 12 & 957 \\ \hline 2,0 & 7,9 \ 9 \\ \hline & \text{£.} 3 \ 19 \ 9 \end{array}$$

CASE II.

$$(4) \begin{array}{r} 12)1762 \text{ at } 1d. \\ \hline 2,0) \ 14,6 \ 10 \\ \hline \text{£.} \ 7 \ 6 \ 10 \end{array}$$

$$(5) \begin{array}{r|l} \frac{1}{4} & \frac{1}{4} 1400 \text{ at } 1\frac{1}{4}d. \\ \hline & 350 \\ \hline 12 & 1750 \\ \hline 2,0 & 145 \ 10 \\ \hline & \text{£.} 7 \ 5 \ 10 \end{array}$$

(6) $d. \begin{array}{l} 1\frac{1}{2} \\ \frac{1}{8} \end{array} \left| \begin{array}{l} 2462 \text{ at } 1\frac{1}{2}d. \\ \hline 2,0 \quad 30,7 \quad 6 \\ \hline \pounds.15 \quad 7 \quad 6 \end{array} \right.$

(7) $d. \begin{array}{l} 1\frac{1}{2} \\ \frac{1}{8} \\ \frac{1}{4} \\ \frac{1}{6} \end{array} \left| \begin{array}{l} 1041 \text{ at } 1\frac{3}{4}d. \\ \hline 130 \quad 1\frac{1}{2} \\ 21 \quad 8\frac{1}{4} \\ \hline 2,0 \quad 15,1 \quad 9\frac{3}{4} \\ \hline \pounds.7 \quad 11 \quad 9\frac{3}{4} \end{array} \right.$

(8) $\begin{array}{l} 2 \\ \frac{1}{6} \end{array} \left| \begin{array}{l} 2490 \text{ at } 2d. \\ \hline 2,0 \quad 41,5 \\ \hline \pounds.20 \quad 15 \end{array} \right.$

(9) $\begin{array}{l} 2 \\ \frac{1}{4} \\ \frac{1}{8} \end{array} \left| \begin{array}{l} 2408 \text{ at } 2\frac{1}{4}d. \\ \hline 401 \quad 4 \\ 50 \quad 2 \\ \hline 2,0 \quad 45,1 \quad 6 \\ \hline \pounds.22 \quad 11 \quad 6 \end{array} \right.$

(10) $\begin{array}{l} 2 \\ \frac{1}{2} \\ \frac{1}{4} \end{array} \left| \begin{array}{l} 640 \text{ at } 2\frac{1}{2}d. \\ \hline 106 \quad 8 \\ 26 \quad 8 \\ \hline 2,0 \quad 13,3 \quad 4 \\ \hline \pounds.6 \quad 13 \quad 4 \end{array} \right.$

(11) $\begin{array}{l} 2 \\ \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{2} \end{array} \left| \begin{array}{l} 1740 \text{ at } 2\frac{3}{4}d. \\ \hline 290 \\ 72 \quad 6 \\ 36 \quad 3 \\ \hline 2,0 \quad 39,8 \quad 9 \\ \hline \pounds.19 \quad 18 \quad 9 \end{array} \right.$

(12) $\begin{array}{l} 3 \\ \frac{1}{4} \end{array} \left| \begin{array}{l} 746 \text{ at } 3d. \\ \hline 2,0 \quad 18,6 \quad 6 \\ \hline \pounds.9 \quad 6 \quad 6 \end{array} \right.$

(13) $\begin{array}{l} 3 \\ \frac{1}{4} \\ \frac{1}{12} \end{array} \left| \begin{array}{l} 1417 \text{ at } 3\frac{1}{4}d. \\ \hline 354 \quad 3 \\ 29 \quad 6\frac{1}{4} \\ \hline 2,0 \quad 38,3 \quad 9\frac{1}{4} \\ \hline \pounds.19 \quad 3 \quad 9\frac{1}{4} \end{array} \right.$

(14) $\begin{array}{l} 3 \\ \frac{1}{2} \\ \frac{1}{6} \end{array} \left| \begin{array}{l} 3091 \text{ at } 3\frac{1}{2}d. \\ \hline 772 \quad 9 \\ 128 \quad 9 \\ \hline 2,0 \quad 90,1 \quad 6\frac{1}{2} \\ \hline \pounds.45 \quad 1 \quad 6\frac{1}{2} \end{array} \right.$

(15) $d.$

3	$\frac{1}{4}$	214 at $3\frac{3}{4}d.$
$\frac{3}{4}$	$\frac{1}{4}$	53 6
		13 $4\frac{1}{2}$
2,0		6,6 $10\frac{1}{2}$
		<u>£.3 6 $10\frac{1}{2}$</u>

(16) $d.$

4	$\frac{1}{3}$	2000 at $4d.$
2,0		66,6 8
		<u>£.33 6 8</u>

(17) $d.$

3	$\frac{1}{4}$	569 at $4\frac{1}{4}d.$
1	$\frac{1}{3}$	142 3
		47 5
2,0		11 $10\frac{1}{4}$
		20,1 $6\frac{1}{4}$
		<u>£.10 1 $6\frac{1}{4}$</u>

(18) $d.$

3	$\frac{1}{4}$	1246 at $4\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{2}$	311 6
		155 9
2,0		46,7 3
		<u>£.23 7 3</u>

(19) $d.$

3	$\frac{1}{4}$	1426 at $4\frac{3}{4}d.$
1	$\frac{1}{3}$	356 6
$\frac{3}{4}$	$\frac{1}{4}$	118 10
		89 $1\frac{1}{2}$
2,0		56,4 $5\frac{1}{2}$
		<u>£.28 4 $5\frac{1}{2}$</u>

(20) $d.$

4	$\frac{1}{3}$	2740 at $5d.$
1	$\frac{1}{4}$	913 4
		228 4
2,0		114,1 8
		<u>£.57 1 8</u>

(21) $d.$

4	$\frac{1}{3}$	2147 at $5\frac{1}{4}d.$
1	$\frac{1}{4}$	715 8
$\frac{1}{4}$	$\frac{1}{4}$	178 11
		44 $8\frac{3}{4}$
2,0		9339 $3\frac{3}{4}$
		<u>£.46 19 $3\frac{3}{4}$</u>

(22) $d.$

4	$\frac{1}{3}$	674 at $5\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{8}$	224 8
		84 3
2,0		30,8 11
		<u>£.15 8 11</u>

(23) $d.$

3	$\frac{1}{4}$	1746 at $5\frac{3}{4}d.$
2	$\frac{1}{6}$	436 6
$\frac{3}{4}$	$\frac{1}{4}$	295 0
		109 $1 = \frac{1}{4}$ of
2,0		83,6 $7\frac{1}{2}$
		<u>£.41 16 $7\frac{1}{2}$</u>

(24) $d.$

6	$\frac{1}{2}$	1741 at $6d.$
2,0		87,0 6
		<u>£.43 10 6</u>

(25) $d.$

4	$\frac{1}{3}$	2142 at $6\frac{1}{4}$
2	$\frac{1}{2}$	714
$\frac{1}{4}$	$\frac{1}{8}$	357
		44 $7\frac{1}{2}$
2,0		111,5 $7\frac{1}{2}$
		<u>£.55 15 $7\frac{1}{2}$</u>

(26) $d.$

6	$\frac{1}{2}$	1040 at $6\frac{1}{2}d.$
$\frac{1}{2}$	$\frac{1}{2}$	520
		43 4
2,0		56,3 4
		<u>£.28 3 4</u>

(27) $d.$

6	$\frac{1}{2}$	1746 at $6\frac{3}{4}d.$
$\frac{3}{4}$	$\frac{1}{8}$	873
		109 $1\frac{1}{2}$
2,0		98,2 $1\frac{1}{2}$
		<u>£.49 2 $1\frac{1}{2}$</u>

(28) $d.$

4	$\frac{1}{3}$	1000 at $7d.$
3	$\frac{1}{4}$	333 4
		250 0
2,0		58,3 4
		<u>£.29 3 4</u>

(29) $d.$

6	$\frac{1}{2}$	1656 at $7\frac{1}{4}d.$
1	$\frac{1}{6}$	828
$\frac{1}{4}$	$\frac{1}{4}$	138
		34 6
2,0		100,0 6
		<u>£.50 0 6</u>

(30) $d.$

6	$\frac{1}{2}$	1420 at $7\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{4}$	710
		177 6
2,0		88,7 6
		<u>£.44 7 6</u>

(31) $d.$

4	$\frac{1}{3}$	674 at $7\frac{3}{4}d.$
3		224 8
	$\frac{1}{4}$	168 6
$\frac{3}{4}$	$\frac{1}{4}$	42 $1\frac{1}{2} = \frac{1}{4}$ of $3d.$
		<u>2,0</u> +3,5 $3\frac{1}{2}$
		<u>£ 21 15 $3\frac{1}{2}$</u>

(32) $d.$

6	$\frac{1}{2}$	2170 at $8d.$
2	$\frac{1}{3}$	1085
		361 8
		<u>2,0</u> 144,6 8
		<u>£.72 6 8</u>

(33) $d.$

6	$\frac{1}{2}$	1700 at $8\frac{1}{4}d.$
2	$\frac{1}{3}$	850
$\frac{1}{4}$	$\frac{1}{8}$	283 4
		35 5
		<u>2,0</u> 116,8 9
		<u>£.58 8 9</u>

(34) $d.$

6	$\frac{1}{3}$	1765 at $8\frac{1}{2}d.$
2	$\frac{1}{3}$	882 6
$\frac{1}{2}$	$\frac{1}{4}$	294 2
		73 $6\frac{1}{2}$
		<u>2,0</u> 125,0 $2\frac{1}{2}$
		<u>£.62 10 $2\frac{1}{2}$</u>

(35) $d.$

6	$\frac{1}{2}$	749 at $8\frac{3}{4}d.$
2	$\frac{1}{3}$	374 6
		124 10
$\frac{1}{3}$	$\frac{1}{6}$	46 $9\frac{1}{4} = \frac{1}{3}$ of $6d.$
		<u>2,0</u> 54,6 $1\frac{3}{4}$
		<u>£.27 6 $1\frac{3}{4}$</u>

(36) $d.$

6	$\frac{1}{2}$	1417 at $9d.$
3	$\frac{1}{2}$	708 6
		354 3
		<u>2,0</u> 106,2 9
		<u>£.53 2 9</u>

(37) $d.$

6	$\frac{1}{2}$	2373 at $9\frac{1}{4}d.$
3	$\frac{1}{2}$	1186 6
$\frac{1}{4}$	$\frac{1}{2}$	593 3
		49 $5\frac{1}{4}$
		<u>2,0</u> 182,9 $2\frac{1}{4}$
		<u>£.91 9 $2\frac{1}{4}$</u>

(38) $d.$

6	$\frac{1}{2}$	476 at $9\frac{1}{2}d.$
2	$\frac{1}{3}$	738
		246
$1\frac{1}{2}$	$\frac{1}{4}$	184 $6 = \frac{1}{4}$ of $6d.$
		<u>2,0</u> 116,8 6
		<u>£.58 8 6</u>

(39) $d.$

6	$\frac{1}{2}$	1760 at $9\frac{1}{4}d.$
3	$\frac{1}{2}$	880
$\frac{3}{4}$	$\frac{1}{4}$	440
		110
	2,0	143,0
		£.71 10

(40) $d.$

6	$\frac{1}{2}$	6000 at $10d.$
		3000
4	$\frac{1}{3}$	2000
	2,0	500,0
		£.250

(41) $d.$

6	$\frac{1}{2}$	1652 at $10\frac{1}{4}d.$
3	$\frac{1}{2}$	826
1	$\frac{1}{2}$	826
	$\frac{1}{4}$	413
		387 8
		96 11
	2,0	397,3 7
		£.198 13 7

(42) $d.$

6	$\frac{1}{2}$	2476 at $10\frac{1}{2}d.$
3	$\frac{1}{2}$	1238
$1\frac{1}{2}$	$\frac{1}{2}$	619
		309 6
	2,0	216,6 6
		£.108 6 6

(43) $d.$

6	$\frac{1}{2}$	2176 at $10\frac{1}{4}d.$
		1088
4	$\frac{1}{3}$	725 $4 = \frac{1}{3}$ of $1s.$
$\frac{3}{4}$	$\frac{1}{8}$	136 $0 = \frac{1}{8}$ of $6d.$
	2,0	194,9 4
		£.97 9 4

(44) $d.$

6	$\frac{1}{2}$	1276 at $11d.$
3	$\frac{1}{2}$	638
		319
2	$\frac{1}{3}$	212 $8 = \frac{1}{3}$ of $6d.$
	2,0	116,9 8
		£.58 9 8

(45) $d.$

6	$\frac{1}{2}$	2142 at $11\frac{1}{4}d.$
		1071
4	$\frac{1}{3}$	714
1	$\frac{1}{4}$	178 6
	$\frac{1}{4}$	44 $7\frac{1}{2}$
	2,0	200,8 $1\frac{1}{2}$
		£.100 8 $1\frac{1}{2}$

(46) $d.$

6	$\frac{1}{2}$	4760 at $11\frac{1}{2}d.$
		2380
4	$\frac{1}{3}$	586 8
$1\frac{1}{2}$	$\frac{1}{4}$	595 $0 = \frac{1}{4}$ of $6d.$
	2,0	456,1 8
		£.228 1 8

(47)

$$\begin{array}{r|l}
 \text{(47)} & d. \\
 6 & \frac{1}{2} \mid 640 \text{ at } 11\frac{1}{4}d. \\
 3 & \frac{1}{2} \mid 320 \\
 & 160 \\
 2 & \frac{1}{3} \mid 106 \quad 8 = \frac{1}{3} \text{ of } 6d. \\
 \frac{3}{4} & \frac{1}{4} \mid 40 \quad 0 = \frac{1}{4} \text{ of } 3d. \\
 \hline
 & 2,062,6 \quad 8 \\
 \hline
 & \text{£. } 31 \quad 6 \quad 8
 \end{array}$$

When the Price is 2, 3, 4, 6, or 8d.—Some Arithmeticians make Use of this Method.—Thus; for 2d. they divide the given Quantity by 120; for 3d. by 80; for 4d. by 60; for 6d. by 40, and for 8d. by 30, which gives the Answer in Pounds. But this Method I do not approve of, for young Beginners, because the Remainder may sometimes be very large, and therefore to value it, will be a greater Task to them, and take more Time than in doing the whole Work; otherwise.

CASE III.

$$\begin{array}{r|l}
 \text{(49)} & d. \\
 \frac{1}{4} & \frac{1}{4} \mid 2140 \text{ at } 1s. \frac{1}{4}d. \\
 12 & 535 \\
 \hline
 & 44 \quad 7 \text{ added to } [2140 \\
 \hline
 2,0 & 218,4 \quad 7 \\
 \hline
 & \text{£. } 109 \quad 4 \quad 7
 \end{array}
 \quad
 \begin{array}{r|l}
 \text{(50)} & d. \\
 6 & \frac{1}{2} \mid 1749 \text{ at } 1s. 11\frac{1}{2}d. \\
 1\frac{1}{2} & \frac{1}{4} \mid 874 \quad 6 \\
 4 & \frac{1}{3} \mid 583 \quad 0 \\
 & 218 \quad 7\frac{1}{2} \\
 \hline
 2,0 & 342,5 \quad 1\frac{1}{2} \\
 \hline
 & \text{£. } 171 \quad 5 \quad 1\frac{1}{2}
 \end{array}$$

$$\begin{array}{r|l}
 \text{(51)} & d. \\
 4 & \frac{1}{3} \mid 2140 \text{ at } 1s. 5d. \\
 1 & \frac{1}{4} \mid 713 \quad 4 \\
 & 178 \quad 4 \\
 \hline
 2,0 & 303,1 \quad 8 \\
 \hline
 & \text{£. } 151 \quad 11 \quad 8
 \end{array}
 \quad
 \begin{array}{r|l}
 \text{(52)} & d. \\
 6 & \frac{1}{2} \mid 1453 \text{ at } 1s. 7\frac{1}{2}d. \\
 1\frac{1}{2} & \frac{1}{4} \mid 726 \quad 6 \\
 & 181 \quad 7\frac{1}{2} \\
 \hline
 2,0 & 236,1 \quad 1\frac{1}{2} \\
 \hline
 & \text{£. } 118 \quad 1 \quad 1\frac{1}{2}
 \end{array}$$

(53) $6 \overline{) 1614}$ at 1s. 10d. $\frac{1}{2}$ 807 $4 \overline{) 538}$ $\frac{1}{3}$ <u>2,0</u> 295,9 <u>£. 147 19</u>	(54) $6 \overline{) 2647}$ at 1s. 11$\frac{3}{4}$d. $\frac{1}{2}$ 1323 6 $\frac{1}{2}$ 661 9 $2 \overline{) 441}$ $2 = \frac{1}{2}$ of 6d $\frac{1}{3}$ 165 5$\frac{1}{4}$ $\frac{1}{4}$ <u>2,0</u> 523,8 10$\frac{1}{4}$ <u>£. 261 18 10$\frac{1}{4}$</u>
---	---

The 53d Example may be performed thus — 1s. 8d. is $\frac{1}{12}$ of a Pound, 2d. is $\frac{1}{60}$ of 1s. 8d.

C A S E IV.

(55) 2476 at 2s.

$\frac{1}{2}$

£. 247 12

(56) 1476 at 4s.

$\frac{1}{4}$

£. 295 4

(57) 276 at 6s.

$\frac{1}{3}$

£. 82 16

(58) 2100 at 8s.

$\frac{1}{8}$

£. 840 0

(59) 274 at 10s.

$\frac{1}{5}$

£. 137 0

(60) 674 at 12s.

$\frac{1}{6}$

£. 404 8

(61) 2680 at 14s.

$\frac{1}{7}$

£. 1876 0

(62) 267 at 16s.

$\frac{1}{8}$

£. 213 12

(63) 1267 at 18s.

$\frac{1}{9}$

£. 1140 6

The 55, 56, and 59th Examples may be performed thus : 2s. is $\frac{1}{10}$, 4s. is $\frac{1}{5}$, and 10s. is $\frac{1}{2}$ of a Pound, therefore the given Quantities being divided by 10, 5, and 2, will give the Answer as above.

C A S E V.

$$(66) \quad 2174 \text{ at } 7s. \\ \underline{7}$$

$$2,0)1521,8$$

$$\underline{\text{£. } 760 \ 18}$$

$$(67) \quad 1427 \text{ at } 9s. \\ \underline{9}$$

$$2,0)1284,3$$

$$\underline{\text{£. } 642 \ 3}$$

$$(68) \quad 647 \text{ at } 11s. \\ \underline{11}$$

$$2,0)711,7$$

$$\underline{\text{£. } 355 \ 17}$$

$$(69) \quad 267 \text{ at } 13s. \\ \underline{13}$$

$$2,0)347,1$$

$$\underline{\text{£. } 173 \ 11}$$

$$(70) \quad 274 \text{ at } 17s. \\ \underline{17}$$

$$2,0)465,8$$

$$\underline{\text{£. } 232 \ 18}$$

$$(71) \quad 1260 \text{ at } 19s. \\ \underline{19}$$

$$2,0)2394,0$$

$$\underline{\text{£. } 1197 \ 0}$$

C A S E VI.

$$(72) \quad 4s. \frac{1}{5})2420 \text{ at } 4s.$$

$$\underline{\text{£. } 484}$$

$$(73) \quad 5s. \frac{1}{4})1764 \text{ at } 5s.$$

$$\underline{\text{£. } 441}$$

$$(74) \quad 1s. 8d. \frac{1}{2})4762 \text{ at } 1s. 8d. \quad (75) \quad 2s. 6d. \frac{1}{8})467 \text{ at } 2s. 6d.$$

$$\underline{\text{£. } 396 \ 16 \ 8}$$

$$\underline{\text{£. } 58 \ 7 \ 6}$$

$$(76) \quad 3s. 4d. \frac{1}{6})1760 \text{ at } 3s. 4d. \quad (77) \quad 6s. 8d. \frac{1}{3})176 \text{ at } 6s. 8d.$$

$$\underline{\text{£. } 293 \ 6 \ 8}$$

$$\underline{\text{£. } 58 \ 13 \ 4}$$

C A S E VII.

$$(78) \quad \begin{array}{l} d. \\ 3 \end{array} \quad \frac{1}{4} \quad 1420 \text{ at } 3s. 3d.$$

$$\underline{3}$$

$$4260$$

$$355$$

$$2,0)461,5$$

$$\underline{\text{£. } 230 \ 15}$$

$$(79) \quad \begin{array}{l} d. \\ 6 \end{array} \quad \frac{1}{2} \quad 427 \text{ at } 5s. 9d.$$

$$\underline{5}$$

$$2135$$

$$213 \ 6$$

$$106 \ 9$$

$$2,0)245,5 \ 3$$

$$\underline{\text{£. } 122 \ 15 \ 3}$$

(80) $d.$ <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px; text-align: right;">6</td> <td style="padding-left: 10px;">$\frac{1}{2}$ 402 at 10s. $8\frac{3}{4}d.$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; text-align: right;">10</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">4020</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px; text-align: right;">2</td> <td style="padding-left: 10px;">$\frac{1}{3}$ 201</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; text-align: right;">67</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px; text-align: right;">$\frac{3}{4}$</td> <td style="padding-left: 10px;">$\frac{1}{8}$ 25 $1\frac{1}{2} = \frac{1}{8}$ of 6d.</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">2,0 431,3 $1\frac{1}{2}$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">£.215 13 $1\frac{1}{2}$</td> </tr> </table>	6	$\frac{1}{2}$ 402 at 10s. $8\frac{3}{4}d.$		10		4020	2	$\frac{1}{3}$ 201		67	$\frac{3}{4}$	$\frac{1}{8}$ 25 $1\frac{1}{2} = \frac{1}{8}$ of 6d.		2,0 431,3 $1\frac{1}{2}$		£.215 13 $1\frac{1}{2}$	(81) $d.$ <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding-right: 10px; text-align: right;">6</td> <td style="padding-left: 10px;">$\frac{1}{2}$ 174 at 17s. $9\frac{1}{2}d.$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; text-align: right;">17</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">2958</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px; text-align: right;">3</td> <td style="padding-left: 10px;">$\frac{1}{2}$ 87</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; text-align: right;">$\frac{1}{6}$ 43 6</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; text-align: right;">7 3</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">2,0 309,5 9</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 10px;"></td> <td style="padding-left: 10px; border-top: 1px solid black; text-align: right;">£.154 15 9</td> </tr> </table>	6	$\frac{1}{2}$ 174 at 17s. $9\frac{1}{2}d.$		17		2958	3	$\frac{1}{2}$ 87		$\frac{1}{6}$ 43 6		7 3		2,0 309,5 9		£.154 15 9
6	$\frac{1}{2}$ 402 at 10s. $8\frac{3}{4}d.$																																
	10																																
	4020																																
2	$\frac{1}{3}$ 201																																
	67																																
$\frac{3}{4}$	$\frac{1}{8}$ 25 $1\frac{1}{2} = \frac{1}{8}$ of 6d.																																
	2,0 431,3 $1\frac{1}{2}$																																
	£.215 13 $1\frac{1}{2}$																																
6	$\frac{1}{2}$ 174 at 17s. $9\frac{1}{2}d.$																																
	17																																
	2958																																
3	$\frac{1}{2}$ 87																																
	$\frac{1}{6}$ 43 6																																
	7 3																																
	2,0 309,5 9																																
	£.154 15 9																																

(82)

$3d$	$\frac{1}{4}$ 273 at 19s. $4\frac{3}{4}d.$
	19
	5187
1	$\frac{1}{3}$ 68 3
	22 9
$\frac{3}{4}$	$\frac{1}{4}$ 17 $0\frac{3}{4} = \frac{1}{4}$ of 3d.
	2,0 5295 $0\frac{3}{4}$
	£.264 15 $0\frac{3}{4}$

(83)

$6d$	$\frac{1}{2}$ 260 at 14s. $11\frac{3}{4}d.$
	14
	3640 = 14s.
3	$\frac{1}{2}$ 130 = 6d.
	65 = 3
2	$\frac{1}{3}$ 43 4 = 2
	$\frac{3}{4}$ $\frac{1}{4}$ 16 3 = $\frac{1}{4}$ of 3d.
	2,0 3894 7
	£.194 14 7

Example 78. may be performed thus—2s. is $\frac{1}{10}$, 1s. is $\frac{1}{2}$ of 2s. and 3d. is $\frac{1}{4}$ of 1s.—Example 79. thus; 5s. are $\frac{1}{4}$, 6d. is $\frac{1}{10}$ of 5s. and 3d. is $\frac{1}{2}$ of 6d. Likewise, Example 80. thus; 10s. is $\frac{1}{2}$ of a Pound, 6d. is $\frac{1}{20}$ of 10s. 2d. is $\frac{1}{3}$ of 6d. and $\frac{3}{4}$ is $\frac{1}{8}$ of 6d. which Parts added together, will give the Answers as before.

C A S E VIII.

(84) 120 at 4£.

4
£.480

(85) 96 at 17£.

17
£.1632

(86)

(86) 100 at 3 £.

$$\begin{array}{r} 3 \\ \hline \text{£. } 300 \\ \hline \end{array}$$

(87) 142 at 42 £.

$$\begin{array}{r} 42 = 6 \times 7 \\ \hline \text{£. } 5964 \\ \hline \end{array}$$

C A S E IX.

(88) 649 at 2 £. 6s.

$$\begin{array}{r} 2 \\ \hline 1298 \\ 194 \text{ } 14 \\ \hline \text{£. } 1492 \text{ } 14 \\ \hline \end{array}$$

(89) 526 at 7 £. 16s.

$$\begin{array}{r} 7 \\ \hline 3682 \\ 420 \text{ } 16 \\ \hline \text{£ } 4102 \text{ } 16 \\ \hline \end{array}$$

(90) 142 at 1 £. 17s.

$$\begin{array}{r} 17 \\ \hline 2,0)241,4 \\ \hline 120 \text{ } 14 \text{ added to } 142 \text{ £.} \\ \hline \text{£. } 262 \text{ } 14 \\ \hline \end{array}$$

(91) 164 at 24 £. 19s.

$$\begin{array}{r} 20 \\ \hline 24 \text{ £. } 19 = 499 \text{ £.} \\ 164 \\ \hline 1996 \\ 7984 \\ \hline 2,0)8183,6 \\ \hline \text{£. } 4091 \text{ } 16 \\ \hline \end{array}$$

(92) 271 at 5 £. 7s.

$$\begin{array}{r} 5 \\ \hline 1355 \text{ £.} \\ \hline 2,0)189,7 = 271 \times 7 \\ \hline 94 \text{ } 17 \text{ added to } 1355 \text{ £.} \\ \hline \text{£. } 1449 \text{ } 17 \\ \hline \end{array}$$

(93) 5 | $\frac{1}{4}$ | 604 at 20 £. 9s.

$$\begin{array}{r} 20 \\ \hline 12080 \\ 151 \\ 4 \left| \frac{1}{5} \right| 120 \text{ } 16 \\ \hline \text{£. } 12351 \text{ } 16 \\ \hline \end{array}$$

(94)

(94) 10s. $\frac{1}{2}$ | 914 at 10 £. 15s.

10	
9140	
5 $\frac{1}{2}$	457
	228 10
£. 9825 10	

(95) 737 at 1 £. 14s.

7	
512	18 added to
[737 £.	
£. 1249	18

C A S E X.

(96) 6s. 8d. $\frac{1}{3}$ | 274 at 7 £. 6s. 8d.

7	
1918	
91	6 8
£. 2009 6 8	

(97) 3s. 4d. $\frac{1}{6}$ | 120 at 12 £. 3 4

12	
1440	
20	
£. 1460	

(98) 1s. 8d. $\frac{1}{12}$ | 97 at 9 £. 1s. 8d.

9	
873	
8	1 8
£. 881 1 8	

(99) 5s. $\frac{1}{4}$ | 512 at 4 £. 5s.

42	
21504	
128	
£. 21632	

C A S E XI.

(100) 1472 at £. 4 6 $7\frac{1}{2}$

4	
5888	
441	12
36	16
9	4
£. 6375 12	

(101) 10s. $\frac{1}{2}$ | 279 at £. 6 11 $9\frac{1}{4}$

6	
1674	0
139	10
13	19
6	19 6
3	9 9
3	9 9
£. 1837 15 $8\frac{1}{4}$	

(102)

10	$\frac{1}{2}$	1420 at £. 19 14 11 $\frac{3}{4}$
		<u>19</u>
		26980
		710
4	$\frac{1}{5}$	284
6	$\frac{1}{8}$	35 10 = $\frac{1}{8}$ of 4s.
3	$\frac{1}{2}$	17 15 = $\frac{1}{2}$ of 6
2	$\frac{1}{3}$	11 16 8
$\frac{3}{4}$	$\frac{1}{4}$	4 8 9
		<u>£. 28043 10 5</u>

(103)

4	$\frac{1}{3}$	2074 at £. 1 17 5 $\frac{1}{2}$
		<u>37</u>
		76738
		691 4
1	$\frac{1}{8}$	259 3
2,0		<u>7768,8 7</u>
		<u>£. 3884 8 7</u>

(104) £. s. d.

27 at 4 11 8 $\frac{1}{4}$

9 × 3 = 27

41 5 6 $\frac{1}{4}$
3

£. 123 16 8 $\frac{1}{4}$

(105) £. s. d.

64 at 12 13 7 $\frac{1}{4}$

8 × 8 = 64

101 8 10
8

£. 811 10 8

CASE XII.

(106) 10C. 3 qrs. 14 lb. at 2 11 10 $\frac{1}{2}$

		£. s. d.
		10
2	$\frac{1}{2}$	25 18 9
1	$\frac{1}{2}$	1 5 11 $\frac{1}{4}$
14	$\frac{1}{2}$	12 11 $\frac{1}{2}$
		6 5 $\frac{3}{4}$
		<u>£. 28 4 1$\frac{1}{2}$</u>

(107)

(107)

qrs. lb. 72 C. 3 qrs. 19 lb. at

£. s. d.

$$2 \quad \begin{array}{r} \frac{1}{2} \\ 3 \quad 17 \quad 4\frac{1}{2} \\ 9 \times 8 = 72 \end{array}$$

$$\begin{array}{r} 34 \quad 16 \quad 4\frac{1}{2} \\ 8 \end{array}$$

$$278 \quad 11 \quad 0$$

$$1 \quad 18 \quad 8\frac{1}{4}$$

$$16 \quad \frac{1}{7} \quad 19 \quad 4$$

$$2 \quad 11 \quad 0\frac{1}{2}$$

$$1 \quad \frac{1}{2} \quad 1 \quad 4\frac{1}{2}$$

$$8\frac{1}{4}$$

$$\underline{\underline{\text{£. 282} \quad 2 \quad 1\frac{1}{2}}}$$

(108)

lb. 12 C. 1 qr. 17 lb. at

£. s. d.

$$1 \quad \frac{1}{4} \quad 4 \quad 4 \quad 4$$

$$12$$

$$0 \quad 12 \quad 0$$

$$14 \quad \frac{1}{2} \quad 1 \quad 1 \quad 1$$

$$2 \quad \frac{1}{7} \quad 10 \quad 6\frac{1}{2}$$

$$1 \quad \frac{1}{2} \quad 1 \quad 6$$

$$9$$

$$\underline{\underline{\text{£. 52} \quad 5 \quad 10\frac{1}{2}}}$$

(109)

lb. 23 C. 18 lb. at

£. s. d.

$$16 \quad \frac{1}{7} \quad 4 \quad 14 \quad 1\frac{1}{2}$$

$$11 \times 2 + 1 = 23$$

$$49 \quad 15 \quad 4\frac{1}{2}$$

$$2$$

$$99 \quad 10 \quad 9$$

$$2 \quad \frac{1}{8} \quad 4 \quad 14 \quad 1\frac{1}{2}$$

$$13 \quad 5\frac{1}{4}$$

$$1 \quad 8$$

$$\underline{\underline{\text{£. 104} \quad 19 \quad 11\frac{3}{4}}}$$

(110)

lb. 94 C. 27 lb. at

£. s. d.

$$16 \quad \frac{1}{7} \quad 6 \quad 19 \quad 6$$

$$10 \times 9 + 4 = 94$$

$$69 \quad 15 \quad 0$$

$$9$$

$$627 \quad 15 \quad 0$$

$$27 \quad 18 \quad 0$$

$$8 \quad \frac{1}{2} \quad 19 \quad 11$$

$$2 \quad \frac{1}{4} \quad 9 \quad 11\frac{1}{2}$$

$$1 \quad \frac{1}{2} \quad 2 \quad 5\frac{3}{4}$$

$$1 \quad 2\frac{3}{4}$$

$$\underline{\underline{\text{£. 657} \quad 6 \quad 7}}$$

(111)

16C. 2qrs. at

qrs.	£.	s.	d.
2	$\frac{1}{2}$	2	6 11
			$4 \times 4 = 16$
			9 7 8
			4
			37 10 8
			1 3 $5\frac{1}{2}$
			<u>£. 38 14 $1\frac{1}{2}$</u>

(112)

48C. 2qrs. 7lb. at

qrs.	£.	s.	d.
2qrs.	$\frac{1}{2}$	74	16 6
			$8 \times 6 = 48$
			598 12 0
			6
			3691 12 0
			37 8 3
			4 13 $6\frac{1}{4}$
			<u>£. 3733 13 $9\frac{1}{4}$</u>

(113) lb. 24lb. at

lb.	£.	s.
16	$\frac{1}{7}$	4 17 per C.
		8 $\frac{1}{2}$ 0 13 $10\frac{1}{4}$
		0 6 11
		<u>£. 1 0 $9\frac{1}{4}$</u>

(114)

17lb. at

lb.	£.	s.	d.
14	$\frac{1}{8}$	3	5 4
			2 $\frac{1}{7}$ 0 8 2
			1 $\frac{1}{2}$ 0 1 2
			0 0 7
			<u>£. 0 9 11</u>

When the Pupil is perfect in all the Rules to the foregoing Cases, he may then learn the Contractions, some of which are as follow.

CONTRACTIONS to CASE I. and II.

2	$\frac{1}{80}$	1276 at $\frac{3}{4}d.$
		15 19
		<u>£. 3 19 9</u>

See Example 3.

3	$\frac{2}{80}$	174,0 at $2\frac{3}{4}d.$
		21 15 0
		1 16 3
		} subtract.
		<u>£. 19 18 9</u>

See Example 11.

*s. d.*2 6 | $\frac{1}{8}$ | 1420 at $7\frac{1}{2}d.$ 7 $\frac{1}{2}$ | $\frac{1}{4}$ | 177 10£. 44 7 6

See Ex. 30.

*s. d.*6 0 | $\frac{3}{10}$ | 1417 at $9d.$

3—See Case 4.

9 | $\frac{1}{8}$ | 425 2£. 53 2 9

See Ex. 36.

CONTRACTIONS TO CASE III. VII. and IX.

*s. d.*5 | $\frac{1}{4}$ | 246 at $15d.$ 1 3 | $\frac{1}{4}$ | 61 10£. 15 7 6*s. d.*1 8 | $\frac{1}{12}$ | 3162 at $1s. 7d.$ 1 | $\frac{1}{20}$ | 263 10 0 } subtract.£. 250 6 6*s. 6d.*6 | $\frac{3}{10}$ | 427 at $5s. 9d.$

3

3 | $\frac{1}{80}$ | 128, 2 0 } subtract.£. 122 15 618s | $\frac{9}{10}$ | 246 at $17s. 10d.$

9

| $\frac{1}{20}$ | 221 8 } subtract.£. 219 72d | $\frac{1}{120}$ | 241 at £. 6 19 10

7

1687
2 0 2 } subtract.£. 1684 19 105s. | $\frac{1}{4}$ | 641 at £. 1 15

2

1282 0 } subtract.

£. 1121 15

18. T A R E and T R E T T.

- (2) First, $6C. 2qrs. 17lb. \times 6 \times 4$ (24 the No. of Hhds.) = $C. 159 2qrs. 16lb.$ Gross, from which take $C. 17 3qrs. 27lb.$ Remains $C. 141 2qrs. 17lb.$ Neat, the Answer.
- (3) Gross $C. 23 2qrs. 19lb.$; Tare $C. 1 12lb.$; Difference, $C. 22 2qrs. 7lb.$ the Neat required.
- (5) First, $C. 2 3qrs. \times 8 = 22C.$ Gross; and $22lb. \times 8 = C. 1 2qrs. 8lb.$; then from $22C.$ take $C. 1 2qrs. 8lb.$ Remains $C. 20 1qr. 20lb.$ Neat, the Answer.
- (6) First, $C. 3 0qr. 27lb. \times 8 \times 5$ (40 the Number of Bales) = $C. 129 2qrs. 16lb.$ Gross, and $18lb. \times 8 \times 5 = C. 6. 1qr. 20lb.$ Tare; then from $C. 129 2qrs. 26lb.$ take $C. 6 1qr. 20lb.$ Remains $C. 123 0qr. 24lb.$ Neat, the Answer.
- (8) First, $C. 2 1qr 17lb. \times 2 \times 11$ (22 Barrels) = $C. 52 3qrs. 10lb.$ Gross, which $\div$ by 8, (because $14lb.$ is $\frac{1}{8}$ of $1Cwt.$) gives $C. 6 2qrs. 11lb. 12oz.$ Tare, which take from the Gross, will remain $C. 46 0qr. 14lb. 4oz.$ Neat.
- (9) First, $C. 6 2qrs. 12lb. \times 9 = C. 59 1qr. 24lb.$ Gross; then $14lb.$ is $\frac{1}{8}$, viz. $C. 7 1qr. 20lb. 8oz.$ and $2lb. \frac{1}{7}$ of $14lb.$ viz. $C. 1 0qr. 6lb. 14oz.$ Also, $1lb.$ is $\frac{1}{2}$ of $2lb.$ viz. $2qrs. 2lb. 7oz.$ which Parts, added together = $C. 9 0qr. 2lb. 13oz.$ Tare, which taken from the Gross, viz. $C. 59 1qr. 24lb. = C. 50 1qr. 21lb. 3oz.$ Neat, the Answer.
- (11) First, from $C. 16 3qrs.$ take $C. 1 1qr. 12lb.$; remains $C. 15 1qr. 16lb.$ Suttle, which, $\div$ by 26, ($4lb.$ being $\frac{1}{26}$ of 104) gives $2qrs. 10lb. 4oz.$ Trett; this, taken from the Suttle, leaves $C. 14 3qrs. 5lb. 12oz.$ Neat, the Answer.
- (12) First, $C. 2 3qrs. 27lb. \times 9 \times 3$ (27 Bags) = $C. 80 3qrs. 1lb.$ Gross, then as $13lb.$ cannot be conveniently divided into aliquot Parts of 112, I take for $16lb.$ which is $\frac{1}{7}$; therefore I $\div$ the Gross by 7, which gives $C. 11 2qrs. 4lb. 2oz.$ Tare, supposing it had been $16lb.$ per C. from which I take Parts with the $13lb.$ viz. $8lb.$ is $\frac{1}{2}$ of $16lb.$ viz. $C. 5 3qrs. 2lb. 10oz.$ and 4 is $\frac{1}{2}$ of 8, viz. $C. 2 2qrs. 3lb. 15oz.$ Also, $1lb.$ is $\frac{1}{4}$ of 4, viz. $2qrs. 24lb. 12oz.$; these, added together, = $C. 9 1qr. 13lb. 13oz.$ Tare, which, taken from the Gross, = $C. 71 1qr. 15lb. 3oz.$ Suttle; this $\div$ by 26 = $C. 2 2qrs. 27lb. 8oz.$ Trett, which taken from the Suttle, leaves $C. 68 2qrs. 15lb. 11oz.$ Neat, the Answer.

14) First, C. 42 1qr. = 172 qrs. these $\div$ by 168 = 1qr. $\frac{1}{2}$ lb. Cloff, which taken from C. 42 3qrs. = C. 42 1qr. 27 $\frac{1}{2}$ lb. Neat, the Answer.

(16) First, the 4 Hogheads added together = C. 20 3qrs. 13 lb. Gross, which $\div$ by 7 = C. 2 3qrs. 25 $\frac{6}{7}$ lb. Tare, supposing it was at 16 lb. per 112; but it is only 8 lb. which is $\frac{1}{2}$ of 16, viz. C. 1 1qr. 26 $\frac{1}{2}$ lb. Tare, taken from the Gross, leaves C. 19 1qr. 14 lb. 8 oz. Suttle; this $\div$ by 26 = C. 0 2qrs. 27 lb. 7 $\frac{2}{3}$ oz. Trett, this taken from the Suttle, leaves C. 18 2qr. 15 lb. 0 $\frac{4}{3}$ oz. Suttle; this, $\div$ by 168, = 12 lb. 6 oz. Cloff, which take from the last Suttle, leaves C. 18 2qrs. 3 lb. 10 $\frac{4}{3}$ oz. Neat, the Answer.

QUESTION I.

	£.	s.	d.	C.	qr.	lb.	
Neat Proceeds,	4	14	6	$\frac{1}{8}$	9	3 10	Gross.
Custom, &c.	2	8	6	0	3	26 $\frac{1}{3}$	Tare.
Freight,	1	2	8				
Factorage,	0	4	9	C. 8	3	11 $\frac{4}{5}$	Net = 4959 Fifths.
							And 1 Cwt. = 560 Ditto.
	£.	8	10	5			= 2045 Pence.
							d.

Stated thus—If 4959 : 2045 :: 560
560

4959) 1145200 (230 $\frac{3}{4}$ d. $\frac{3}{4}$ $\frac{6}{5}$ $\frac{4}{3}$, or £. 0 19s. 2 $\frac{1}{4}$ d.
 $\frac{3}{4}$ $\frac{6}{5}$ $\frac{4}{3}$ the Answer.

QUEST. 2.—First, 1180 $\times$ 80 = 94400 Inches, which, $\div$ by 231, (the Number of Inches in a Gallon) gives 408 $\frac{1}{2}$ $\frac{2}{3}$ Gallons; these, $\times$ 7 $\frac{1}{2}$ lb. (the Number of lb. in a Gallon) gives 3064 $\frac{2}{3}$ $\frac{1}{3}$ $\frac{6}{5}$ lb. Gross, which, $\div$ by 10, = 306 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{4}{5}$ lb. Tare; this, taken from the Gross, = 2758 $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{5}$ lb. Neat.

Then, stated thus—If 112 lb. : 4s. 6d. or 54d. :: 2758 $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{5}$ lb. :
1329 $\frac{3}{4}$ d. $\frac{2}{5}$ $\frac{2}{8}$ $\frac{3}{7}$ $\frac{2}{2}$, or £. 5 10s. 9 $\frac{3}{4}$ d. $\frac{2}{5}$ $\frac{2}{8}$ $\frac{3}{7}$ $\frac{2}{2}$, the Answer.

19. SIMPLE INTEREST.

- (2) First, £.824 18s. 2d. Principal, $\times$ by 4, Rate per Cent. = £.3299 12s. 8d. which, $\div$ by 100, gives £.32 19s. 11d. $\frac{12}{100}$, the Answer.
- (3) First, 500£. $\times$ 5 = 2500£. which, $\div$ by 100, = 25£. Interest for 1 Year; this, $\times$ by 4, (the Number of Years) = 100£. the Answer.
- (4) First, £.526 18s. 8d. $\times$ 4, the Rate = £.2107 14s. 8d. this $\times$ 9, (the Number of Years) = £.18969 12s. which $\div$ by 100 = £.189 13s. 11d. $\frac{4}{100}$, the Interest; this added to £.526 18s. 8d. Principal, gives £.716 12s. 7d. $\frac{4}{100}$ the Amount required.
- (5) First, £.264 0s. 4d. $\times$ by 5 = £.1320 1s. 8d. this, $\times$ by 12 (the Time) = 15841£. which, $\div$ by 100, = £.158 8s. 2 $\frac{1}{4}$ d. $\frac{6}{100}$ Interest; which, added to the Principal, gives £.422 8s. 6 $\frac{1}{4}$ d. $\frac{6}{100}$, the Amount required.
- (7) First, £.520 10s. 10d. $\times$ by 3 $\frac{1}{2}$ = £.1821 17s. 11d. which, $\div$ by 100 = £.18 4s. 4 $\frac{1}{2}$ d. Interest for a Year; this $\times$ by 3 = £.54 13s. 1 $\frac{1}{2}$ d. Interest for 3 Years, added to the Principal, gives £.575 3s. 11 $\frac{1}{2}$ d. the Amount.
- (8) First, £.140 10s. $\times$ 4 $\frac{3}{4}$ = £.667 7s. 6d. $\div$ 100 = £.6 13s. 5 $\frac{1}{2}$ d. $\frac{4}{5}$ Interest for a Year, which, $\times$ by 7, = £.46 14s. 3 $\frac{3}{4}$ d. Interest for 7 Years, the Answer.
- (9) First, £.470 $\times$ 3 $\frac{1}{4}$ = £.1527 10s. $\div$ 100 = £.15 5s. 6d. Interest for a Year; which, $\times$ 5, = £.78 7s. 6d. Interest for 5 Years; this, added to the Principal, viz. £.470 = £.548 7s. 6d. the Amount required.
- (11) First, £.742 12s. 6d. $\times$ 2 $\frac{1}{2}$ = £.1856 11s. 3d. this, $\div$ by 100 = £.18 11s. 3 $\frac{3}{4}$ d. the Answer.
- (12) First, £.374 19s. 10 $\frac{3}{4}$ d. $\times$ 2 = £.749 19s. 9 $\frac{1}{2}$ d. which $\div$ by 100 = £.7 9s. 11 $\frac{3}{4}$ d. $\frac{9}{100}$, the Answer.

B R O K A G E.

- (14) First, 1000£. $\div$ 100 = 10£. then, by Practice,

$$\begin{array}{r} \text{s. d. } \text{£.} \\ 4 \text{ } 0 \text{ } \frac{1}{3} \overline{) 10 \text{ at } 4\text{s. } 6\text{d.}} \end{array}$$

$$\begin{array}{r} 6 \text{ } \frac{1}{8} \overline{) 2 \text{ } 0} \\ \underline{0 \text{ } 5} \end{array}$$

Answer, £. 2 5

- (15) First, $\pounds.540 \text{ } 10\text{s.} \div 100 = \pounds.5 \text{ } 8\text{s. } 1\text{d.}$ then

s. d.	$\pounds.$	s.	d.
10 0	$\frac{1}{2}$ 5	8	1 at 13 $10\frac{1}{2}$
3 4	$\frac{3}{4}$ 2	14	$0\frac{1}{2}$
		0	18 0
6	$\frac{1}{20}$ 0	2	$8\frac{1}{4}$
Answ. $\pounds.3 \text{ } 14 \text{ } 8\frac{3}{4}$			

- (16) First, $\pounds.2474 \text{ } 15 \div 100 = \pounds.24 \text{ } 14\text{s. } 11\frac{1}{4}\text{d.}$ then

s. d.	$\pounds.$	s.	d.
10 0	$\frac{1}{2}$ 24	14	$11\frac{1}{4}$ at 19 $9\frac{1}{2}$
		12	7 $5\frac{1}{2}$
6 8	$\frac{1}{3}$ 8	4	$11\frac{3}{4}$
2 6	$\frac{1}{8}$ 3	1	$10\frac{1}{4}$
	$7\frac{1}{2}$	0	15 $5\frac{1}{2}$
Answ. $\pounds.24 \text{ } 9 \text{ } 9$			

I N S U R A N C E.

- (18) First, $\pounds.2460 \times 10\frac{3}{4} = \pounds.26445$, which $\div 100 = \pounds.264 \text{ } 9\text{s.}$ the Insurance required.
- (19) First, $\pounds.2500 \times 6\frac{7}{8} = \pounds.17187 \text{ } 10\text{s.}$ which, $\div$ by 100 = $\pounds.171 \text{ } 17\text{s. } 6\text{d.}$ the Insurance required.
- (20) First, $\pounds.7406 \text{ } 17\text{s. } 6\text{d.} \times 15\frac{3}{4} = \pounds.116658 \text{ } 5\text{s. } 7\frac{3}{4}\text{d.}$ which $\div 100 = \pounds.1166 \text{ } 11\text{s. } 7\frac{3}{4}\text{d.}$ the Insurance required.

P U R C H A S I N G of S T O C K S.

- (22) First, $\pounds.460 \times 87\frac{3}{4} = 403,65\pounds.$ which $\div$ by 100 = $\pounds.403 \text{ } 13\text{s.}$ the Purchase required.
- (23) First, $\pounds.2470 \text{ } 17\text{s. } 10\text{d.} \times 3\frac{1}{2}$ (the Excess) = $\pounds.8648 \text{ } 2\text{s. } 5\text{d.}$ which $\div 100 = \pounds.86 \text{ } 9\text{s. } 7\frac{1}{4}\text{d.}$ $\frac{96}{100}$; this, added to $\pounds.2470 \text{ } 17\text{s. } 10\text{d.}$ the Stock, gives $\pounds.2557 \text{ } 7\text{s. } 5\frac{1}{4}\text{d.}$ $\frac{96}{100}$ the Answer.
- (24) First, $876\pounds. \times 14\frac{5}{8}$ (the Excess) = $\pounds.12811 \text{ } 10\text{s.}$ which $\div$ by 100 = $\pounds.128 \text{ } 2\text{s. } 3\frac{1}{2}\text{d.}$ $\frac{4}{10}$; this added to $876\pounds.$ the Stock, gives $\pounds.1004 \text{ } 2\text{s. } 3\frac{1}{2}\text{d.}$ $\frac{4}{10}$ the Purchase required.

(26)	$\pounds.$	s.	d.
	246	12	6
Year.			5
$\frac{1}{4}$	$\frac{1}{4}$ 1233	2	6 = 1 Year.
	308	5	$7\frac{1}{2} = \frac{1}{4}$

$\pounds.15,41 \text{ } 8 \text{ } 1\frac{1}{2}$; this $\div$ by 100 = $\pounds.15 \text{ } 3\frac{1}{2}\text{d.}$ the Answer.

(27) £. s. d.

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 298 & 11 \ 0 \\ & \times 4 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{4} & \frac{1}{2} \\ \hline 1195 & 12 \ 0 \\ 149 & 9 \ 0 \\ 74 & 14 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 1419 & 15 \ 6 \\ & \times 4 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 5679 & 2 \ 0 \\ 709 & 17 \ 9 \\ \hline \end{array}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

$$\frac{1}{2} \frac{1}{2} 1419 \ 15 \ 6 = 1 \text{ Year.}$$

(28) £. s. d.

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 604 & 17 \ 6 \\ & \times 5 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 3024 & 7 \ 6 \\ 302 & 8 \ 9 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 3326 & 16 \ 3 \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 9980 & 8 \ 9 \\ 1663 & 8 \ 1\frac{1}{2} \\ 831 & 14 \ 0\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} \\ \hline 124,75 & 10 \ 11\frac{1}{4} \\ & \times 3 \\ \hline \end{array}$$

£. 63,88 19 9; this ÷ by 100 £. 63 17s. 8½d. $\frac{84}{1000}$, the Interest, which, added to the Principal, gives £. 362 15s. 8½d. $\frac{84}{1000}$, what is due, the Answer.

£. 124,75 10 11¼ = 3½ Years, this ÷ by 1,00 = £. 124 15s. 1d. the Interest, which added to the Principal gives £. 729 12s. 7d. the Amount required.

(30) First, £. 600 × 3¼ = 22,50£. this ÷ by 1,00 = £. 22 10s. the Interest for 1 Year; then, by the Rule of Three,

$$\text{If } 52 \text{ W.} : 22£. \ 10s. :: 26 \text{ W.}$$

Here £. 22 10s. = 450s. these × 26 = 11700s. which ÷ by 52 = 225s. or £. 11 5s. the Interest for 26 Weeks; this added to the 600£. gives £. 611 5s. the Amount required.

(31) First, 740£. × 5 = 37,00£. ÷ 1,00 = 37£. Interest for a Year; then

$$\text{If } 52 \text{ W.} : 37£. :: 42 \text{ W.}$$

Here 37 × 42 = 1554£. these ÷ by 52 = £. 29 17s. 8¼d. $\frac{12}{52}$, the Interest for 42 Weeks, which, added to 37£. × 4, viz. 148£. the Interest for 4 Years, gives £. 177 17s. 8¼d. $\frac{12}{52}$, the Interest for 4 Years, 42 Weeks, the Answer.

(32) First, 200£. × 4½ = 9,00£. ÷ 1,00 = 9£. Interest for a Year; then

$$\text{If } 51 \text{ W.} : 9£. :: 50 \text{ W.}$$

Here $9 \times 50 = 450 \div 52 = \text{£ } 8 \text{ } 13\text{s. } 0\frac{1}{2}\text{d. } \frac{3}{5}\frac{6}{12}$, Interest for 50 Weeks; which added to the Interest for a Year gives $\text{£ } 17 \text{ } 13\text{s. } 0\frac{1}{2}\text{d. } \frac{3}{5}\frac{6}{12}$, the whole Interest; this added to $200\text{£.} = \text{£ } 217 \text{ } 13\text{s. } 0\frac{1}{2}\text{d. } \frac{3}{5}\frac{6}{12}$, the Amount required.

(34) First, $\text{£ } 340 \text{ } 10\text{s.} \times 5 = \text{£ } 17,02 \text{ } 10\text{s.} \div 1,00 = \text{£ } 17 \text{ } 0\text{s. } 6\text{d.}$ the Interest for a Year.

And from Jan. the 1st to July the 18th, = 199 Days by the Table, then,

D. £. s. d. D.
— If 365 : 17 0 6 :: 199

Here $\text{£ } 17 \text{ } 0\text{s. } 6\text{d.} = 4068$ Pence; these $\times 199 = 813114\text{d.}$ which $\div 365 = 2227\frac{1}{2}\text{d. } \frac{3}{5}\frac{0}{6}\frac{6}{12}$, or $\text{£ } 9 \text{ } 5\text{s. } 7\frac{1}{2}\text{d. } \frac{3}{5}\frac{0}{6}\frac{6}{12}$ Answ.

(35) First, $500 \text{£.} \times 4 = 20,00\text{£.} \div 1,00 = 20\text{£.}$ Interest for a Year.

And from December the 4th, 1772, to March the 10th, 1774 = 1 Year, 96 Days; then,

D. £. D.
— If 365 : 20 :: 96

Here $96 \times 20 = 1920\text{£.}$ this, $\div 365 \text{£ } 5 \text{ } 5\text{s. } 2\frac{1}{4}\text{d. } \frac{3}{5}\frac{1}{6}\frac{5}{12}$ Interest for 96 Days; which, added to 20£. (Interest for a Year) = $\text{£ } 25 \text{ } 5\text{s. } 2\frac{1}{4}\text{d. } \frac{3}{5}\frac{1}{6}\frac{5}{12}$, the Interest required.

(37) First, $\text{£ } 4 \text{ } 10\text{s.}$ Rate $\times 9\frac{1}{2}$ Years, Time = $\text{£ } 42 \text{ } 15\text{s.}$ Interest of 100£. for $9\frac{1}{2}$ Years; which Interest added to $100\text{£.} = \text{£ } 142 \text{ } 15\text{s.}$ the Amount; then

£. s. £. £. s. d.
— If 142 15 : 100 :: 85 6 10

Here $\text{£ } 142 \text{ } 15\text{s.} = 2855\text{s.}$ and $\text{£ } 85 \text{ } 6 \text{ } 10 = 17130\text{s.}$ which $\times 100 = 1713000\text{£.}$ these $\div 2855 = 600\text{£.}$ the Answer.

(38) First, 3£. Rate, $\times 7\frac{1}{2}$, Time, = $13.20\text{£.} \div 1,00 = \text{£ } 23 \text{ } 5\text{s.}$ Interest; which, added to 100, Principal, = $\text{£ } 123 \text{ } 5\text{s.}$ the Amount; then,

£. s. £. £. s. d.
— If 123 5 : 100 :: 614 3 11

Here $\text{£ } 123 \text{ } 5\text{s.} = 29580\text{d.}$ and $\text{£ } 614 \text{ } 3\text{s. } 11\text{d.} = 147407\text{d.}$ $\times 100 = 14740700\text{£.}$ $\div 29580 = \text{£ } 498 \text{ } 6\text{s. } 8\text{d.}$ the Answer.

(40) First, $600\text{£.} \times 4\frac{1}{2} = 2700\text{£.} \div 100 = 27\text{£.}$ Interest of 600£. for a Year; and from $\text{£ } 856 \text{ } 10\text{s.}$ take 600£. remains $\text{£ } 256 \text{ } 10\text{s.}$ Interest of 600£. for the whole Time; then,

If

£. Y. £. s.

If 27 : 1 :: 256 10

20 20

54,0 1513,0 (9½ Years, Answer.

- (41) First, £. 498 6s. 8d. $\times 3 =$ £. 1459 $\div 100 =$ £. 14 19s. Interest of the given Principal for a Year; and £. 614 3s. 11d. — £. 498 6s. 8d. = £. 115 17s. 3d. Interest of the Principal for the whole Time; then,

£. s. Y. £. s. d.

If 14 19 : 1 :: 115 17 3

Here £. 14 19s. = 2588d. and £. 115 17s. 3d. = 27807d. which $\div 2588 = 7\frac{3}{4}$ Years, the Answer.

- (43) First, from £. 856 10s. take 600£. Remains £. 256 10s. Interest of the 600£. for 9½ Years.

£. £. s. s. £.

If 600 : 256 10 or, 5130 :: 100

100

6,00)5130,00(855s. Interest of 160 £. for 9½ Years, which $\div 9\frac{1}{2}$, viz. 19 Half-Years, gives 45s. or 2 £. 5s. Interest of 100 £. for Half a Year, which $\times 2 =$ £. 4 10s. per Cent. the Answer.

- (44) First, from £. 614 3s. 11d. take £. 498 6s. 8d. Remains £. 115 17s. 3d. Interest; then,

£. s. d. £. s. d. £.

If 498 6 8 : 115 17 3 :: 100

Reduced, 119600d. : 27807d. :: 2400d.

2400

1196,00)6673680,00(5580, or 465s.

And $7\frac{3}{4} = 31$ Qrs. then, 31)465(15s. $\times 4 = 3$ £. per Cent. the Answer.

- (45) First, 5000£. $\times 4\frac{1}{2} = 225,00$ £. $\div 1,00 = 225$ £. Interest for a Year; which $\div 4 =$ £. 56 5s. Interest due to Lady-Day, which is a Quarter, and in this Manner proceed with each new Principal for the Interest,

1771.

£.

Christmas, lent - 5000 at $4\frac{1}{2}$ per Cent.

1772. Inter. due to Lady-D. 56 5

Amount, - - 5056 5

Drew out, - - 194 5 = 185 Guineas.

Remains - - 4862 0 New Principal.

Interest of which to Midf. 54 13 $11\frac{1}{4}$ Amount, - - 4916 13 $11\frac{1}{4}$

Paid in 500 Moidores, = 675 0 0

Sum, - - 5591 13 $11\frac{1}{4}$

Drew out, - - 700 0 0

Remains - - 4891 13 $11\frac{1}{4}$ New Principal.Interest to Michaelmas, 55 0 $7\frac{1}{2}$ Amount, - - 4946 14 $6\frac{3}{4}$

Paid in, - - 569 17 0

Answer, £. 5516 11 $6\frac{3}{4}$

(56) Here multiply each new Principal by the Number of Days it has continued, and divide the Sum of all the Products by 7300, viz. 100×365 , the Quotient will be the Interest required.

Mr. RALPH NEWLANDS, Dr.

		£.	Days	Pro-ducts.
1771.				
May	1. Lent per Bill at one Day's Date,	500	13	6500
	13. Received in Part, - - -	50		
	Balance, - - -	450	22	9900
June	4. Received in Part, - - -	56		
	Balance, - - -	394	40	15760
Ju'y	14. Received in Part, - - -	44		
	Balance, - - -	350	9	3150
	Carried over, Sum			35310

Simple Interest.

141

	£.	Days	Pro- ducts.
Brought over,	350		35310
July 23. Received in Part,	50		
Balance,	300	26	7800
Aug. 18. Received in Part,	87		
Balance,	213	12	2556
30. Received in Part,	13		
Balance,	200	22	4400
Sep. 21. Received in Part,	30		
Balance,	170	27	4590
Oct. 18. Received in Part,	30		
Balance,	140	11	1540
29. Received in Part,	40		
Balance,	100	13	1300
Nov. 11. Received in Part,	50		
Balance,	50	47	2350
Dec. 28. Received in full of Principal,	50		
Sum of the Products,			59846

Then, $73,00)598,46(8\text{£. } 3\text{s. } 11\frac{1}{2}\text{d. } \frac{116}{730}$ Interest due on this Account.

(47) JOHN JAMESON, Dr.

	£.	s.	d.	Days	Products.
To a Bill at one Day's Date,	878	19	10	40	35159 13 4
Feb. 27. Received in Part,	57	15	7		
Ballance,	821	4	3	19	15603 0 9
Mar. 18. Received in Part,	37	14	0		
Ballance,	783	10	3	42	32907 10 6
April 29. Received in Part,	34	11	0		
Carried over,	748	19	3	13	9736 10 3

		£.	s.	d.	Days	Products.	£.	s.	d.
	Balance, -	748	19	3		93406	14	10	
May 12.	Received in Part,	136	15	7					
	Balance, -	612	3	8	38	23292	19	4	
June 19.	Received in Part, -	67	13	4					
	Balance, -	544	10	4	26	14157	8	8	
July 15.	Received in Part,	15	15	6					
	Balance, -	528	14	10	10	5207	8	4	
25.	Received in Part,	111	11	11					
	Balance, -	417	2	11	70	29200	4	2	
Oct. 3.	Received in Part, -	78	7	4					
	Balance, -	338	15	7	47	15922	12	5	
23.	Received in Part,	100	0	0					
	Balance, -	238	15	7	4	955	2	4	
	Received in Part,	100	0	0					
	Balance, -	138	15	7	37	5134	16	2	
Dec. 30.	Received in full of the Principal,	138	15	7					

Sum of the Products, £. 187327 6 8

Then $73,00 \over 187327$ £. 6s. 8d. (25 £. 13s. $2\frac{1}{2}$ d. $\frac{604}{730}$, the Interest required.

	£.	s.	d.
(48) First, from 109 Moidores take 2s. 6d. Rem.	147	0	6
Amount of the Bond,			
And 109 Guineas	-	-	-
Value of Ditto,	=	114	9 0
Difference or Interest,	£.	32	11 6

Also, £. 114 9s. Principal, $\times 4$ Rate, = £. 4,57 16s.; this $\div 1,00$, = £. 4 11s. $6\frac{1}{2}$ d. Interest for a Year.

Then

£. s. d. Yea. £. s. d.

Then—If 4 11 6 $\frac{1}{2}$: 1 :: 32 11 6

Reduced 2197 Halfpence : 1 :: 15636 Halfpence ; therefore
2197)15636(7 Years, 42 Days, the whole Time.

July hath 31 Days } Sum 49—42=7th of July, the An-
Till Aug. 18th, } swer.

(49) First, £.39 19s. 8d.=9596 Pence ; then, per Sect. XV.

Prin. Yrs. Interest.

Placed thus { 100 : 12 : 9596
1 : 0 : 240

Then $240 \times 12 \times 100 = 288000$ Dividend, which $\div$ by 9596
=30 Years, $4\frac{5}{9}\frac{4}{59}\frac{16}{96}$ Days, the Answer.

(50) First, from Aug. 7, 1766, to May 11, 1771=4 Years,
277 Days.

£. s. d.

1st Bond was made for - 1114 10 0 at 6 per Ct.
Interest of which for 4Y. 277 D. is 318 4 6 $\frac{1}{4}$

Sum, 1432 14 6 $\frac{1}{4}$ Amount.

May 11. Paid off, - 140 0 0

2d Bond, - 1272 14 6 $\frac{1}{4}$ at 5 $\frac{1}{4}$
Due Sept. 5, (See next Page). 21 16 8 Interest.

Sum, 1394 11 1 $\frac{1}{4}$ Amount.

Sept. 5. Paid off, - 87 11 9

3d Bond, - 1226 19 5 $\frac{1}{4}$ } subtract.
Sept. 11. Received in full, - 1409 16 8

Interest, 182 17 2 $\frac{3}{4}$ Difference.

Then, from September 5, 1776, to September 11, 1777.=
6 Years, 6 Days, or 2196 Days, 3d Bond or Princi-
pal, viz. £.1226 19s. 5 $\frac{1}{4}$ d.=1177893 Farthings.

£.182 17s. 2 $\frac{1}{2}$ d.=175547 Farthings. £.100=96000
Farthings, and a Year =365 Days ; then, by Case 7,
or rather by Sect. 15, thus—

Prin.

Simple Interest.

<i>Prin.</i>	<i>Time.</i>	<i>Interest.</i>
1177893	2196	175547
96000	365	

Then $1177893 \times 2196 = 2586658028$ the Divisor.

And $175547 \times 365 \times 96000 = 6151166880000$ the Dividend.

Therefore, 2586658028,61511668800000(2378 $\frac{94289416}{2586658028}$
Farthings, or £.2 9s. 6½d. the Answer required.

The Time of the 2d Bond's Continuance is found, thus—

P.	Time.	R.	
100	365	5 5 0	} Red. 96000 365 1260
1292 14 6 $\frac{1}{4}$	—	21 16 8	
			1241017 — 5240

Then $1241017 \times 1260 = 1563681420$ the Divisor.

$$5240 \times 365 \times 96000 = 18360960000 \text{ the Dividend.}$$

1563681420)1836c9600000(117 Days, since May 11,
which answers to Sept. 5, as before mentioned.

(51)

	£.		£.
If he puts in	$\begin{Bmatrix} 100 \\ 200 \\ 300 \end{Bmatrix}$	to have	$\begin{Bmatrix} 40 \\ 55 \\ 70 \end{Bmatrix}$
			per Year.

Then from $\left\{ \begin{smallmatrix} \text{£.} \\ 55 \\ 70 \end{smallmatrix} \right\}$ take $\left\{ \begin{smallmatrix} \text{£.} \\ 40 \\ 55 \end{smallmatrix} \right\} = 15 \text{ per C. for his Money.}$

And $\begin{matrix} \text{£.} \\ 40 \\ 55 \\ 70 \end{matrix} \Bigg\} - \begin{matrix} \text{£.} \\ 15 \\ 30 \\ 45 \end{matrix} \Bigg\} = 25\text{£. per Year for his Attendance.}$

(52)

	£.	s.	d.
£. 900 at $111\frac{3}{8}$ per Cent. per Case II.	1002	7	6
Brokerage of 900 £. at 2s. 6d. per Cent.	1	2	6

	£.	1003	10	0
Midsummer Dividend, at 2 per Cent.	-	18	0	0

Interest of £.1003 10s. for 45 Days, at 5 } £. 985 10 0
per Cent. - - - - - } 6 14 8

Brokerage of £. 400 at 2s. 6d. per Cent.	0	10	0
--	---	----	---

Carried over, £. 992 14 8

Simple Interest.

145

	£.	s.	d.
Brought over,	£. 992	14	8
Sold 400£. at 92½ per Cent,	370	0	0
	£. 622	14	8
Interest for ½ Year, due Feb. 10, 1746,	15	11	4¼
	£. 638	6	0¼
Dividend received at that Time,	10	0	0
	£. 628	6	0¼
Interest due to Aug. 10,	15	14	1¾
	644	0	2
Dividend received at that Time,	10	0	0
	£. 634	0	2
Interest due to Feb. 1747,	15	17	0
	£. 649	17	2
Dividend received then,	10	0	0
	£. 639	17	2
Interest to the 10th of August,	15	19	11
	£. 655	17	1
Midsummer Dividend, received August 10,	10	0	0
	£. 645	17	1
Sold off 500£. at 102½ per Cent.	512	2	6
	£. 133	14	7
Brokerage,	0	12	6
	£. 133	2	1
To my Damage in the Whole,			

20. COMPOUND INTEREST.

(2) First, $150\text{£} \times 4 = 6,00\text{£} \div 100 = 6\text{£}$. Interest, which added to $150 = 156\text{£}$. second Year's Principal; then $156\text{£} \times 4 = 624\text{£} \div 100 = 6\text{£}$, 4s . $9\frac{1}{2}\text{d}$. Interest, + $156\text{£} = \text{£}162$ 4s . $9\frac{1}{2}\text{d}$. third Year's Principal, which $\times 4 = \text{£}648$ 19s . 2d . $\div 1,00 = \text{£}6$ 9s . $9\frac{1}{2}\text{d}$. Interest, which added to $\text{£}162$ 4s . $9\frac{1}{2}\text{d} = \text{£}168$ 14s . 7d . = fourth Year's Principal, which $\times 4 = \text{£}674$ 18s . 4d . $\div 100 = \text{£}6$ 14s . $11\frac{3}{4}\text{d}$. fourth Year's Interest, which added to $\text{£}168$ 14s . $7\text{d} = \text{£}175$ 9s . $6\frac{3}{4}\text{d}$. = fifth Year's Principal, which $\times 4 = \text{£}701$ 18s . 3d . this $\div 100 = \text{£}7$ 0s . $4\frac{1}{2}\text{d}$. fifth Year's Interest, which added to $\text{£}175$ 9s . $6\frac{3}{4}\text{d} = \text{£}182$ 9s . $11\frac{1}{4}\text{d}$. the Amount.

(3) £ . £ . s . d .
 $5 \frac{1}{20} 440 \ 16 \ 0 = 1\text{st Year's Principal}$.
 $22 \ 0 \ 9\frac{1}{2} = \text{Interest}$.

$\frac{1}{20} 462 \ 16 \ 9\frac{1}{2} = \text{second Year's Principal}$.
 $23 \ 2 \ 10 = \text{Interest}$.

$\frac{1}{20} 485 \ 19 \ 7\frac{1}{2} = \text{third Year's Principal}$.
 $24 \ 5 \ 11\frac{3}{4} = \text{Interest}$.

$\frac{1}{20} 510 \ 5 \ 7\frac{1}{4} = \text{fourth Year's Principal}$.
 $25 \ 10 \ 3\frac{1}{4} = \text{Interest}$.

[mount of 4 Years.

Mo. $\frac{1}{20} 535 \ 15 \ 10\frac{1}{2} = \text{fifth Year's Principal, or A-}$
 $6 \ \frac{1}{2} 26 \ 15 \ 9\frac{1}{2} = \text{Interest}$.

$\frac{1}{6} 13 \ 7 \ 10\frac{3}{4}$
 $15 \text{ D. } \frac{1}{2} 2 \ 4 \ 7\frac{3}{4}$
 $1 \ 2 \ 3\frac{3}{4}$

$16 \ 14 \ 10\frac{1}{4} = \text{Interest for 7 Mo. 15 Days}$.

$535 \ 15 \ 10\frac{1}{2} = 4 \text{ Year's Amount}$.

$552 \ 10 \ 8\frac{3}{4} = \text{Amount for 4 Y. 7 Mo. 15 D.}$

$440 \ 16 \ 0 = \text{Principal}$.

Answer $\text{£} \ 111 \ 14 \ 8\frac{3}{4} \text{ Compound Interest}$.

21. REBATE OR DISCOUNT.

(2) First, $100 + 6 = 106$ £. Amount of 100 £. for a Year.
Then—If 106 £. : 100 £. :: 100 £.

$$\begin{array}{r} 100 \\ 106 \overline{) 10000} \end{array} \begin{array}{l} \text{£. } 94 \text{ } 6 \text{ s. } 9 \frac{1}{2} \text{ d. } \frac{4}{100} \end{array} \text{ Answ.}$$

(3) Mo. £. £. s. d. £. £. s. d.
6 $\frac{1}{2}$ 4 then—If $103 \text{ } 6 \text{ } 8$: 100 :: $600 \text{ } 10 \text{ } 6$
2
4 $\frac{1}{3}$ 1 6 8
24800d. : 100 :: 144126d.

Int. for 10 M. 3 6 8
Principal, 100 0 0
Amount, 103 6 8

248,00)144126,00(£. 581 3s. 1d.
present Worth, which, taken
from the Principal, leaves
£. 19 7s. 5d. Answer.

(4) First, from December 12, to July 27, = 227 Days.
Then—If 365 D. : 5 £. :: 227 Days.

$$\begin{array}{r} 227 \\ 365 \overline{) 1135} \end{array} \begin{array}{l} \text{£. } 3 \text{ } 2 \text{ s. } 2 \frac{1}{4} \text{ d. } \end{array} \text{ Rebate of } 100 \text{ £. for } 227 \text{ Days.}$$

£. s. d. £. s. d. £. s.
Again,—If $103 \text{ } 2 \text{ } 2 \frac{1}{4}$: $3 \text{ } 2 \text{ } 2 \frac{1}{4}$:: $890 \text{ } 16$
 $98985 \text{ qrs.} : 2985 \text{ qrs.} :: 855168$
 855168

$98985)2552676480(25787 \text{ qrs. or } \text{£. } 26 \text{ } 17 \text{ s. } 3 \text{ d.}$
the Answer.

(5) First, one-half of 430 £. = 215 £.

Mo. £. £. s. d. £. s. d. £.
4 $\frac{1}{3}$ 5 If $101 \text{ } 13 \text{ } 4$: $1 \text{ } 13 \text{ } 4$:: 215

Interest, 1 13 4 24400d. : 400d. :: 51600d.
Prin. 100 0 0 51600

Amount, 101 13 4 244,00)206400,00(845 $\frac{3}{4}$ d. or £. 3 10s.
5 $\frac{3}{4}$ d. Disc. of one-half for 4 Months.

Again, £. 1 13s. 4d. $\times 2 =$ £. 3 6s. 8d. Interest of 100£. for 8 Months.

$$\begin{array}{r} \text{Then—If } \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \end{array} \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \end{array} \text{£.} \\ 103 & 6 & 8 : 3 & 6 & 8 :: 215 \\ \text{or, } 24800\text{d.} : 800\text{d.} :: 51600\text{d.} \\ \hline & & 51600 \end{array}$$

248,00)412800,00(1664 $\frac{1}{2}$ d. or £. 6 18s. 8 $\frac{1}{2}$ d.
Discount of Half for 8 Months, which, added to that for 4 Months, gives £. 10 9s. 2 $\frac{1}{4}$ d. Answer.

(6) First, from May 21, to Christmas, = 218 Days; then—
If 365D. : 5£. :: 218 Days.

218
365)1090(£. 2 19s. 8 $\frac{1}{2}$ d. Interest of 100£. for 218 Days.

$$\begin{array}{r} \text{Again,—If } \begin{array}{ccc} \text{£.} & \text{s.} & \text{d.} \end{array} \text{£.} \text{£.} \\ 102 & 19 & 8 : 100 :: 550 \\ \text{or, } 98866\text{d.} : 100 :: 52800\text{d.} \\ \hline & & 52800 \end{array}$$

98866)5280000(£. 534 1s. 1 $\frac{1}{4}$ d. Answer.

(7) Mo. £. £. s. £. £.
3 $\frac{1}{4}$ 6 then—If 101 10 : 100 :: 100

Interest, 1 10
Prin. 100 0

$$\begin{array}{r} 2030\text{s.} : 100 :: 2000\text{s.} \\ \hline 2000 \end{array}$$

Amount, 101 10

203,0)20000,0(£. 98 10s. 10 $\frac{1}{4}$ d.
present Worth of 100£. for 3 Months.—Again,

Mo. £. £. s. £. £.
4 $\frac{1}{3}$ 6 then—If 102 10 : 100 :: 60

1 $\frac{1}{4}$ 2 0
0 10

$$\begin{array}{r} 2050\text{s.} : 100 :: 1200 \\ \hline 1200 \end{array}$$

Interest, 2 10
Princip. 100 0

205,0)1200,0(£. 58 10s. 8 $\frac{1}{2}$ d.
present Worth of 60£. for 5 Months.

Amount, 102 10

Now,

Now, $100 + 60 = 160$ £. And 3220 £. $- 160$ £. $= 3060$ £.
for 9 Months.

Mo. £. s. £. s. £. s.
 $6 \frac{1}{2}$ 6 If 104 10 : 100 :: 3060

$3 \frac{1}{2}$ 3 2090 : 100 :: 61200 s.

1 10

612

Interest, 4 10

Prin. 100 0

Amt. 104 10

$209,0)6120,0$ (£. 2928 4 s. 7 d., present
Worth of 3060 £. for 9 Months; there-
fore all the present Worths, added to-
gether, will be £. 3085 5 s. 9 d. the
Answer.

(8) First $\frac{1}{3}$ th of 400 £. $= 80$ £. for three Months.

Mo. £. s. £. s. d. £. s.
 $3 \frac{1}{4}$ 4 10 then — If 101 2 6 : 100 :: 80

Interest, 1 2 6

Princip, 100 0 6

Amount 101 2 6

4045 : 100 :: 3200 Six-pences.

$4045)320000$ (£. 79 2 s. $2\frac{1}{2}$ d. the
present Worth of 80 £. for 3
Months.

Again, $\frac{1}{2}$ of 400 £. $= 200$ £. for 6 Months.

Mo. £. s. £. s. £. s.
 $6 \frac{1}{2}$ 4 10 then — If 102 5 : 100 :: 200

Interest, 2 5

Princip. 100 0

Amount 102 5

2045 s. : 100 :: 4000 s.

$2045)400000$ (£. 195 11 s. $11\frac{1}{2}$ d.
the present Worth of 200 £. for
6 Months.

Now, $80 + 200 = 280$ £. And $400 - 280 = 120$ £. to conti-
nue for three times 3 Months, which is 9 Months.

Mo. £. s. £. s. d. £. s.
 $6 \frac{1}{2}$ 4 10 then — If 103 7 6 : 100 :: 120

$3 \frac{1}{2}$ 2 5

1 2 6

4135 : 100 :: 4800 Six-pences.

$4135)480000$ (£. 116 2 s. $7\frac{1}{2}$ d. the
present Worth of 120 £. for 9
Months.

Interest, 3 7 6

Principal 100 0 0

£. 103 7 6

0 3

Now

Now all the present Worths, added together, gives £. 390 15s. 9½d. which, taken from 400 £. Value of the Goods, leaves £. 9 4s. 1½d. the Rebate.

(9) First, $\frac{1}{3}$ of 360£. = 120£. to continue 5 Months.

Mo.	£.		£.	s.	£.	£.
4 $\frac{1}{3}$	3		then—If	101	5	: 100 :: 120

1 $\frac{1}{4}$	1	0
	0	5

2025s. : 100 :: 240s.

2025)240000(£. 118 10s. 4¼d. the present Worth of 120£. for 5 Months.

Interest,	1	5
Princip.	100	0

Amount, 101 5

Again, 360—120=240£. to continue 10 Months.

Mo.	£.		£.	s.	£.	£.
6 $\frac{1}{2}$	3		then—If	102	10	: 100 :: 240

1	10
4 $\frac{1}{3}$	10

2050s. : 100 :: 4800s.

205,0)4800000(£. 234 2s. 11d. present Worth of 240£. for 10 Mo. which, added to that for 5 Mo. will make £. 352 13s. 3¼d. the present Worth required.

Interest,	2	10
Princip.	100	0

Amount, 102 10

(10) 5£. $\times \frac{1}{20}$ 500£. Principal.

25 Interest of 500£. for one Year; then
25£. $\times 12$ = 300£. Interest of 12 Years.

Again, 5 $\times 12$ = 60£. Interest of 100£. for 12 Years.

If 160£. : 60£. :: 500£. : 187£. 10s.

∴ 300—£. 187 10s. = 112£. 10s. Advantage to Interest.

(11) *Mo. £. s. d.* then — If 112£. 6s. 8 $\frac{1}{4}$ d. : 100£.

6 $\frac{1}{2}$ 4 7 6 = 4 $\frac{3}{8}$ £. :: 13377£. 13s. 4d.
2 or, 107841 grs. : 100 :: 12842560 grs.

107841) 1284256000 (£. 11908 15s. 9 $\frac{3}{4}$ d. $\frac{105}{107}$ $\frac{801}{841}$, the ready Money required.

	$\frac{1}{2}$	4	7	6	=	4 $\frac{3}{8}$ £.
				2		
2	$\frac{1}{3}$	2	3	9		
1	$\frac{1}{3}$	0	14	7		
15 D.	$\frac{1}{2}$	0	7	3 $\frac{1}{2}$		
		0	3	7 $\frac{3}{4}$		
10	$\frac{1}{3}$	0	2	5		

Inter. 12 6 8 $\frac{1}{4}$
Prin. 100 0 0

Amt. 112 6 8 $\frac{1}{4}$

22. EQUATION OF PAYMENTS.

(2) £. Mo. Prod.

$$200 \times 3 = 600$$

$$100 \times 4 = 400$$

$$300 \times 6 = 1800$$

$$\underline{\quad\quad\quad}$$

$$6,00 \quad 28,00)$$

(3) £. Mo. Prod.

$$200 \times 7 = 1400$$

$$260 \times 5 = 1300$$

$$\underline{\quad\quad\quad}$$

$$46,00$$

$$270,00 (5 M. 26 $\frac{4}{6}$ D.$$

the Answer.

Answer, 4 Months, 20 Days.

(4) Here suppose 120£. to be the Sum owed.

£. Mo. Prod.

$$\text{then } \left. \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{array} \right\} \text{ of } 120 = \left\{ \begin{array}{l} 60, \times 3 = 180 \\ 40, \times 4 = 160 \\ 20, \times 9 = 180 \end{array} \right.$$

$$\underline{\quad\quad\quad}$$

$$12,0$$

$$\underline{\quad\quad\quad}$$

$$520$$

$$) 520 (4 \text{ Mo. } 12 \text{ D. Answer.}$$

(5) Here, as the Debt is to be paid at four equal Payments, and $\frac{1}{4}$ being paid down, there remains $\frac{3}{4}$ to be paid, at three equal Payments; consequently, the Sum of the different

different Times that each Payment is to be made, being divided by 3, will give the Answer; thus
 $4+5+6=15$, this, $\div 3=5$ Months, the Answer.

(6) £. Mo. Prod.

Owed, $240 \times 5 = 1200$

Paid down, 40

Remains, $2,000 - 1200 = 800$ (6 Months, the Answer.)

23. SINGLE FELLOWSHIP.

(2) First, $320+340=660$ £. C and D put in; then, from 824£. take 660£. remains 164£. E's Stock, and $824+70=894$ £. their whole Gain; therefore,

	£.	£.	s.	d.	Rem.	
As 824 : 894 ::	320	347	3	$8\frac{1}{4}$	128	C's
	320	368	17	8	72	D's
	164	177	18	$7\frac{1}{2}$	642	E's

} Gain.

(3) Here, suppose 600£. to be their Stock; then

	£.		£.
$\left. \begin{matrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{5} \end{matrix} \right\}$ of 600£.	300	A's	} Stock.
	200	B's	
	150	C's	
	120	D's	
	770	Sum.	

Stated thus, £. = £. As 770 : 120 ::

300	
200	
150	
120	

} Ans. as below.

By Fractions thus— $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, $= \frac{60}{120}$, $\frac{40}{120}$, $\frac{30}{120}$, and $\frac{24}{120}$ —Then, $60+40+30+24=154$.

	£.	£.	s.	d.	Rem.	
∴ As 154 : 120 ::	60	46	15	$0\frac{3}{4}$	9	A's
	40	31	3	$4\frac{1}{2}$	6	B's
	30	23	7	$6\frac{1}{4}$	43	C's
	24	18	14	$0\frac{1}{4}$	19	D's

} Loss.

(4) First, $30+48+42=120$ £. their whole Gain.—Then,

	£.	£.	
As 120 : 400 ::	30	100	D's
	48	160	E's
	42	140	F's

} Stock.

(5)

£.	s.	
First, 1000	= 20000	A's
640	= 12800	B's
900	= 18000	C's
842 16	= 16856	D's

} Debt.

Sum, 3382 16 = 67656 the whole Debt.

And £. 2420 17s. 6d. = 581010 Pence, his whole Worth.

Then,—

	s.	d.	£.	s.	d.
As 67656 : 581010 ::	20000	: 171754	= 715	12	10
	12800	: 109922 $\frac{1}{2}$	= 458	0	2 $\frac{1}{2}$
	18000	: 154578 $\frac{1}{4}$	= 644	1	6 $\frac{1}{4}$
	16856	: 144754 $\frac{1}{4}$	= 603	2	10 $\frac{1}{4}$

Rem.

45504	A	} Received.
360	B	
360	C	
44088	D	

- (6) Here, suppose 420, as it will divide by 3, 4, 5, 6, and by 7, and have no Remainder, then $\frac{1}{3}$ of 420 = 140 A's $\frac{1}{4}$ = 105 B, $\frac{1}{5}$ = 84 C, $\frac{1}{6}$ = 70 D, and $\frac{1}{7}$ = 60 E, therefore, added together, = $\frac{459}{28}$, then, neglecting the Denominator, the Statings will stand thus;

£.	£.	s.	d.	
As 459 : 500 ::	140	: 152	10	1 $\frac{1}{4}$ 105 A's
	105	: 114	7	6 $\frac{3}{4}$ 423 B's
	84	: 91	10	0 $\frac{3}{4}$ 63 C's
	70	: 76	5	0 $\frac{1}{2}$ 282 D's
	60	: 65	7	2 $\frac{1}{4}$ 45 E's

} Share.

Or by Fractions, thus, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$, = $\frac{840}{2520}$, $\frac{630}{2520}$, $\frac{504}{2520}$, $\frac{420}{2520}$, and $\frac{360}{2520}$. Here neglecting the Denominator, and the Sum of the Numerators will be the first Term, and each Numerator the third; then proceed as above, will give the Answer.

- (7) First, $\frac{3}{8} + \frac{3}{7} = \frac{21}{56} + \frac{24}{56} = \frac{45}{56} = A + B's = \text{Part.}$ Then, $\frac{5}{6} - \frac{45}{56} = \frac{11}{56}$ C's Part.

£.	£.	s.	d.	
As 11 : 140 ::	21	: 267	5	5 $\frac{1}{4}$ $\frac{7}{11}$ A
	24	: 305	9	1 $\frac{4}{11}$ B

} paid.

(8) First, $10 + 7 = 17$; then—If $17 : 52 \text{ } 10 :: 10$
 $\frac{10}{17}$
 $17) 525 \text{ } 0 (\text{£. } 30 \text{ } 17 \text{ } s. \text{ } 7 \frac{1}{2} d.$
 $\frac{1}{2}$, A's Gain.

And,—If $45 : 100 :: 30 \text{ } 17 \text{ } 7 \frac{1}{2} \frac{1}{17}$
 $734400 : 100 :: 504000$ Seventeenths.
 $7344,00) 504000,00 (\text{£. } 68 \text{ } 12 \text{ } s. \text{ } 6 \frac{1}{2} d. \frac{2}{7} \frac{5}{3} \frac{9}{4} \frac{2}{4}$ A's.
 Adventure.

Again,—If $17 : 52 \text{ } 10 :: 7$
 $\frac{7}{17}$
 $17) 367 \text{ } 10 (\text{£. } 21 \text{ } 12 \text{ } s. \text{ } 4 d. \frac{1}{17} \frac{6}{7}$ B's Gain.

Also—If $45 : 100 :: 21 \text{ } 12 \text{ } 4 \frac{1}{17} \frac{6}{7}$
 $734400 : 100 :: 352800$
 $7344,00) 352800,00 (\text{£. } 48 \text{ } 0 \text{ } s. \text{ } 9 \frac{1}{4} d. \frac{4}{7} \frac{5}{3} \frac{7}{4} \frac{2}{4}$ B's.
 Adventure.

(9) First, 50 Guineas = $\text{£. } 52 \text{ } 10 \text{ } s.$ B's Gain, which take from $\text{£. } 74 \text{ } 11 \text{ } s.$ A's Gain, leaves $\text{£. } 22 \text{ } 1 \text{ } s.$ Difference.
 And $\text{£. } 52 \text{ } 10 \text{ } s. + \text{£. } 74 \text{ } 11 \text{ } s. = \text{£. } 127 \text{ } 1 \text{ } s.$ the whole Gain.

Then,—If $22 \text{ } 1 : 19 \text{ } 19 \text{ } 8 :: 127 \text{ } 1$
 $441 \text{ } s. : 4796 d. :: 2541 \text{ } s.$
 $\frac{2541}{441}$

$441) 12186636 (27634 d. \text{ or } \text{£. } 115 \text{ } 2 \text{ } s. \text{ } 10 d. \frac{4}{4} \frac{2}{1}$
 Cost, which added to the Gain, viz. $\text{£. } 127 \text{ } 1 \text{ } s. =$
 $\text{£. } 242 \text{ } 3 \text{ } s. \text{ } 10 d.$ or 58126 Pence, which $\div$ by 55s. viz.
 660 Pence, will give 88 Anchors, and 3s. 10d. over;
 then, $\text{£. } 127 \text{ } 1 \text{ } s. \div 88 = \text{£. } 1 \text{ } 8 \text{ } s. \text{ } 10 \frac{3}{4} d.$ gained per Anchor.

(10) First, from 14s. 6d. take 8s. 6d. remains 6s. A gains more than B; then,

$$\begin{array}{r} s. \quad s. \quad s. \quad d. \\ \text{If } 6 : 35 :: 14 \quad 6 \\ 12 : 35 :: 29 \\ \hline 29 \end{array}$$

12) 1015 (84s. 7d. or
£. 4 4s. 7d. A's Stock.

$$\begin{array}{r} s. \quad s. \quad s. \quad d. \\ \text{Again, If } 6 : 35 :: 8 \quad 6 \\ 12 : 35 :: 17 \\ \hline 17 \end{array}$$

12) 595 (49s. 7d. or
£. 2 9s. 7d. B's Stock.

(11) First, $210 + 312 = 522$ £. Sum of A and B's Stock, and
 $140 - £. 37 \text{ 10s.} = £. 102 \text{ 10s.}$ their Gain, then,

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad s. \quad \text{£.} \quad \text{£.} \quad s. \quad d. \quad \text{Rem.} \\ \text{If } 522 : 102 \text{ 10s.} :: 310 : 41 \quad 4 \quad 8\frac{1}{2} \quad 108 \text{ A's} \\ 522 : 102 \text{ 10s.} :: 312 : 61 \quad 5 \quad 3\frac{1}{4} \quad 414 \text{ B's} \end{array} \left. \vphantom{\begin{array}{r} \text{£.} \quad \text{£.} \quad s. \quad \text{£.} \quad \text{£.} \quad s. \quad d. \quad \text{Rem.} \\ \text{If } 522 : 102 \text{ 10s.} :: 310 : 41 \quad 4 \quad 8\frac{1}{2} \quad 108 \text{ A's} \\ 522 : 102 \text{ 10s.} :: 312 : 61 \quad 5 \quad 3\frac{1}{4} \quad 414 \text{ B's} \end{array}} \right\} \text{Gain.}$$

If £. 102 10s. : 522 :: £. 37 10s. : £. 190 19s. 6d. $\frac{20}{5}$ C's
Stock.

(12) First, $8 + 5 = 13$ £. their Gain per Cent.

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \quad \text{£.} \quad s. \quad d. \\ \text{Then—If } 13 : 154 :: 8 : 94 \quad 15 \quad 4\frac{1}{2} \quad \frac{6}{13} \text{ A's} \\ 13 : 154 :: 5 : 59 \quad 4 \quad 7\frac{1}{4} \quad \frac{7}{13} \text{ B's} \end{array} \left. \vphantom{\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \quad \text{£.} \quad s. \quad d. \\ \text{Then—If } 13 : 154 :: 8 : 94 \quad 15 \quad 4\frac{1}{2} \quad \frac{6}{13} \text{ A's} \\ 13 : 154 :: 5 : 59 \quad 4 \quad 7\frac{1}{4} \quad \frac{7}{13} \text{ B's} \end{array}} \right\} \text{Gain.}$$

Anf. A had for his Trouble, $35 \quad 10 \quad 9 \quad \frac{12}{13}$

A B, A C, B C, A B, A C, B C,

(13) First, $\frac{2}{7}, \frac{3}{8}, \frac{3}{10} = \frac{160}{560}, \frac{210}{560}, \frac{168}{560}$, by Case 5,
Sect. 38; then rejecting the Denominator, we shall
have,

First, $210 - 160 = 50$ C's Share more than B's.

And $210 - 168 = 42$ A's

Also, $160 - 42 = 118$ by A and B equally.

$\therefore 118 \div 2 = 59$ B's Share.

And $59 + 42 = 101$ A's

Also, $59 + 50 = 109$ C's

$\left. \vphantom{\begin{array}{l} 59 + 42 = 101 \text{ A's} \\ 59 + 50 = 109 \text{ C's} \end{array}} \right\} = 269 \text{ their Sum.}$

$$\begin{array}{r} s. \quad d. \quad \text{Rem.} \\ \text{If } 269 : 30s. :: \left\{ \begin{array}{l} 59 : 6 \quad 6\frac{3}{4} \quad 225 \text{ B's} \\ 101 : 11 \quad 3 \quad 180 \text{ A's} \\ 109 : 12 \quad 1\frac{3}{4} \quad \frac{133}{269} \text{ C's} \end{array} \right\} \text{Share of the} \\ \hline \hline 30s. \end{array}$$

Meal. Malt. Meal. Malt.

(14) First,—If $3 : 5 :: 8 : 13\frac{1}{3} = 4\frac{2}{3}$
 Then, $8 + 7 + 13\frac{1}{3} = 28\frac{1}{3} = 8\frac{5}{3}$, their Sum.

Qrs. Bush.

$$8\frac{5}{3} : 40\frac{0}{3} :: \frac{100}{1} : 8\frac{00}{17} = 47\frac{1}{17} = 376\frac{8}{17} = 6\frac{400}{17} \text{ Malt.}$$

$$8\frac{5}{3} : \frac{8}{1} :: \frac{100}{1} : 4\frac{80}{17} = 28\frac{4}{17} = 225\frac{15}{17} = 3\frac{840}{17} \text{ Meal.}$$

$$8\frac{5}{3} : \frac{7}{1} :: \frac{100}{1} : 4\frac{20}{17} = 24\frac{12}{17} = 197\frac{11}{17} = 3\frac{360}{17} \text{ Oatmeal.}$$

$$\left. \begin{array}{l} 8\frac{00}{17} \times \frac{2}{3} = 1\frac{200}{17} \\ 4\frac{80}{17} \times \frac{1}{2} = 2\frac{10}{17} \end{array} \right\} \text{Price of the } \left\{ \begin{array}{l} \text{Malt.} \\ \text{Meal.} \\ \text{Oatmeal.} \end{array} \right.$$

Here the Denominator (17) may be omitted, and the Numerators divide by 30, the Quotient will still retain the same Proportion.

$$3,0) 120,0 + 48,0 + 21,0 (= 40 + 16 + 7 = 63 \text{ Sum.})$$

£. s. d.

$$\text{As } 63 : 142 :: \left\{ \begin{array}{l} 40 : 5\frac{680}{63} = 90 \quad 3 \quad 2\frac{2}{3} \text{ Malt.} \\ 16 : 2\frac{272}{63} = 36 \quad 1 \quad 3\frac{5}{6} \text{ Meal.} \\ 7 : 9\frac{94}{63} = 15 \quad 15 \quad 6\frac{14}{63} \text{ Oatmeal.} \end{array} \right\} \text{Cost.}$$

s. d.

$$\left. \begin{array}{l} \text{Then } 5\frac{680}{63} \div 6\frac{400}{17} = 40\frac{320}{63} \text{ £.} = 4 \quad 9\frac{30}{63} \text{ Malt.} \\ \text{And } 2\frac{272}{63} \div 3\frac{840}{17} = 24\frac{192}{63} = 3 \quad 2\frac{20}{63} \text{ Meal.} \\ \text{Also } 9\frac{94}{63} \div 3\frac{360}{17} = 21\frac{98}{63} = 1 \quad 7\frac{10}{63} \text{ Oatmeal.} \end{array} \right\} \text{per Bush.}$$

(15) First, $\frac{12}{11}$ of $\frac{3}{8} = \frac{36}{88}$, or $\frac{9}{22}$, B's Part.

$$\text{And } \frac{4}{13}, \frac{9}{22}, \frac{1}{6} = \frac{528}{1716}, \frac{702}{1716}, \frac{286}{1716}, \text{ or } \frac{264}{858}, \frac{351}{858}, \frac{143}{858}.$$

$$\text{Then } \frac{351}{858} - \frac{264}{858} = \frac{86}{858} = \text{Difference betwixt A's and B's.}$$

$$\text{And } \frac{143}{858} + \frac{86}{858} = \frac{230}{858} \text{ C's Part.}$$

$$\text{Also } \frac{264}{858} + \frac{351}{858} + \frac{230}{858} = \frac{845}{858} = \text{A's} + \text{B's} + \text{C's.}$$

$$\frac{858}{858} - \frac{845}{858} = \frac{12}{858} \text{ D's Part.}$$

£. s. d. Rem.

$$\text{As } 858 : 400 :: \left\{ \begin{array}{l} 264 : 123 \quad 1 \quad 6\frac{1}{4} \quad 726 \text{ A's} \\ 351 : 163 \quad 12 \quad 8\frac{1}{2} \quad 780 \text{ B's} \\ 230 : 107 \quad 4 \quad 6\frac{1}{4} \quad 54 \text{ C's} \\ 12 : 6 \quad 1 \quad 2\frac{1}{2} \quad 156 \text{ D's} \end{array} \right\} \text{Share.}$$

(16) See Question IX. in Exercise for Fractions.

24. DOUBLE FELLOWSHIP;

O R,

FELLOWSHIP WITH TIME.

(2) $4 \times 4 \times 50 = 800$ Officers Pay and Time.
 $8 \times 4 \times 40 = 1280$ Midshipmen's ditto.
 $120 \times 3 \times 28 = 10080$ Sailors ditto.

12160 Sum.

As 12160 : 4000 :: $\left\{ \begin{array}{l} 800 : 263 \text{ } 3 \text{ } 1\frac{3}{4} \text{ } 704 \text{ Officers.} \\ 1280 : 421 \text{ } 1 \text{ } 0\frac{1}{2} \text{ } 640 \text{ Midship.} \\ 10080 : 3315 \text{ } 15 \text{ } 9\frac{1}{4} \text{ } 1088 \text{ Sailors.} \end{array} \right.$

$\begin{array}{l} \text{£. } s. \text{ } d. \\ 263 \text{ } 3 \text{ } 1\frac{3}{4} \div 4 = 65 \text{ } 15 \text{ } 9\frac{1}{4} \text{ Officers.} \\ 421 \text{ } 1 \text{ } 0\frac{1}{2} \div 8 = 52 \text{ } 12 \text{ } 7\frac{1}{2} \text{ Midsh.} \\ 3315 \text{ } 15 \text{ } 9\frac{1}{4} \div 120 = 27 \text{ } 12 \text{ } 7\frac{1}{2} \text{ Sailors.} \end{array} \left. \vphantom{\begin{array}{l} 263 \\ 421 \\ 3315 \end{array}} \right\} \text{each, Fractions rejected.}$

(3) $60 \times 4 = 240$ A's Stock and Time.
 $40 \times 5 = 200$ B's ditto.
 $30 \times 3 = 90$ C's ditto.

530 Sum of their Stocks and Times.

As 530 : 40 :: $\left\{ \begin{array}{l} 240 : 18 \text{ } 2 \text{ } 3 \text{ } 36 \text{ A's} \\ 200 : 15 \text{ } 1 \text{ } 10\frac{1}{2} \text{ } 30 \text{ B's} \\ 90 : 6 \text{ } 15 \text{ } 10 \text{ } 40 \text{ C's} \end{array} \right\} \text{Part.}$

£. Mo.

(4) $\left. \begin{array}{l} 400 \times 6 = 2400 \\ 200 \times 6 = 1200 \end{array} \right\} = 3600 \text{ A's Stock and Time.}$
 $\left. \begin{array}{l} 360 \times 7 = 2520 \\ 460 \times 2 = 920 \end{array} \right\} = 4460 \text{ B's ditto.}$
 $340 \times 3 = 1020$

£. Mo.

$$\left. \begin{array}{l} 190 \times 8 = 1520 \\ 300 \times 2 = 600 \\ 200 \times 2 = 400 \end{array} \right\} = 2520 \text{ C's ditto.}$$

then $3600 + 4460 + 2520 = 10580$ their Sum.

	£.	s.	d.	Rem.	
As 10580 : 460 ::	3600	156	10	5	920 A's
	4460	193	18	3	552 B's
	2520	109	11	$3\frac{1}{2}$	$105\frac{4}{8}$ C's
					} Share.

(5) Reciprocally, As 19 : 84 12 $\frac{6}{7} :: 7$
 $6 \times 3 + 1 = 19$

$$7 \overline{) 1607 \ 17 \ 6}$$

Answer, B's Adventure, £. 229 13 $11\frac{1}{7}$

(6) First, $20 + £. 26 \ 5s. + 32 = £. 78 \ 5s.$ Sum of their Gain.

	£.	s.	£.	s.	d.	Rem.	
As 78 5 : 640 ::	20	0	163	11	$6\frac{3}{4}$	225	A's
	26	5	214	13	11	980	B's
	32	0	261	14	6	$13\frac{60}{65}$	C's
							} Stock and Time.

Therefore,—

£.	s.	d.	£.	s.	d.	
163	11	$6\frac{3}{4} \div 9 = 18$	3	6		A's
214	13	$11 \div 7 = 30$	13	5		B's
261	14	$6 \div 5 = 52$	6	$10\frac{3}{4}$		C's
						} Stock, Fract. rejected.

(7) First, $£. 72 \ 10s. = 1450s.$ C's Stock; then by Sect. 15.
 S. T. G.

1450 9 23 Here $23 \times 6 = 138$ Divisor.

And $1450 \times 9 \times 13 = 169650$ Dividend.

$138 \overline{) 169650} (1229s. \ 4d. \ 1\frac{2}{3} = 61 \ £. \ 9s. \ 4d. \text{ A's Stock.}$

Again,—

Again,—

S. T. G.

1450 9 23 Here $23 \times 5 = 115$ Divisor.
 — 5 18 And $1450 \times 9 \times 18 = 234900$ Dividend.

£. s. d.

115)234900(2042s. $7\frac{1}{4}d. \frac{25}{115} = 102 \quad 2 \quad 7\frac{1}{4}$ B's Stock.
 74 10 0 C's —.
 61 9 4 A's —.

Answer, the whole Stock, £. 236 1 11 $\frac{1}{4}$

(8) Here suppose X's Gain to be 2
 Then by the Question Y's will be 3 } = 9 their Sum.
 And C's - - - 4 }

£. s. d. Pence.

Now as 9 : 420 :: { 2 : 93 6 8 = 22400 X's
 3 : 140 0 0 = 33600 Y's } Gain.
 4 : 186 13 4 = 44800 Z's }

Therefore, $22400 \times 4 = 89600$ X's
 $33600 \times 6 = 201600$ Y's } Gain and Time.
 $44800 \times 9 = 403200$ Z's }

694400 their Sum.

Now rejecting the 0's we shall have the following Proportion.

£. s. d. Rem.

As 6944 : 4262 :: { 896 : 549 18 8 $\frac{1}{2}$ 448 X's
 2016 : 1237 7 1 4480 Y's } Stock.
 4032 : 2474 14 2 $\frac{1}{4}$ 2016 Z's }

(9) First, $\frac{1}{5}, \frac{2}{3} = \frac{3}{15}, \frac{10}{15} = \frac{13}{15}$, then $\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$ B's Gain;
 and rejecting the Denominator, we shall have A's Gain
 = 3, B's 2, and C's 10; then by Sect. 15.

S. T. G.

400 7 10 Here $10 \times 5 = 50$ Divisor.
 — 5 3 And $400 \times 7 \times 3 = 8400$ Dividend.
 5,0)840,0(168 £. A's Stock.

Again,

S. T. G.

400 7 10 Here $10 \times 8 = 80$ Divisor.
 — 8 2 And $7 \times 2 \times 400 = 5600$ Dividend.
 8,0)560,0(70£. B's Stock.

(10)

	Days.	Days.
From Feb. 10, to June 10,	= 122	} = 208 A's Time.
Jan. 17, to April 30,	= 61	
July 14, to 14 after St. James's,	= 24	
Aug. 2, to Nov. 13,	= 104	} = 179 B's.
May 1, to July 24,	= 55	
Sept. 30, to October 19,	= 20	
		387 Horse — in Use.

	D.	£.	s.	d.	Rem.	
D. £. s.	208	4	0	$7\frac{1}{4}$	$\frac{297}{387}$	A's
As 387 : 7 10 ::	179	3	9	$4\frac{1}{2}$	$\frac{90}{387}$	B's

} Share.

25. B A R T E R.

(2) $4d.\frac{1}{3}$ 45 at 1s. 4d. then per Sect. 10. $60 \div 18 = 3\frac{1}{3}$ Yds.

15
 —
 60s.

(3) First, 30 Cwt. = 360 lb. at $7\frac{1}{2}d.$ per lb.

$6\frac{1}{2}d.$ | 3360
 —
 $1\frac{1}{2}d.$ | $\frac{1}{3}$ 1680
 —
 420

d. d. s. s.
 then—If 6 : $7\frac{1}{2}$:: 36 : 45

Again,—If 45s. : 1 Cwt. :: 2100s.

45)2100(46 Cwt. 2 qrs. $18\frac{2}{3}lb.$ the

2,0)210,0 = Answer.

105 £. = 2100s.

s. d. s. s. d.
 (4) If 8 6 : 10 :: 1 6
 17 3 3
 —

17)30(1s. 9d. $\frac{3}{7}$ per lb. Answer.

(5) First, $20\text{£} = 4800$ Pence dealt for.

d. d. d.

Then—If $6 : 4 :: 4800$
 $\underline{4800}$

6) $19200(3200d. = \text{£} 13 \text{ } 6s. \text{ } 8d.$ real Value of B's Currants. Now $20\text{£} \div 2 = 10\text{£}$, or $200s.$ and $6s. \text{ } 8d. = 80d.$

If $7s. : 80d. :: 200s.$

$\underline{200}$

7) $16000($

$\underline{\text{£} \text{ } s. \text{ } d.}$
 $2285\frac{1}{2}d \frac{6}{7} = 9 \text{ } 10 \text{ } 5d. \frac{16}{27}$ real Value of B's Candles.
 $10 \text{ } 0 \text{ } 0$ gave in ready Money.

A received $19 \text{ } 10 \text{ } 5d. \frac{16}{27}$
 For $13 \text{ } 6 \text{ } 8$

Answer, $\text{£} 6 \text{ } 3 \text{ } 9d. \frac{16}{27}$ A got of B.

(6) Cwt. $\text{£} \text{ } s.$ $\text{£} \text{ } s.$ $\text{£} \text{ } s.$

First, 18 at 1 11 per C. $= 1 \text{ } 11 \times 3 \times 6 = 27 \text{ } 18$ real Value.

And ditto at 2 $\underline{2} = 2 \text{ } 2 \times 3 \times 6 = 37 \text{ } 16$ Adv. Value.

$\therefore$ A advanced his Sugar, $\underline{9 \text{ } 18}$

Also, 3) $37 \text{£} \text{ } 16s. (= 12 \text{£} \text{ } 12s.$ A received in Cash, and

$\text{£} 37 \text{ } 16s. - \text{£} 12 \text{ } 12s. = 25 \text{£} \text{ } 4s.$ in Paper.

Again, $15s. \text{ } 6d. - 14s. = 1s. \text{ } 6d.$ B advanced his Paper.

Therefore,

s. d. d. $\text{£} \text{ } s.$

If $15 \text{ } 6 : 18 :: 25 \text{ } 4$

$\times 2$

$\underline{31}$

$\times 40$ Six-pences in a $\text{£}.$

$\underline{1008}$

$\times 18$

31) $18144(585d. \text{ or } \text{£} 2 \text{ } 8s. \text{ } 9\frac{1}{4}d. \frac{5}{11}$ B's Advance on his Paper.

Then from £. 9 18s. take £. 2 8s. $9\frac{1}{4}d. \frac{5}{31}$. Rem. £. 7 9s.
 $2\frac{1}{2}d. \frac{2}{31}$ in A's Favour, the Answer.

£. s. £. s. £. s.
 (7) First, 5 6—4 5=1 1 Gain per Piece.
 And 5 6÷2 =2 13 required down.

Also, 4 5—2 13=1 12 Value of the Half remaining.
 +1 1

Made £. 2 13 of the Half remaining.

£. s. £. s. d.
 Then,—If 1 12 : 2 13 :: 3
 or 384d. : 53s. :: 3d.
 ×3 ×12.

384)1908($4\frac{3}{4}d. \frac{3}{8}\frac{6}{4}$ per lb. Answer.

£. s.
 (8) First, $\left. \begin{array}{r} 13 \ 0 \\ 11 \ 10 \end{array} \right\} \times 50 = \left\{ \begin{array}{l} 650 \text{ advanced Val. of the Cloth.} \\ 575 \text{ real Value.} \end{array} \right.$

75 £. Gain by the Cloth.

s. d.
 Again, 2s. 6d. ×2=5 0 adv. Value of the Wool per Tod.
 —4 2 real Value.

0 10 Gain per Tod, or 5d. per Stone.

s. d. St. £.
 If 2 6 : 1 :: 650
 ×2 ×40
 ———
 5 5)26000

$4d. \frac{1}{60}$ 5200 Stone at 5d. per Stone.

$1\frac{1}{4}$ 86 13 4
 21 13 4

108 6 8 gained by the Wool.

A Sack = 26 Stone; therefore $5200 \div 26 = 200$ Sacks, which will pay for the Cloth.

$\therefore \text{£. } 108 \text{ } 6s. \text{ } 8d. - 75\text{£.} = \text{£. } 33 \text{ } 6s. \text{ } 8d.$ B's Gain by this Affair.

d. d. s. d.

(9) If $10 : 16 :: 20 \text{ or } 240$

240

10) 3840

12) 384

32s. advanced Value of the Malt.
20 real Value.

12s. B's Gain per Quarter.

Here 30 Guineas $\times 21 = 630$ Shillings, which $\div$ by 12 = $52\frac{1}{2}$ Quarters, or 420 Bushels, the Answer.

(10) 720 Ells at 5s. per E. or $\frac{1}{4} = 180\text{£.}$ real. } Value of the
Ditto — 6s. 8d. or $\frac{1}{3} = 240$ adv. } Holland.
240£. at 10£. per C. or $\frac{1}{10} = 24$ Discount.

Then $240 - 24 = 216\text{£.}$

And $216 \div 2 = 108$ paid in ready Money.

£. £. s.
Then,—If $216 : 180 :: 36$
20 180

4320) 6480 (£. 15s. real Value per lb.

Now $108\text{£.} = 2160s.$ which $\div 30 = 72\text{ lb.}$ the Quantity delivered.

s. d. s. d.

(11) If 8 or 96 : 10 :: 6

6

—
60

12

96) 720 ($7\frac{1}{2}d.$ Barter Price of the Pamphlets.

£.

100 Reams at 8s. = 40 real } Value of the
 Ditto, at 10s. = 50 advanced } Paper.
 $\frac{1}{4} = 50 \text{ £.} \div 4 = 2 \text{ £. } 10\text{s.}$ B to have in Cash.
 40 £. Value of B's Pamphlets.
 $\times 40$ Six-pences in a £.

1600 Pamphlets to be delivered.

From 40 £. take 12 £. 10s. Rem. 27 £. 10s. what they then
 stood him in, so that the Advantage to B is 27 £. 10s.

(12) First, 140lb. 110z = 16910z.

5s. $\left| \frac{1}{4} \right|$ 1691 at 6s. 4d.

1 $\left| \frac{1}{5} \right|$ 422 15
 4d. $\left| \frac{1}{3} \right|$ 84 11
 28 3 8

£. 535 9 8 real Value
 of the Plate.

6 8 $\left| \frac{1}{3} \right|$ 1691 at 7s. 2d.

6 $\left| \frac{1}{40} \right|$ 563 13 4
 42 5 6

£. 605 18 10 adv. V.

£. s. s. £. s. d.
 From 605 18 10 take 535 9 8 Remains £. 70 9s. 2d. the
 whole Advantage of A's Plate.

$\frac{1}{7}$ £. 605 18 10 —
 86 11 $3\frac{1}{7}$ received in ready Specie.

£. 519 7 $6\frac{6}{7}$

7C. 2qrs. 18lb. = 858lb. at 11s. 2d. =

10s. $\left| \frac{1}{2} \right|$ 858 at 11s. 2d.

1 $\left| \frac{1}{10} \right|$ 429
 2d. $\left| \frac{1}{6} \right|$ 42 18
 7 3

479 1 adv. Val. of
 B's Tea.

5s. $\left| \frac{1}{4} \right|$ 858 at 9s. 6d.

4 $\left| \frac{1}{5} \right|$ 214 10
 6d. $\left| \frac{1}{8} \right|$ 171 12
 21 9

407 11 real Value of
 B's Tea.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 519 \quad 7 \quad 6\frac{6}{7} \\ -479 \quad 1 \quad 0 \\ \hline \end{array}$$

$$\text{£.} \quad 40 \quad 6 \quad 6\frac{6}{7} \text{ Dif. allowed by A.}$$

A's Advantage by the Rise of his Plate, 70 9 2

$$\text{B's whole Advantage,} \quad - \quad - \quad \text{£.} \quad 41 \quad 7 \quad 4\frac{5}{7}$$

(13) 14C. 2qrs. 25lb. at 3£. 3s. per C. = 46£. 7s. 6 $\frac{3}{4}$ d. advance Value of A's Hops.

$$\begin{array}{r} \text{s.} \quad \text{s.} \quad \text{s.} \\ \text{If } 59 : 63 :: 6 \\ \quad \quad 6 \end{array}$$

59)378(6s. 4 $\frac{3}{4}$ d. $\frac{31}{59}$ advanced Price of B's Wine per Gallon. Now 1 $\frac{1}{2}$ bbd. = 94 $\frac{1}{2}$ gal. at 6s. 4 $\frac{3}{4}$ d. $\frac{31}{59}$
 $10 \times 9 + 4\frac{1}{2} = 94\frac{1}{2}$

$$\begin{array}{r} \text{B's Wine comes to} \quad 39 \quad 5 \quad 5\frac{1}{4}d. \frac{9}{59} \\ \text{A's Hops} \quad - \quad 46 \quad 7 \quad 6\frac{3}{4} \\ \hline \end{array}$$

$$\text{Answ.} \quad \text{£.} \quad 16 \quad 2 \quad 1\frac{1}{4}d. \frac{59}{59}$$

26. LOSS and GAIN.

(2) First, 9C. 2qrs. 18lb. = 1082lb. and 46£. + 12£. 12s. = £. 58 12s. or 1172s. sold for; therefore,
 1082)1172(1s. 0 $\frac{3}{4}$ d. $\frac{107}{1082}$ per lb. the Answer.

(3) First, 10s. 6d. — 8s. 6d. = 2s. Gain by 8s. 6d. — then

$$\begin{array}{r} \text{s.} \quad \text{d.} \quad \text{s.} \quad \text{£.} \\ \text{If } 8 \quad 6 : 2 :: 100 \\ \quad \quad 2 \quad \quad 40 \\ \hline 17 \quad \quad 4000 \\ \hline \quad \quad 2 \end{array}$$

$$17)8000(470s. 7d. $\frac{1}{17}$ = 23£. 10s. 7d. $\frac{1}{17}$ Answ.$$

(4)

(4) First, $100\text{£} + 8 = 108\text{£}$. Amount; then

If $\begin{array}{c} s. \\ 5 \end{array} : \begin{array}{c} s. \ d. \\ 108 : : 6 \ 3 \end{array}$ then $135 - 100 = 35\text{£}$. Answer.
 $\begin{array}{r} 20d. \ 25 \\ \hline \end{array}$

Mr. WEBSTER's Answer is 10£ .

$2,0)2700(135\text{£}$.

(5) First, $100 + 7\text{£}$. $10\text{£} = 107\text{£}$. 10s . Amount.

If $\begin{array}{c} s. \ \text{£} \end{array} : \begin{array}{c} s. \ \text{£} \end{array} : : \begin{array}{c} s. \ d. \\ 5 \ 9 \end{array}$
 $\begin{array}{r} 4 \quad 20 \quad 4 \\ \hline 20 \quad 2150 \quad 23 \\ \times 23 \\ \hline \end{array}$

$2,0)49450(2472\text{s. } 6d. = \text{£} .123 \ 12\text{s. } 6d.$ Amount per C.
 Then $\text{£} .123 \ 12\text{s. } 6d. - 100 = \text{£} .23 \ 12\text{s. } 6d.$ the Answer.

Mr. STONEHOUSE's Answer is $\text{£} .8 \ 12\text{s. } 6d.$

(6) First, $100 + 15 = 115\text{£}$. Amount; then

If $\begin{array}{c} s. \ d. \\ 11 \ 6 \end{array} : \begin{array}{c} s. \\ 115 : : 12 \end{array}$ Here $115 \times 24 = 2760 \div 23 =$
 $\begin{array}{r} 2 \quad 2 \\ \hline 23 \quad 24 \end{array}$ $\text{£} .120$ Amount per Cent.

Therefore $120 - 100 = 20\text{£}$. per Cent. Answer.

Mr. HILL makes the Answer $\text{£} .15 \ 13\text{s. } 0\frac{1}{2}d. \frac{2}{3}$

(7) $3d. \left| \frac{1}{4} \right| 500$ at $1\text{s. } 3d.$ Now $100 - 9 = 91\text{£}$. or 1820s . then

$\begin{array}{c} s. \ \text{£} \end{array} : \begin{array}{c} s. \ \text{£} \end{array} : : \begin{array}{c} s. \\ 1820 : 100 : : 625 \end{array}$
 $625 = 31 \ 5$

$1820)62500(\text{£} .34 \ 6\text{s. } 9\frac{3}{4}d. \frac{6}{18} \frac{2}{3}$ Amt.
 Therefore $\text{£} .34 \ 6\text{s. } 9\frac{3}{4}d. \frac{6}{18} \frac{2}{3} - 31\text{£} .5\text{s.} = \text{£} .3 \ 1\text{s. } 9\frac{3}{4}d. \frac{6}{18} \frac{2}{3}$
 the Answer.

Mr. DILWORTH's Answer is $\text{£} .2 \ 16\text{s. } 3d.$

(8) First, $100 + 25 = 125\text{£}$. Amount; then

$\begin{array}{c} \text{£} \ s. \end{array} : \begin{array}{c} \text{£} \end{array} : : \begin{array}{c} \text{£} \end{array} : \begin{array}{c} \text{£} \end{array} : : \begin{array}{c} s. \ d. \\ 6 \ 15 : 125 : : 8 : 148 \ 2 \ 11\frac{1}{2} \frac{3}{5} \end{array}$ Amount per Cent.

Then

Then £. 148 2s. 11½d. $\frac{30}{135}$ — 100 = £. 48 2s. 11½d. $\frac{30}{135}$ the Answer.

Mr. WALKINGHAM's Answer is £. 29 12s. 7d. $\frac{1}{8}$

(9) First $100 + 30 = 130$ £. Amount; then

£. s. £. £. s. £. s. d.
If 3 10 : 130 :: 4 5 : 157 17 1½ $\frac{6}{8}$ Amount per Cent.
Then £. 157 17s. 1½d. $\frac{6}{7}$ — 100 = £. 57 17s. 1½d. Answer.

(10) First, $100 - 17 = 83$ £. and $100 + 20 = 120$ £. then

£. £. £. s. £. s.
If 83 : 100 :: 52 10 : 63 5 $\frac{5}{8}$
£. £. £. s. £. s. d.
Again, If 100 : 120 :: 63 5 $\frac{5}{8}$: 75 18 0 $\frac{3}{4}$ Worth.
Sold for 52 10 0

Answer, £. 23 8 0 $\frac{3}{4}$ $\frac{78}{100}$

(11) 112 lb. at 2s. 11d. $\frac{2}{3}$ = £. 16 12s. 10d. $\frac{2}{3}$ sold for at Turkey.

Then, £. 16 12s. 10d. $\frac{2}{3}$ ÷ 2 = £. 8 6s. 5d. $\frac{1}{3}$ cost him.

Therefore, from £. 8 6s. 8d. $\frac{1}{3}$, take 3 £. Rem. 6s. 5d. $\frac{1}{3}$ Loss, the Answer.

(12) First, 4s. 3d + 2d. = 4s. 5d. prime Cost and Charges.

s. d. s. £. s. d.
Then—If 4 5 : 6 :: 100 : 27 16 11 $\frac{3}{4}$ $\frac{5}{8}$
Again, $100 - 12 = 88$ £.

£. £. s. d.
Also—If 100 : 88 :: 27 16 11 $\frac{3}{4}$ $\frac{5}{8}$

£. s. d.
Or, as 5088000 : 88 :: 6912000 : 119 10 11 $\frac{1}{4}$ $\frac{1440}{5088}$ Amt.
Then, £. 119 10s. 11 $\frac{1}{4}$ d. $\frac{1440}{5088}$ — 100 = £. 19 10s. 11 $\frac{1}{4}$ d. $\frac{1440}{5088}$, gained per lb. the Answer.

(13) First, $800 \times 14 = 11200$ lb. at 12½d. per lb.
Then $11200 \times 12\frac{1}{2} = 140000$ d. Value of the Anchovies;
which ÷ by 749,0, gives 18½d. $\frac{574}{749}$ Amount of 12½d.
Again, $100 + 17 = 117$ £. Amount; then

If

£. £. d.

If 117 : 100 :: $18\frac{1}{2}\frac{574}{749}$

Or, as 84127680 : 100 :: 5542600 : 1s. $3\frac{3}{4}d.$ $\frac{8085216}{8412768}$, the Answer.

(14) First, £. 41 3s. 4d. = 9880d. and 3s. 1d. = 37d.

Then, $9880 \div 37 = 267\frac{1}{37}lb.$ at 3s. 1d. per lb. bought.

Again, £. 34 2s. 6d. = 8190d. and 4s. 6d. = 54d.

Then, $8190 \div 54 = 151\frac{2}{3}lb.$ sold at 4s. 6d. per lb.

Therefore, $267\frac{1}{37} - 151\frac{2}{3} = 267\frac{1}{37} - 151\frac{74}{111} = 115\frac{40}{111}lb.$

Spoiled at 3s. 1d. = 37d. so that $115\frac{40}{111} \times 37 = 4268\frac{37}{111}d.$

or £ 17 15s. $8\frac{1}{4}d.$ $\frac{37}{111}$ prime Cost of the Goods spoiled.

(15) First, $\frac{3}{8}$ of 11s. = $3\frac{3}{8}$, or 4s. $1\frac{1}{2}d.$ gained per Thousand when he sold them at 11s. $\therefore$ 11s. - 4s. $1\frac{1}{2}d.$ = 6s. $10\frac{1}{2}d.$ per Thousand prime Cost; then

As 6s. $10\frac{1}{2}d.$: 11s. :: 100£. : 160£. Amount of 100£.

Again, as 11s. : 160£. :: 13s. 6d. : 196£. 7s. $3\frac{1}{4}d.$ $\frac{1}{11}$ Amount per Cent. at 13s. 6d.

Then, £. 196 7s. $3\frac{1}{4}d.$ $\frac{1}{11}$ - 100 = £. 96 7s. $3\frac{1}{4}d.$ $\frac{1}{11}$ gained per Cent. the Answer.

(16)

£.	£.	s.	d.
5	$\frac{1}{20}$	500	16 8
1	$\frac{1}{5}$	25	0 10
$\frac{1}{2}$	$\frac{1}{2}$	5	0 2
		2	10 1

 } C's Profit.

Then 500 16 8 - 32 11 1 = 468 5 7 Wine cost C.
Also, 468 5 7 - 38 11 6 = 429 14 1 ——— cost B.

Again, £. 20 $\frac{1}{3}$ 429 14 1 at $4\frac{1}{3}$ per Cent.

£.	s.	d.
4	$\frac{1}{3}$	85 18 $9\frac{3}{4}$
$\frac{1}{3}$	$\frac{1}{2}$	17 3 9
		1 8 $7\frac{3}{4}$

 } A's Profit.

Then

£. 429 14 1 - 18 12 $4\frac{3}{4}$ = 411 1 $8\frac{1}{4}$ cost A.

Also,

Also, £.411 1s. 8d. = 8221s. 8 $\frac{1}{4}$ d. and 15 Pipes $\times$ 126 = 1890 Gallons; then 1890)8221s. 8 $\frac{1}{2}$ d. (4s. 4 $\frac{1}{2}$ d. $\frac{3}{8}$ per Gallon, the Answer.

(17) First, $\frac{2}{7}$ of 480£. 12s. = £.961 4s. = £.137 6s. 3d. $\frac{3}{7}$
Cost of the damaged Goods; then £.137 6s. 3d. $\frac{3}{7}$ — £.48 18s. = £.88 8s. 3d. $\frac{3}{7}$ made of the damaged Goods.

s. d. Yds. £. s. d.

$\therefore$ As 5 6 :: 1 :: 88 8 3 $\frac{3}{4}$: 321 $\frac{2}{3}$ $\frac{3}{4}$ Yards damaged.

321 $\frac{2}{3}$ $\frac{3}{4}$ $\times$ 7 = 2250 $\frac{2}{3}$ $\frac{5}{2}$ Yards, which $\div$ 2 = 1125 $\frac{1}{4}$ $\frac{2}{6}$ Yards, bought in all.

Then 1125 $\frac{1}{4}$ $\frac{2}{6}$ — 321 $\frac{2}{3}$ $\frac{3}{4}$ = 803 $\frac{3}{4}$ $\frac{2}{6}$ Yards undamaged.

Again,—From £.480 12s. take £.88 8s. 3d. $\frac{3}{7}$. Rem. £.392 3s. 8d. $\frac{4}{7}$ to be made of the undamaged Goods.

Yds. £. s. d. Ell.

$\therefore$ If 803 $\frac{3}{4}$ $\frac{2}{6}$: 392 3 8 $\frac{4}{7}$:: 1

Or, as 37 $\frac{1}{4}$ $\frac{3}{6}$ $\frac{1}{2}$: 658 $\frac{8}{7}$ 72 :: $\frac{5}{4}$: 152 $\frac{1}{10}$ $\frac{9}{39}$ $\frac{4}{67}$ $\frac{3}{08}$ = 12 s. 2 $\frac{1}{4}$ d.
 $\frac{1}{2}$ $\frac{4}{5}$ $\frac{7}{8}$ $\frac{5}{6}$ $\frac{7}{67}$ $\frac{5}{7}$ per Ell.

27. ALLIGATION MEDIAL.

Gal. at s. s.

(2) 14 $\times$ 8 = 112
12 $\times$ 6 = 72
10 $\times$ 7 = 70
20 $\times$ 4 = 80
8 $\times$ 9 = 72

As 64 : 406 :: 1 Gal. : 6s. 4 $\frac{1}{2}$ d. the Answer.

lb. at s. s.

(3) 20 $\times$ 12 = 240
12 $\times$ 8 = 96
16 $\times$ 6 = 96
12 $\times$ 4 = 48

As 60 : 480 :: 1 lb. : 8s. per lb. Answer.

Car. lb. Car.

(4) $22 \times 4 = 88$

$20 \times 3 = 60$

$18 \times 3 = 54$

— — —

10

As 10 : 202 :: 120 $\frac{1}{5}$ Carrats fine, the Answer.

£. £. d. d.

(5) $13 \times 80 = 1040$ | If 100 : 110 :: 2600 : 11 10 $\frac{3}{4}$ or 2860
 $20 \times 60 = 1200$ | then 2850 — 2600 = 260. Also 2600
 $10 \times 36 = 360$ | + 260 = 2860.

— *qrs.* —

Sum 43 = 172, 2600 ∴ As 172 : 2860 :: 1 : 16 $\frac{1}{2}$ d. $\frac{88}{172}$, the Answer.

28. ALLIGATION ALTERNATE.

(2)

Bush.

s. d.

60d. $\left\{ \begin{array}{l} 36 \\ 48 \\ 54 \\ 72 \end{array} \right\}$ | — — — 12 | at $\left\{ \begin{array}{l} 3 \text{ } 0 \\ 4 \text{ } 0 \\ 4 \text{ } 6 \\ 6 \text{ } 0 \end{array} \right\}$ Answer.
 24 + 12 + 6 = 42

(3) *Car.*

Car.

Car.

20 *Car.* $\left\{ \begin{array}{l} 18 \\ 13 \\ 19 \\ 16 \end{array} \right\}$ | 2 + 1 + 4 = 7 | at $\left\{ \begin{array}{l} 8 \\ 23 \\ 19 \\ 16 \end{array} \right\}$ Carrats fine, the Answer.

(4)

lb.

d.

6d. $\left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\}$ | — — — 1 | of $\left\{ \begin{array}{l} 10 \\ 7\frac{1}{2} \\ 5 \\ 4\frac{1}{2} \end{array} \right\}$ per lb. Answer.
 — — — 1 $\frac{1}{2}$
 — — — 4
 — — — 1 $\frac{1}{2}$

As Examples of this Nature will admit of as many different Answers as there are different Ways of linking together a larger Price and a lesser than middle or mean Rate proposed, so consequently the last will admit of seven different Ways or Answers.

29. ALLIGATION PARTIAL.

(2) *d.* *Diff.* *Diff. Gal.* *Diff. Gal.*

$$\begin{array}{r} d. \left\{ \begin{array}{l} 84 \\ 48 \end{array} \right\} \\ 72 \left\{ \begin{array}{l} 78 \\ 96 \end{array} \right\} \end{array} \quad \begin{array}{r} - \\ 2+6+24=42 \\ - \\ - \\ - \end{array} \quad \begin{array}{r} 24 \\ 24 \\ 24 \end{array} \quad \left| \begin{array}{l} \text{If } 24 : 28 :: 42 : 49 \text{ at } 4s. \\ \text{per Gal. and 28 Gal. of} \\ \text{each Sort of the other.} \end{array} \right.$$

(3) *d.* *Diff.* *Diff. lb.* *s. d.*

$$\begin{array}{r} 96 \left\{ \begin{array}{l} 144 \\ 108 \\ 90 \\ 78 \end{array} \right\} \end{array} \quad \begin{array}{r} 6 \\ 18 \\ 48 \\ 12 \end{array} \quad \left| \begin{array}{l} \text{Diff. lb.} \\ \text{If } 6 : 36 :: \end{array} \right. \quad \begin{array}{l} \left\{ \begin{array}{l} 18 : 108 \\ 48 : 288 \\ 12 : 172 \end{array} \right\} \text{ of } \left\{ \begin{array}{l} 9 \text{ } 0 \\ 7 \text{ } 6 \\ 6 \text{ } 6 \end{array} \right\} \text{ per } \\ \text{lb.} \\ \text{An.} \end{array}$$

(4) *d.* *Diff.* *Diff. lb.* *Diff. lb.*

$$\begin{array}{r} d. \left\{ \begin{array}{l} 30 \\ 20 \end{array} \right\} \\ 24 \left\{ \begin{array}{l} 18 \\ 15 \end{array} \right\} \end{array} \quad \begin{array}{r} 4+6+9=19 \\ - \\ - \\ - \end{array} \quad \begin{array}{r} 6 \\ 6 \\ 6 \end{array} \quad \left| \begin{array}{l} \text{If } 19 : 120 :: 6 : 37\frac{1}{3}, \text{ so that} \\ \text{with 120 lb. of the given} \\ \text{Quantity, there must be } 37\frac{1}{3} \\ \text{lb. of each Sort of the} \\ \text{other, the Answer.} \end{array} \right.$$

30. ALLIGATION TOTAL.

(2) *d.* *Diff. lb.* *d.*

$$\begin{array}{r} 6d. \left\{ \begin{array}{l} 3 \\ 4 \\ 7 \\ 8 \end{array} \right\} \end{array} \quad \begin{array}{r} 1 \\ 2 \\ 3 \\ 2 \end{array} \quad \left| \begin{array}{l} \text{Sum. lb.} \\ \text{If } 8 : 112 :: \end{array} \right. \quad \begin{array}{l} \left\{ \begin{array}{l} 1 : 14 \\ 2 : 28 \\ 3 : 42 \\ 2 : 28 \end{array} \right\} \text{ at } \left\{ \begin{array}{l} 3 \\ 4 \\ 7 \\ 8 \end{array} \right\} \text{ per lb. the An-} \\ \text{swer.} \end{array}$$

Sum, 8 the whole Quan. 112 lb.

(3) *Diff. oz.* *Car fine.*

$$\begin{array}{r} 20 \left\{ \begin{array}{l} 16 \\ 18 \\ 23 \end{array} \right\} \end{array} \quad \begin{array}{r} 3 \\ 3 \\ 4+2=6 \end{array} \quad \left| \begin{array}{l} \text{Sum. oz.} \\ \text{If } 12 : 60 :: \end{array} \right. \quad \begin{array}{l} \left\{ \begin{array}{l} 3 : 15 \\ 3 : 15 \\ 6 : 30 \end{array} \right\} \text{ of } \left\{ \begin{array}{l} 16 \\ 18 \\ 23 \end{array} \right\} \text{ An.} \end{array}$$

Sum 12 the whole Quan. 60 oz.

(4) *s.* *Diff.* *Diff. lb.* *s.*

$$\begin{array}{r} 7s. \left\{ \begin{array}{l} 5 \\ 6 \\ 8 \\ 9 \end{array} \right\} \end{array} \quad \begin{array}{r} 1 \\ 2 \\ 2 \\ 1 \end{array} \quad \left| \begin{array}{l} \text{Sum. lb.} \\ \text{As } 6 : 168 :: \end{array} \right. \quad \begin{array}{l} \left\{ \begin{array}{l} 1 : 28 \\ 2 : 56 \\ 2 : 56 \\ 1 : 28 \end{array} \right\} \text{ at } \left\{ \begin{array}{l} 5 \\ 6 \\ 8 \\ 9 \end{array} \right\} \text{ per lb. the} \\ \text{Answer.} \end{array}$$

Sum 6

Q₂

EX.

31. E X C H A N G E.

1. With F R A N C E.

Cro. d. Cro. sol. din.

(3) If $1 : 54\frac{1}{2} :: 644 \ 12 \ 8$ or, as $720 : 109 :: 460952 : 69783\frac{8}{20}$, or £. 145 7s. $7\frac{1}{4}$ d.
 $\frac{1}{90}$ the Answer.

Or thus, by Practice.

4 s.	$\frac{1}{5}$	640	at 4s. $6\frac{1}{2}$ d. and 12 Sols. 12 Din. at 4s. $6\frac{1}{2}$ d. per		
	$\frac{1}{8}$	128			
	$\frac{1}{2}$	16			
			1	6	8
			0	0	$11\frac{1}{2}$

10 so.	$\frac{1}{6}$	4	$6\frac{1}{2}$
	$\frac{1}{5}$	0	9 — 2 Rem.
	$\frac{1}{3}$	0	$1\frac{3}{4}$ — 1
		0	$0\frac{1}{2}$ — 1
			11 $\frac{1}{2}$

An. £. 145 7 $7\frac{1}{2}$ as before.

d. Cro. £. s. d.

(4) If $54\frac{1}{2} : 1 :: 145 \ 7 \ 7\frac{1}{2} \ 7\frac{8}{20}$ or, as $78480 : 1 :: 50243768 : £. 640 \ 12s. \ 8d.$ the Answer.

2. With S P A I N.

d. Piece. £. s. d.

(5) If $56 : 1 :: 856 \ 6 \ 8$ or, as $56 : 1 :: 205690 : 3670$ Pieces, Answer.

P. d. Piece. re. mar.

(6) If $1 : 54\frac{1}{4} :: 1426 \ 4 \ 26$ or, as $272 : 217 :: 84203378 : 309571\frac{6}{72}$ qrs. or £. 322
9s. $4\frac{3}{4}$ d. $\frac{66}{72}$, the Answer.

By Practice, thus,

4 s.	$\frac{1}{5}$	1426	at 4s. $6\frac{1}{4}$. — then for 4 R. 26 Mar. thus,		
	$\frac{1}{2}$	285	4	0	
	$\frac{1}{2}$	23	15	4	
	$\frac{1}{8}$	11	17	8	
			1	9	$8\frac{1}{2}$
			0	2	$8\frac{1}{4}$

4 R.	$\frac{1}{2}$	4	$6\frac{1}{4}$
			Rem.
17 M.	$\frac{1}{8}$	2	3 $\frac{1}{2}$
	$\frac{1}{2}$		$3\frac{1}{4}$ $\frac{1}{2}$
			$1\frac{1}{2}$ $\frac{1}{7}$
			2 $8\frac{1}{4}$ nearly.

£. 322 9 $4\frac{3}{4}$ as before.

3. With I T A L Y.

(7)

$$\begin{array}{r|l}
 4s. & \frac{1}{5} | 640 \text{ at } 4s. \text{ } 4d. \text{ per Dollar.} \\
 4d. & \frac{1}{12} | \begin{array}{r} 128 \quad 0 \quad 0 \\ 10 \quad 13 \quad 4 \end{array}
 \end{array}$$

Answer, £. 138 13 4

(8) First, £. 138 13s. 4d. = 33280 Pence.
Then, as 52d. : 1 Dol. :: 33280 : 640 Dollars, the Answer.

(9) 2s. $\frac{1}{10}$ | 8644 at 3s. 11½d. per Pez.

$$\begin{array}{r|l}
 1 & \frac{1}{3} | 864 \quad 8 \\
 6d. & \frac{1}{2} | 432 \quad 4 \\
 & 216 \quad 2 \\
 4 & \frac{1}{3} | 144 \quad 1 \quad 4 \\
 1\frac{1}{2} & \frac{1}{4} | 54 \quad 0 \quad 6 \\
 2s. \text{ } 6d. & \frac{1}{8} | 0 \quad 0 \quad 5\frac{3}{4} \frac{6}{8} = \frac{1}{8} \text{ of } 47\frac{1}{2}d.
 \end{array}$$

Answer, £. 1710 16 3¼ ⅙

(10) First, £. 1710 16s. 4d. = 8211926 Half-pence, and
47½d. = 95 ditto.

Then—If 95d. : 1 Pez. :: 821192d. : 8944 Pez. Answer.

4. With P O R T U G A L.

(11) 5s. $\frac{1}{4}$ | 4760 at 5s. 4d.

$$\begin{array}{r|l}
 4d. & \frac{1}{60} | \begin{array}{r} 1190 \quad 0 \quad 0 \\ 79 \quad 6 \quad 8 \end{array}
 \end{array}$$

Answer, £. 1269 6 8

(12) First, £. 1566 6s. 8d. = 375920d.

Then—If 64d. : 1 Mil. :: 375920d. : 5873 Mil. 720 Reas,
the Answer.

5. With HOLLAND, FLANDERS, and GERMANY.

(13) First, £. 564 10s. 6d. = 135486 Pence, and 34s. 4d. = 412d.

Then, as 412d. : 1£. :: 135486d. : £. 328 16s. 11 $\frac{3}{4}$ d. $\frac{220}{412}$ the Answer.

(14) If $\begin{matrix} \text{£.} & \text{s.} & \text{d.} \\ 1 & 34 & 4 \end{matrix} :: \begin{matrix} \text{£.} & \text{s.} & \text{d.} \\ 328 & 16 & 11\frac{3}{4} \end{matrix}$

or, as 960 grs. : 412 d. :: 315695 grs. : 135485 $\frac{3}{4}$ d. $\frac{1}{12}$, or £. 564 10s. 5 $\frac{3}{4}$ d. $\frac{1}{12}$ the Answer.

By Practice, thus,

10 s.	1/2	328 at £. 1 14s. 4d.
4	1/5	164 0
4d.	1/2	65 12
		5 9 4
		1 9 1 $\frac{1}{4}$
		Answ. £. 564 10 5 $\frac{1}{4}$

10s.	1/2	34 4
5	1/2	17 2
1	1/5	8 7
6d.	1/2	1 8 $\frac{1}{2}$
3	1/2	0 10 $\frac{1}{4}$
2	1/3	0 5
3/4	1/4	0 3 $\frac{1}{4}$
		0 1 $\frac{1}{4}$
		£. 1 9 1 $\frac{1}{4}$

(15) s. d. £. Guil. Stiv. Pen.

If 33	3	1	4200	12	8
12			20		
399			84012		
			2		

399) 168025 (421 £. 2s. 3 $\frac{1}{2}$ d. $\frac{269}{399}$ Answer.

£. s. d. £. s. d.

(16) If 1 : 33 3 :: 421 2 3
or, as 240 : 399d. :: 101067d.

× 399

24,0) 40325733 (168023d.

213d.

8

24,0) 1704 (7 $\frac{24}{40}$ Pen.

Now $168023 \div 40 = 4200$ Guil. 11 Stiv. $15\frac{24}{40}$ Pen. the Answer.

(17) First, £. 242 13s. 6d. = 58242 Pence ; then
4,0)5824,2(1456 Guil. 1 Stiv. the Answer.

6. To Change CURRENT MONEY into BANCO.

(18)

G.	£.	G.	St.
105	:	100	::
20		495	18
21,00		)9918,00	(472 5 11 $\frac{9}{11}$, Answer.

Rem. 6

20

21)120(5 Stivers.

Rem. 15d.

× 16

21)240(11 $\frac{9}{11}$ Pin.

(19)

G.	G.	St.	G.	St.
100	:	470 8	::	105 12 $\frac{1}{2}$
20		20		20
2000		9408		2112
2		2		2
4000				4225
				4908

4,000)39748,800(

2,0)993,7 Stiv. 3 $\frac{1}{5}$ Pin.

Answer, Guild. 496 17 3 $\frac{1}{5}$

7. With V E N I C E.

Du. d.

Du. sol. den.

(20) If 1 : 47 $\frac{6}{8}$:: 4700 10 8

Or, as 240 : 382 :: 1128128 : 224450 $\frac{1}{4}$ d. $\frac{1}{8}$, or, £. 935 4s.
2 $\frac{1}{4}$ d. $\frac{1}{8}$ the Answer.

Or by Practice thus,—

<i>Du.</i> <i>d.</i> $ \begin{array}{r} 4\frac{1}{2} \overline{) 4700} \text{ at } 47\frac{6}{8}; \\ \times 47 \\ \hline 220900 \\ 2350 \\ 1175 \\ 25\frac{2}{8} \\ \hline 12 \overline{) 224450\frac{1}{4}} \\ \hline 2,0 \overline{) 1870,4} - 2 \\ \hline \text{Ans. } \underline{\underline{\text{£. 935 } 4 \frac{2}{4}}} \end{array} $	then for 10 Sol. 8 Den. at <table border="0"> <tr> <td style="vertical-align: top;"> <i>St.</i> <i>d.</i> $\begin{array}{r} 5 \overline{) 47\frac{6}{8}} \\ \hline 11\frac{7}{8} \\ 4 \overline{) 9\frac{4}{8}} \\ \hline 2\frac{3}{8} \\ 4d. \overline{) 2\frac{3}{8}} \\ \hline 4d. \overline{) 2\frac{3}{8}} \\ \hline 25\frac{2}{8} \end{array}$ </td> </tr> </table>	<i>St.</i> <i>d.</i> $ \begin{array}{r} 5 \overline{) 47\frac{6}{8}} \\ \hline 11\frac{7}{8} \\ 4 \overline{) 9\frac{4}{8}} \\ \hline 2\frac{3}{8} \\ 4d. \overline{) 2\frac{3}{8}} \\ \hline 4d. \overline{) 2\frac{3}{8}} \\ \hline 25\frac{2}{8} \end{array} $
<i>St.</i> <i>d.</i> $ \begin{array}{r} 5 \overline{) 47\frac{6}{8}} \\ \hline 11\frac{7}{8} \\ 4 \overline{) 9\frac{4}{8}} \\ \hline 2\frac{3}{8} \\ 4d. \overline{) 2\frac{3}{8}} \\ \hline 4d. \overline{) 2\frac{3}{8}} \\ \hline 25\frac{2}{8} \end{array} $		

8. With POLAND and PRUSSIA.

G.P. £. Flor.

(21) If 270 : 1 :: 4684

Or, as 270 : 1 :: 140520 : 520£. 8s. 10d. $\frac{2}{3}$ Flemish.Again, If 34s. 4d. : 1£. :: 520£. 8s. 10d. $\frac{2}{3}$ Or, as 1236 Thirds : 1 :: 374720 Thirds : 303£. 3s. 5d. $\frac{204}{1236}$ $\frac{1080}{1236}$ the Answer.

(22) *d.* *s.* *d.* | Now 270 Gros. = 9 Florins;
 $4\frac{1}{3} \overline{) 390} \text{ at } 33 \ 4$ | Then—If 1£. : 9 Fl. :: 650£.;
 $\times 33$ | 5850 Florins, the Answer.

$$\begin{array}{r}
 12870 \\
 130 \\
 \hline
 2,0 \overline{) 1300,0} \\
 \hline
 \underline{\underline{650 \text{ £. Flemish.}}}
 \end{array}$$

9. With

9. With R U S S I A.

$$(23) \quad \begin{array}{r} \text{Rub. cop.} \\ 6420 \ 42 \\ \times 100 \\ \hline \end{array}$$

$$122)642042(5262\frac{78}{22} \text{ Rix-dollars.} \\ \times 2\frac{1}{2}$$

$$\left. \begin{array}{r} 10525\frac{34}{22} \\ 2631\frac{39}{22} \end{array} \right\} \text{Florins Current.} \quad (8s)$$

Fl. Cur. Fl. B.

$$\text{If } 103 : 100 :: 13156\frac{73}{22} \text{ Flor. Banco.}$$

$$\text{Or, as } 12566 : 100 :: 1605105 : 12773\frac{2491}{6283}$$

$$\text{Now } 12773\frac{2491}{6283} \times 40 = 510935\frac{5395}{6283} \text{ Pence, and } 34s. 6d. = 414d.$$

$$\text{Then—If } 414d. : 1\text{£} :: 510935\frac{5395}{6283}d.$$

$$\text{Or, as } 2601162 : 1\text{£} :: 3210210000 : \text{£} 1234 \ 2s. 10\frac{1}{2}d. \\ \frac{2087964}{2601162}, \text{ the Answer.}$$

10. With I R E L A N D.

$$(24) \quad \begin{array}{cccc} \text{£.} & \text{£.} & \text{£.} & s. \ d. \\ \text{If } 112 : 100 :: 740 & 14 & 6 \end{array}$$

$$\text{Or, as } 4480d. : 100 :: 29629d. : 661\text{£} \ 7s. 2\frac{1}{4}d. \frac{54}{448} \text{ the Answer.}$$

$$(25) \quad \begin{array}{cccc} \text{£.} & \text{£.} & s. & d. \\ 10 \overline{) 10651} & 14 & 11\frac{3}{4} & \text{at } 112 \text{ per } 100 \\ 2 \overline{) 65} & 3 & 5\frac{3}{4} & \\ & 13 & 0 & 8\frac{1}{4} \end{array}$$

$$\text{Answer, } \text{£} 729 \ 19 \ 1\frac{3}{4}$$

11. With AMERICA and the WEST-INDIES.

$$(26) \quad \begin{array}{cccc} \text{£.} & \text{£.} & \text{£.} & s. \\ \text{If } 164 : 100 :: 1474 & 16 \end{array}$$

$$\text{Or, as } 3280s. : 100 :: 29496s. : \text{£} 899 \ 5s. 4\frac{1}{4}d. \frac{184}{328} \text{ the Sterl. required.}$$

(27)

(27)

£.	£.	s.	d.	
50 $\frac{1}{2}$	943	17	$5\frac{1}{4}$	at 164 per 100
10 $\frac{1}{5}$	471	18	$8\frac{1}{2}$	
2 $\frac{1}{5}$	94	7	$8\frac{3}{4}$	
2 $\frac{1}{5}$	18	17	$6\frac{1}{2}$	
	18	17	$6\frac{1}{2}$	

Answer, £. 1547 18 $11\frac{1}{2}$ Currency.

(28) First, $100 + 30 + 5 = 135$ Amount.

Then—If $135 \text{ £.} : 100 \text{ £.} :: 987 \text{ £.} 12 \text{ s.}$

Or, as $2700 \text{ s.} : 100 :: 19752 \text{ s.} : \text{£. } 731 \text{ } 11 \text{ s. } 1\frac{1}{4} \text{ d. } \frac{1}{3}$ remitted.
 Configned - 640 16 9

Gained - £. 90 14 $4\frac{1}{4} \frac{1}{3}$ Sterling.

Therefore,—If $\text{£. } 640 \text{ } 16 \text{ } 9 : \text{£. } 90 \text{ } 14 \text{ } 4\frac{1}{4} \frac{1}{3} :: 100$

Or, as $153792 \text{ d.} : 261268 \text{ Thirds} :: 24000 \text{ d.} : 6 \text{ £. } 12 \text{ s. } 2\frac{1}{4} \text{ d. } \frac{1}{3}$
 gained per Cent.

(29)

£.	£.	s.	d.	
25 $\frac{1}{4}$	1470	12	8	at $136\frac{1}{2}$ per 100
	367	13	2	
10 $\frac{1}{10}$	147	1	$3\frac{1}{5}$	
1 $\frac{1}{10}$	14	14	$1\frac{2}{5}$	
$\frac{1}{2}$ $\frac{1}{2}$	7	7	$0\frac{3}{5}$	
	<u>2007</u>	<u>8</u>	<u>$3\frac{1}{5}$</u>	

s. d. £. d.

(30) If $34 \text{ } 4 : 1 :: 52$

Or, as $412 \text{ d.} : 240 \text{ d.} :: 52 \text{ d.} : 30 \text{ d. } \frac{30}{103}$ for 400 Reas,

$\therefore 30 \text{ } \frac{30}{103} \times 2\frac{1}{2} = 75 \text{ } \frac{75}{103}$ Pence Sterling for 1000 Reas.

(31) *Crowns.*

$$\begin{array}{r|l}
 4s. & \frac{1}{5} | 1200 \text{ at } 4s. 7d. \text{ per Crown,} \\
 6d. & \frac{1}{8} | 240 \\
 1 & \frac{1}{6} | 30 \\
 & 5 \\
 \hline
 \pounds. & 275 \quad 0 \quad 0
 \end{array}$$

$\pounds. \quad s. \quad \pounds.$

If $100 : 10 :: 275 :: 27s. 6d.$ Comm'sion.

Therefore, $275 \pounds. + 1 \pounds. 7s. 6d. = \pounds. 276 \quad 7 \quad 6 = 66330d.$

Then, as $56d. : 1Cr. :: 66330d. : 1184\frac{1}{2}\frac{3}{8}Cr.$

$\therefore 1200 - 1184\frac{1}{2}\frac{3}{8} = 15\frac{1}{2}\frac{5}{8}$ A's Gain.

d. Sols. d. Sols.

(32) As $67 : 32 :: 70 : 30\frac{2}{9}$ Lubeck, per Florin.

(33) Recip.—If $54d. : 33s. 6d. :: 54\frac{1}{2}d.$

Or, as $108 : 402 :: 109 : 398\frac{3}{4}d. = 33s. 2d. \frac{2}{9}$ Flemish, the Answer.

$\pounds. \quad \pounds. \quad s. \quad d.$

(34) If $100 : 102 :: 33 \quad 4$

Or, as $6000 : 102 :: 100 : 1 \pounds. 14s.$ Flem. per $\pounds.$ Sterling.

(35) If $100\frac{3}{5} : 100 :: 91 : 90\frac{1730}{2003} = 90\frac{520730}{602903}.$

Also, $100 \frac{1}{3} : 100 :: 93 : 92 \frac{208}{301} = 92\frac{416624}{602903}.$

Then $92\frac{416624}{602903} - 90\frac{520730}{602903} = 1\frac{438357}{602903}$, Gained per Cent. for two Months; therefore $1\frac{438357}{602903} \times 6 = 10\frac{218530}{602903} = 10 \pounds. 19s. 1\frac{3}{4}d.$ Gain per Cent. per Ann. the Answer.

(36) First, $2000 \times 40 = 80000$ Flemish Pence.

Or, 160000 Half-pence, $90\frac{1}{2}d. = 181$ Ditto, and $89\frac{1}{2}d. = 179.$

$181) 160000 (883\frac{171}{187} \}$ Crowns.

$179) 160000 (893\frac{153}{179} \}$ Tournois.

Or, $104 : 100 :: 883\frac{171}{187} : 849\frac{2303}{2353} = 849\frac{8656977}{8844927}.$

Also, $105 : 100 :: 893\frac{153}{179} : 851\frac{1061}{3759} = 851\frac{4473123}{8844927}.$

$851\frac{4473123}{8844927} - 849\frac{8656977}{8844927} = 1\frac{4661073}{8844927} = 1$ Cro. 18 sols.

9 din. in Favour of B.

32. COMPARISON of WEIGHTS and MEASURES.

(2) $\begin{array}{r} \text{lb.} \quad \text{lb.} \\ \text{First, } 104 \quad 84\frac{1}{2} \\ \quad 100 \quad 108 \\ \quad 64 \quad \text{---} \end{array} \left| \begin{array}{l} \text{Then } 104 \times 100 \times 64 = 665600 \\ \text{And } 108 \times 84\frac{1}{2} = 9126 \\ 9126 \overline{)665600} (72 \text{ lb. } 14\frac{8}{9} \times \frac{84}{126} \text{ oz. the Ans.} \end{array} \right.$

(3) $\begin{array}{r} \text{Yds.} \quad \text{Ells.} \\ 100 \quad 78 \\ 78 \quad 133\frac{1}{3} \\ 100 \quad \text{---} \end{array} \left| \begin{array}{l} \text{Then } 100 \times 78 \times 100 = 7800,00 \\ \text{And } 133\frac{1}{3} \times 78 = 104,00 \\ 104 \overline{)7800} (75 \text{ Yards, the Answer.} \end{array} \right.$

(4) $\begin{array}{r} \text{Canes.} \quad \text{Ells.} \\ 100 \quad 191\frac{1}{3} \\ 78 \quad 131\frac{2}{3} \\ 100 \quad \text{---} \end{array} \left| \begin{array}{l} \text{Then } 78 \times 100 \times 100 = 780000 \\ \text{And } 191\frac{1}{3} \times 131\frac{2}{3} = 25267\frac{30}{9} \\ \text{Also } 780000 \div 25267\frac{30}{9} = 30\frac{4}{5} \\ \text{Canes, nearly the Answer.} \end{array} \right.$

(5) $\begin{array}{r} \text{lb.} \quad \text{lb.} \\ 100 \quad 92 \\ 100 \quad 110 \\ \text{---} \quad 60 \end{array} \left| \begin{array}{l} \text{Then } 92 \times 110 \times 60 = 607200 \\ \text{And } 100 \times 100 = 10000 \\ 10000 \overline{)607200} (60\frac{72}{100} \text{ lb. the Answer.} \end{array} \right.$

(6) $\begin{array}{r} \text{Yds.} \quad \text{Bra.} \\ 74 \quad 100 \\ 100 \quad 30 \\ \text{---} \quad 100 \end{array} \left| \begin{array}{l} \text{Then } 100 \times 100 \times 30 = 300000 \\ \text{And } 74 \times 100 = 7400 \\ \text{Also } 7400 \overline{)300000} (40\frac{40}{74} \text{ Canes, the Answ.} \end{array} \right.$

33. SINGLE POSITION.

(2) Suppose she had 10 | Then $40 - 10 = 30$
 Then as many 10 | If $25 : 10 :: 30 : 12$ her Flock,
 One half as many 5 | the Answer.

Sum, 25, for $12 + 12 + 6 + 10 = 40$, Proof.

Single Position.

181

(3) Suppose 10 to be C's Age.

$$\begin{array}{l} \therefore 10 + 4 = 14 \text{ A's.} \\ \text{And } 10 + 14 + 9 = 33 \text{ B's.} \end{array} \quad \left| \quad \begin{array}{l} \text{Then } 4 + 4 + 9 = 17 \\ 45 - 17 = 28 \end{array} \right.$$

$$\text{Sum } 47 - 17 = 40 : 10 : 28 :$$

$$\begin{array}{l} \text{And } 40) 280 (7 \text{ C's Age.} \\ 7 + 4 = 11 \text{ A's.} \\ \text{Also } 7 + 11 + 9 = 27 \text{ B's.} \end{array}$$

$$\text{Proof, } 45 \text{ D's.}$$

(4)

	<i>Miles.</i>	
Suppose Andrew goes	30	469
Then Ben. will go $30 \times 3 + 3 =$	93	25
And Christopher $93 \times 2 + 16 =$	202	444
	Sum 325	

$$\text{Then } 3 + 3 \times 2 + 16 = 25$$

$$M. \quad M. \quad M. \quad \text{Miles.} \quad 300 \text{ Diff.}$$

$$\therefore 300 : 30 :: 444 : 44\frac{2}{3} \text{ Andrew.}$$

$$\text{Also } 44\frac{2}{3} \times 3 + 3 = 136\frac{1}{3} \text{ Benjamin.}$$

$$\text{And } 136\frac{1}{3} \times 2 + 16 = 288\frac{2}{3} \text{ D. Christopher.}$$

$$\text{Proof, } 469$$

(5)

	£. s.	£. s.
Suppose A paid	200 0	45 15
Then B $200 \times 2\frac{1}{2} - 45 \text{ £. } 10 \text{ s.} =$	654 5	$\times 2$
And C $200 + 654 5 + 26 10 =$	880 15	91 10
	Sum 1735 0	26 10
		Difference 65 0

R.

Now

Now $2000 + 65 = 2065$, and $65 + 1735 = 1800$

	£.	£.	£.	£.	s.		
Then—If	1800	:	200	::	2065	:	
	229		8 $\frac{16}{18}$		A	} paid.	
∴	229		8 $\frac{16}{18}$	×	3 $\frac{1}{2}$		—
	45		15	=	757		6 $\frac{2}{18}$
And	229		8 $\frac{16}{18}$	+	757	6 $\frac{2}{18}$	+
	26		10	=	1013	5	C

Proof, £. 2000 0

(6) Suppose they will empty the Cistern in 30 Minutes, then the first will empty $\frac{1}{2}$ or 30 Gallons.

The second - $\frac{1}{4}$ or 15

And the third - $\frac{1}{6}$ or 10

Sum, 55

Gal. Min. G. M. Sec.

∴ If $55 : 30 :: 60 : 32\ 43\frac{1}{11}$, the Time sought.

(7) First, $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$ —

Suppose he had 240, then $240 \div 8 \times 3 = 90$

∴ If $90 : 240 :: 900 : 2400$ Men, the Answer.

(8) See Question the 28th in Double Position.

(9)	Suppose he had	12		If	37	:	12	::	333
	Then as many	12			333				
	$\frac{1}{2}$ as many	6			—				
	$\frac{1}{3}$ Ditto	4		37)	3996	(	108	Scholars,	the
	$\frac{1}{4}$ Ditto	3			Answer.				
	Sum,	37							

34. DOUBLE POSITION.

(11)	Suppose he had 8	Again, suppose he had 7
	Then $8 + 8 = 16$	Then $7 + 7 = 14$
	Also, $16 - 6 = 10$	$14 - 6 = 8$
	$10 + 10 = 20$	$8 + 8 = 16$
	$20 - 6 = 14$	$16 - 6 = 10$
	$14 + 14 = 28$	$10 + 10 = 20$
	$28 - 6 = 22$	$20 - 6 = 14$
	too much.	too much.

Sup. Errors.

$$\begin{array}{r} 8 \\ 7 \end{array} \times \begin{array}{r} 22 \\ 14 \end{array} \left| \begin{array}{l} \text{Then } 22 - 14 = 8, \text{ and } 154 - 112 = 42 \\ \therefore 42 \div 8 = 5\frac{1}{4} d. \text{ the Answer.} \end{array} \right.$$

(12)	Crs.	
Suppose	28 A's	Again, suppose A had 34
Then	$28 + 16 = 44$ B's	Then B $34 + 16 = 50$
And	$28 + 44 \div 6 = 12$ C's	C $84 \div 6 = 14$
	Sum, $84 = 28$	Sum, $98 = 14$
	too little.	too little.

Sup. Errors.

$$\begin{array}{r} 28 \\ 34 \end{array} \times \begin{array}{r} 28 \\ 14 \end{array} \left| \begin{array}{l} \text{Then } 952 - 392 = 560, \text{ and } 28 - 14 = 14 \text{ Diff.} \\ \text{of Errors.} \\ \therefore 14 \overline{) 560} \quad \begin{array}{l} 40 \text{ A's} \\ = 56 \text{ B's} \\ = 16 \text{ C's} \end{array} \\ \text{And } 40 + 16 \\ \text{Also } 40 + 56(96) \div 6 = 16 \text{ C's} \end{array} \right. \text{Crowns.}$$

(13)	Suppose he had 60	Again, suppose he had 76
	Tom took $30 - 10 = 20$	Then $38 - 10 = 28$
	Rem. 40	Rem. 48
	Ned took $20 - 4 = 16$	$24 - 4 = 20$
	Rem. 24	Rem. 28
	Jack took $12 - 1 = 11$	$14 - 1 = 13$
	Rem. 13 = 5	Rem. 15 = 3
	too little.	too little.

$$8 = 3$$

$$8 = 3$$

$$8 = 3$$

Sup. Er.

$$\begin{array}{r} 60 \times 5 \\ 76 \times 3 \\ \hline 580 \quad 180 \end{array}$$

Then $580 - 180 = 200$, and $5 - 3 = 2$
 $\therefore 200 \div 2 = 100$, what he had at first.

(14) Suppose the Father's Age to be 40
 Then the Son will be $40 \div 5 = 8$
 And 4 Years ago was $8 - 4 = 4$
 which is 12 too little.

Again, suppose the Father was 60
 Son will be $60 \div 5 = 12$
 And 4 Years since was $12 - 4 = 8$, which is 4
 too little.

Sup. Er.

$$\begin{array}{r} 40 \times 12 \\ 60 \times 4 \\ \hline 720 \quad 160 \end{array}$$

Then $720 - 160 = 560$, and $12 - 4 = 8$ $\therefore 560 \div 8 = 70$ Years, the Father's Age; and $70 \div 5 = 14$ the Son's.

For $14 - 4 = 10 = \frac{1}{7}$ of 70 (the Age of the Son 4 Year's ago) and $14 \times 5 = 70$, the Proof.

(15) Suppose the Body to be 24 Inches.

Then $24 \div 2 + 9 = 21$ Tail; also $21 + 9 = 30$, which is 6 too much.

Again, suppose the Body to be 26, then $26 \div 2 + 9 = 22$ Tail; also $22 + 9 = 31$, which is 5 too much.

Sup. Er.

$$\begin{array}{r} 24 \times 6 \\ 26 \times 5 \\ \hline 156 \quad 120 \end{array}$$

Then $156 - 120 = 36$, and $6 - 5 = 1$
 $\therefore 36$ Inches is the Length of the Body.

And $30 \div 2 + 9 = 27$ Tail.

Therefore $36 + 27 + 9 = 72$ Inches, the Answer.

(16)

Suppose the No. to be 4
 Then $4 + 4 = 8$
 And $8 \times 4 = 32$
 Also $32 - 4 = 28$
 Likewise $28 \div 4 = 7$
 Therefore $13 - 7 = 6$
 too little.

Again, suppose the No. to be 5
 Then $5 + 5 = 10$
 And $10 \times 5 = 50$
 Also $50 - 5 = 45$
 $45 \div 5 = 9$
 $13 - 9 = 4$
 too little.

Brought

Brought over,

Sup. Er.

$\begin{array}{r} 4 \times 6 \\ 5 \times 4 \\ \hline 30 \quad 16 \\ \hline \end{array}$	Then $30 - 16 = 14$, and $6 - 4 = 2$. $\therefore 14 \div 2 = 7$, the Number sought. For $7 + 7 \times 7 - 7 = 91$, which $\div 7 = 13$, the Proof.
---	---

(17)

Suppose she 14 Then he $14 \times 3 = 42$ $14 + 10 + 5 = 29$ $42 + 10 + 5 = 57$ $29 \times 2 = 58$	}	Again, suppose she 16 Then he will be $16 \times 3 = 48$ $16 + 10 + 5 = 31$ $48 + 10 + 5 = 63$ $31 \times 2 = 62$	}	sub.
too little by 1		too much by 1		

Sup. Er.

$\begin{array}{r} 14 \times 1 \\ 16 \times 1 \\ \hline 16 \quad 14 \\ \hline \end{array}$	Then $16 + 14 = 30$, and $1 + 1 = 2$ $\therefore 30 \div 2 = 15$ Years, her Age. And $15 \times 3 = 45$ his, Answer. For $15 + 10 + 5 = 30$ her, when married 15 Years. And $45 + 10 + 5 = 60$ his, _____ For as $8 : 16 :: 30 : 60$ Proof.
---	--

(18)

Beg. Beg. Sup. 8 8 $\times 4 \quad \times 6$ 32 48 $+ 16 \quad - 12$ 48 36 =	}	Beg. Beg. Again, suppose 10 10 $\times 4 \quad \times 6$ 40 60 $+ 16 \quad - 12$ 56 48 = 8 too little.
12 too lit.		

Sup. Er.

$\begin{array}{r} 8 \times 12 \\ 10 \times 8 \\ \hline 120 \quad 64 \\ \hline \end{array}$	Then $120 - 64 = 56$, and $12 - 8 = 4$. $\therefore 56 \div 4 = 14$, the Number of Beggars. For $14 \times 4 + 16 = 14 \times 6 - 12 = 72d$. what he gave, Proof.
--	--

(19) Proceed with this as with Example 11, and the Answer will be the same as that, viz. $5\frac{1}{4}d$.

(20)

Suppose the Number to be 15

$$\text{Then } 15 \times 3 = 45$$

$$\text{And } 45 - 5 = 40$$

$$\text{Also } 40 \div 2 = 20$$

$$20 + 15 = 35$$

$$40 - 35 = 5$$

too little.

Again, suppose 25

$$\text{Then } 25 \times 3 = 75$$

$$\text{And } 75 - 5 = 70$$

$$\text{Also } 70 \div 2 = 35$$

$$35 + 25 = 60$$

$$60 - 40 = 20$$

too much.

Sup. Er.

$$\begin{array}{r} 15 \times 5 \\ 25 \times 20 \\ \hline 125 \quad 300 \end{array}$$

$$\text{Then } 125 + 300 = 425, \text{ and } 20 + 5 = 25$$

$$\therefore 425 \div 25 = 17, \text{ the Number required.}$$

$$\text{For } 17 \times 3 - 5 = 46, \text{ and } 46 \div 2 + 17 = 40, \text{ Proof.}$$

(21)

Suppose B had 6

Then A must have 4

$$\text{For } 6 - 1 = 4 + 1 = 5$$

$$\text{Again } 4 - 1 = 3$$

$$\text{And } 6 + 1 = 7$$

$$\text{Also, } 3 \times 2 = 6$$

$$7 - 6 = 1 \text{ too lit.}$$

Again, suppose B had 8

Then A must have 6

$$\text{For } 8 - 2 = 6 + 1 = 7$$

$$\text{Again, } 6 - 1 = 5$$

$$\text{And } 8 + 1 = 9$$

$$\text{Also, } 5 \times 2 = 10$$

$$10 - 9 = 1, \text{ too much.}$$

Sup. Er.

$$\begin{array}{r} 6 \times 1 \\ 8 \times 1 \\ \hline 8 \quad 6 \end{array}$$

$$\text{Then } 8 + 6 = 14, \text{ and } 1 + 1 = 2 \therefore 14 \div 2 = 7 \text{ B's}$$

$$\text{Guineas, and } 5 \text{ A's; for } 7 - 2 = 5 + 1 = 6 \text{ A's,}$$

$$\text{when he had received 1 of B. Also, } 5 - 1 \times 2$$

$$= 7 + 1 = 8 \text{ B's, when he had received 1 of A's,}$$

Proof.

(22)

Suppose Father's Age 37

$$\text{Then } 37 - 5 = 32$$

$$\text{And } 32 \div 8 = 4$$

$$4 \times 3 = 12 \text{ Son's Age.}$$

$$\text{Then } 12 + 2 = 14$$

$$\text{And } 14 \times 3 = 42$$

$$\text{Also, } 42 - 7 = 35$$

$$\therefore 37 - 35 = 2 \text{ too little.}$$

Again, suppose Father's 45

$$\text{Then } 45 - 5 = 40$$

$$\text{And } 40 \div 8 = 5$$

$$\therefore 5 \times 3 = 15 \text{ Son's Age.}$$

$$\text{Then } 15 + 2 = 17$$

$$\text{And } 17 \times 3 = 51$$

$$\text{Also, } 51 - 7 = 44$$

$$\therefore 45 - 44 = 1 \text{ too little.}$$

Sup.

Sup. Er.

$$\begin{array}{r} 37 \times 2 \\ 45 \times 1 \\ \hline 90 \end{array} \begin{array}{l} 2 \\ 1 \\ 37 \end{array}$$

Then $90 - 37 = 53$, and $2 - 1 = 1$
 $\therefore 53$ is the Father's Age; then $53 - 5 = 48$, and
 $48 \div 8 = 6 = \frac{1}{3}$ of the Son's Age. $\therefore 6 \times 3 =$
 18 the Son's Age.

(23)

Suppose B had £. 360
 Then $360 \div 3 \times 2 = 240$
 And $1200 - 240 = 960$ A's.
 Then $960 \div 4 \times 3 = 720$
 And $360 + 720 = 1080$ A's and B's.
 $\therefore 1200 - 1080 = 120$ too little.

Again, suppose B had £. 420
 Then $420 \div 3 \times 2 = 280$
 And $1200 - 280 = 920$ A's.
 Then $920 \div 4 \times 2 = 690$
 And $420 + 690 = 1110$
 $\therefore 1200 - 1110 = 90$ too little.

Sup. Er.

$$\begin{array}{r} 360 \times 120 \\ 420 \times 90 \\ \hline 50400 \end{array} \begin{array}{l} 120 \\ 90 \\ 32400 \end{array}$$

Then $50400 - 32400 = 18000$
 And $120 - 90 = 30 \therefore 18000 \div$
 $30 = 600$ £. B's Money; and
 $600 \div 3 \times 2 = 400$. Also $1200 - 400 = 800$ £.
 A's Money, the Answer.

For $600 + 800 \div 4 \times 3 = 800 + 600 \div 3 \times 2 = 1200$, the Proof.

greater. lesser.
 (24) Suppose 28 and 22
 $28 \div 7 = 4$. Also $22 \times 3 = 66$
 $4 + 66 = 70$, then $70 -$
 $50 = 20$ too much.

Again, gr. less.
 Suppose 42 and 8
 $42 \div 7 = 6$; also $8 \times 3 = 24$
 $6 + 24 = 30 \therefore 50 - 30 = 20$ too
 little.

Sup.

Sup. Er.

$$\begin{array}{r}
 28 \times 20 \\
 42 \times 20 \\
 \hline
 840 \quad 560
 \end{array}$$

Then $840 + 560 = 1400$, and $20 + 20 = 40$. $\therefore 1400 \div 40 = 35$, the greater Number: and $50 - 35 = 15$ the lesser; for $35 \div 7 + 15 \times 3 = 50$, the Proof.

(25) Suppose he worked 140 Days, $\therefore 390 - 140 = 250$ Days he was idle.

Then $140 \times 12 = 1680$
 And $250 \times 8 = 2000$ } Subt. = 320 too little.

Again, suppose he worked 150 Days, $\therefore 390 - 150 = 240$ Days idle.

Then $150 \times 12 = 1800$
 And $240 \times 8 = 1920$ } Subt. = 120 too little.

Sup. Er.

$$\begin{array}{r}
 140 \times 320 \\
 150 \times 120 \\
 \hline
 48000 \quad 16800
 \end{array}$$

Then $48000 - 16800 = 31200$
 And $320 - 120 = 200$ $\therefore 31200 \div 200 = 156$
 Days worked, and $390 - 156 = 234$
 Days idle, the Answer.
 For $156 \times 12 = 234 \times 8 = 1872$ Proof.

(26)

Suppose his Age was 24

Then $24 \div 3 \times 2 \times 4 = 64$ And $64 + 12 + 50 = 126$ Then $126 - 100 = 26$ And $100 - 24 = 76$ $\therefore 76 - 26 = 50$ too little.

Suppose he was 30

Then $30 \div 3 \times 2 \times 4 = 80$ And $80 + 15 + 50 = 145$ Then $145 - 100 = 45$ And $100 - 30 = 70$ $\therefore 70 - 45 = 25$ too little.*Sup. Er.*

$$\begin{array}{r}
 24 \times 50 \\
 30 \times 25 \\
 \hline
 1500 \quad 600
 \end{array}$$

Then $1500 - 600 = 900$, and $50 - 25 = 25$ $\therefore 900 \div 25 = 36$ Years, his Age required.
 For $36 \div 3 \times 2 \times 4 + 18 + 50 - 100 = 100 - 36 = 64$, Proof.

£.

(27) Suppose the first Horse to be worth 24

Then the Trappings must be $50 - 24 = 26$ And the second Horse must be $24 + 26 = 50$

Now $50 + 26 = 76$, and $24 \times 2 = 48$; also $76 - 48 = 28$, too little.

Again,

Again, suppose the first Horse to be worth £. 28
 Then the Trappings must be $50 - 20 = 22$
 And the second Horse must be $28 + 22 = 50$
 Now $50 + 22 = 72$, and $28 \times 2 = 56$; also, $72 - 56 = 16$, too little.

Sup. Er.

$\begin{array}{r} 24 \times 28 \\ 28 \times 16 \\ \hline 784 \quad 384 \\ \hline \end{array}$	Then $784 - 384 = 400$ And $28 - 16 = 12 \therefore 400 \div 12 = 33\frac{1}{3}$ £. Value of the first Horse; and $50 - 33\frac{1}{3} = 16\frac{2}{3}$ £. Value of the Trappings. Also, $33\frac{1}{3} + 16\frac{2}{3} = 50$ £. value of the second Horse, for $50 - 16\frac{2}{3} = 33\frac{1}{3} \times 2 = 66\frac{2}{3}$ the Proof.
---	--

(28)

Suppose he left	£. 5000
Interest for 11 Years	= 2700

Amount 7700

Education, &c. $100 \times 11 = 1100$

Neat 6600

$\therefore 7400 - 1600 = 750$ too little.

Again, suppose he left	£. 6000
Interest for 11 Years	3300

Amount 9300

Education, &c. $100 \times 11 = 1100$

Neat 8200

$\therefore 8200 - 7400 = 800$ too much.

Sup.

Er.

$\begin{array}{r} 5000 \\ 9000 \\ \hline 4500000 \end{array}$	$\begin{array}{r} \times 750 \\ \times 800 \\ \hline 4000000 \end{array}$	Then, $4500000 + 4000000 = 8500000$, and $750 + 800 = 1550 \therefore 8500000$ $\div 1550 = 5483$ £. 17 s. 5 d. $\frac{1}{3}$, the Answer required.
---	---	---

35. ARITHMETICAL PROGRESSION.

(2) Thus, $1 + 12 \times 6 = 78$ Strokes, Answer.

(3) First $2 + 188 = 190$ Sum of the Extremes.

And $94 \div 2 = 37 =$ half the Number of Terms.

Product 8930 Feet, a Mile $= 5280$ Feet.

Then $5280 \overline{) 8930}$ (1 Mile, 5 Feet, 21p. $3\frac{1}{2}$ Feet, the Answer.

(4) First $2 + 200 = 202$ Sum of the Extremes.

And $100 \div 2 = \times 50$ half the Numb. of Terms.

A Mile $=$ yds. $1760 \overline{) 10100}$ (5m. 5fur. 36p. 2yds. the Answer.

(5) First $9\text{£}. 19\text{s}. = 199$ Shillings.

Then $1 + 199 = 200$ Sum of the Extremes.

And $100 \div 2 = \times 50$ half the Number of Terms.

$2,0 \overline{) 1000,0}$ (500£. the Answer.

(7) First $60 - 6 = 54$ Diff. of the Extremes.

Then $19 - 1 = 18$ Number of Terms, less one.

Then $54 \div 18 = 3$ the common Difference sought.

Then 3 added to 6 and every other Term respectively, gives each Day's Journey as follows, $6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 + 33 + 36 + 39 + 42 + 45 + 48 + 51 + 54 + 57 + 60 = 627$ Miles from London.

(8) First $50 - 5 = 45$ Difference of the Extremes.

And $10 - 1 = 9$ Number of Terms, less one.

Then $45 \div 9 = 5$ Common Difference.

Then each Payment and the whole Debt will be as follows, viz. $5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 = 275$ £. the whole Debt.

(10) First $60 - 6 = 54$ Diff. of the Extremes.

And $54 \div 3 = 18 \therefore 18 + 1 = 19$ Days, the Answer.

(12) First $10 - 1 = 9$ Number of Terms, less one.

And $9 \times 2 = 18$. Then $23 - 18 = 5$, the Answer.

(14) First $100 \div 2 = 50$ what each travelled.

And $50 \div 5 = 10$. Also $5 - 1 = 4$. Then $4 \times 2 = 8$. Likewise $8 \div 2 = 4 \therefore 10 - 4 = 6$ his first Day's Journey, and $6 + 2 = 8$ the 2d. 10 the 3d. 12 the 4th. and 14 the 5th. Sum of which $= 50$.

Again.

Again, $4)50(12\frac{1}{2}$. Also $4-1=3$. Then $3 \times 3=9$. And $9 \div 2=4\frac{1}{2}$ $\therefore 12\frac{1}{2}-4\frac{1}{2}=8$ his first Day's Journey, and $8+3=11$ his 2d. 14 his 3d. and 17 his 4th. Sum of which = 50 Leagues.

(16) First $100 \times 2=200$. Then $200-2=198$. And $198+1=199$, the last Term required.

(17) First $30 \times 2=60$. Also $60-2=58$.

Again $58+6=64$. And $64 \times 30=1920$. $\therefore 1920 \div 2=960s.=48 \text{ } \text{£}$. due, Answer.

(19) First $300 \times 2=600$. And $600 \div 9=66\frac{2}{3}$. Also $66\frac{2}{3}-4 \times 2(8)=58\frac{2}{3}$ $\therefore 58\frac{2}{3} \div 8=7\frac{1}{3}$ common Difference. Then the Miles he travelled each Day will be as follows, viz.

1st. 2d. 3d. 4th. 5th. 6th. 7th. 8th. 9th.
4, $11\frac{1}{3}$, $18\frac{2}{3}$, 26, $33\frac{1}{3}$, $40\frac{2}{3}$, 48, $55\frac{1}{3}$, and $62\frac{2}{3}$, which added together = 300, the whole Distance, Proof.

QUESTIONS.

(21) For A's Race thus.

First $4+40=44$, Sum of the first and last Terms. Then 44×5 , half the Number of Terms = 220 Yards. A's first Race per Prop. the first.

Then $901 \times 4=3604$ yds. A's first Term of the last Race.

Also $910 \times 4=3640$ A's last Term.

their Sum 7244 which $\times 5=36220$ A's last Race.

Then to find his whole Ground, put 220 first Term. And 36226 the last Term, which (by Prop. 1.) is $36440 \times 5=182200$ Yards gone by A.

For B's, thus.

First $11 \times 4=44$ first Term. And $20 \times 4=80$ last Term. Then $44+80=124$ their Sum, which $\times 5=620$ Yards, B's. first Race.

$\therefore 911 \times 4=3644$ yds. B's first Term of the last Race.

$920 \times 4=3680$ B's last Race.

their Sum 7324 which $\times 5=36620$ yds. B's last Race.

Then $620+36620=37240$ Sum of the first and last Terms. which $\times 5=186200$ Yards, gone by B.

Deduct 182200 ditto by A.

4000 Yards, common Difference.

Which

Which added continually to each of their shares, shew that

	<i>Yds.</i>	<i>Miles. fur. yds.</i>
A in all ran	182200 =	103 4 40
B	186200 =	105 6 80
C	190200 =	108 0 120
D	194200 =	110 2 160
E	198200 =	112 4 200
F	202200 =	114 7 20
G	206200 =	117 1 60
H	210200 =	119 3 100
I	214200 =	121 5 140
K	218200 =	123 7 180
Sums	2002000 =	1137 4 0

If 2002000 : 300£. :: 182200
182200

2002,000) 54660,000 (27£. 6s. 0½d. 590 A's part.

If 2002000 : 300£. :: 186200
186200

2002,000) 55860,000 (27 18 0½ 14 B's part.
27 6 0½ 590

Common Difference 11 11¼ 425

Which by continually adding to each we shall have

	£.	s.	d.		
First	27	6	0½	590	= A's
Then	27	18	0½	14	= B's.
	28	10	0¼	439	= C's.
	29	2	0	864	= D's.
	29	14	0	288	= E's.
	30	5	11¾	713	= F's.
	30	17	11¾	137	= G's.
	31	9	11½	562	= H's.
	32	1	11¼	987	= I's.
	32	13	11¼	411	= K's.

} Part.

Proof 300 0 0

From

From London to York suppose 180 Miles, which $\times 4 = 720$ Miles, twice a back

Their Sum Total 1137 m. 4 fur.

Distance of York 720

Short of the Undertaking $417\frac{1}{2}$ Miles.

(22) By the Nature of the Question, the Number of Calves that were calved at the End of these Years, will be as follows

B { 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16.
1. 1. 1. 2. 3. 4. 6. 9. 13. 19. 28. 41. 60. 88.

17. 18. 19. 20. Years.
120. 189. 277. 406. respectively, which are found by adding the last to the last but two.

Then of the whole Terms $1+1+1+2+3+4+6+9+13+19+ \dots +D+E+F+G$ be represented by S, when D, E, F, and G, denote the four last Terms, we shall then have $1+1+1+2+3+4+6+9+13+19+28+ \dots +D=S-E-F-G$, which being taken from the above, we have $1+1+1+1+2+3+4+6+9+13+ \dots +F=E+G+F$; and by adding G to both Sides of the Equation, we then get $1+1+1+1+2+3+4+6+9+13+ \dots +F-G=E+F+2G$; which will be the Man's Stock of Cows and Calves at the End of any Number of Years; which in this Case $E=189+F=77+2G=812$. Their Sum will be 1278, the Answer.

From the above Solution, it appears that the whole Stock of Cows and Calves at the End of any Number of Years, will be equal to the Number of Cows that would calve at the End of three Years after the given Time.

36. GEOMETRICAL PROGRESSION.

(2) First { 0. 1. 2. 3. 4. 5. 6.
1. 2. 4. 8. 16. 32. 64.

Then $64 \times 64 = 4096 = 646 = 12$ th Term.

And $4096 \times 4096 = 16777216 = 24$ th Term.

Also $64 \times 32 = 2048 = 5+6 = 11$ th Term.

$\therefore 16777216 \times 2048 = 34359738368$ Farthings, or 36th Term, which reduced to Pounds, will give 35791394 £. 2s. 8d. the Answer.

$$(4) \quad \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 500. & 750. & 1125 & 16875. & 2531. & 25. & 3796. & 875 \end{array}$$

$5 + 4 = 9$ Number of Terms less 1.

Then $3796, 875 \times 2531, 25 = 9610839, 84375.$

Which $\div 500$ the first Term = £. 19221. 13. 7. 125.

$$(6) \quad \text{First } \begin{cases} 0. & 1. & 2. & 3. & 4. & 5. & 6. \\ 1. & 2. & 4. & 8. & 16. & 32. & 64. \end{cases}$$

Then $6 + 6 + 6 = 18$ Number of Terms less 1.

$64 \times 64 \times 64 = 262144$ last Term.

$262144 - 1 \div 2 - 1 = 262143, + 262144 = 524287$ Farthings; or, 546 £. 2s. $7\frac{3}{4}d.$ what 20 Horses came to.

$\therefore 20)546 \text{ } 2 \text{ } 7\frac{3}{4}(27\text{£. } 6\text{s. } 3\frac{1}{2}d. \text{ } \frac{7}{8} \text{ per Head, the Answer.}$

$$(7) \quad \text{First } \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 2. & 6. & 18. & 54. & 162. & 486 \end{cases}$$

Then $486 \times 486 = 236196 = 5 + 5 = 10\text{th Term.}$

And $486 \times 162 = 78732 = 5 + 4 = 9\text{th Term.}$

Product 18596183472 last Term.
— 2 First.

Ratio $3 - 1 = 2)18596183470$

9298091735 this added to the last Term gives

1,00)27894275207 the Sum of all the Terms at $\frac{1}{4}$ per [100.

Value of the Pins 278942752 $\frac{7}{800}$ Qrs. = 290565 $\frac{7}{4}$ £. s. d.
Lace comes to £. 8. 1. $8 \times 5 \times 4 (20) =$ 161 13 4

Answer gained 290403 14 0

$$(8) \quad \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 10. & 100. & 1000. & 10000. & 100000. & 1000000. \end{array}$$

Then $1000000 \times 1000000 = 1000000000000$

— 10

Ratio $10 - 1 = 9)9999999999990$

Pints.

768,0)111111111111,0(14467592 $\frac{455}{8}$,

A Bush-

A Bushel = pts. 64) 14467592 (226056 $\frac{1}{8}$ Bushels at 4s.
 226056 $\frac{1}{8}$ at 3s. 4d.

$$\begin{array}{r} 2 \\ \hline 45211.4 \\ -6 = \frac{1}{8} \text{ of } 4s. \end{array}$$

Answer £. 452114.6

- (10) First $12 \times 12 = 144$ Square of the first Term.
 And $12 - 10 = 2$ Diff. of the first and second Term.
 Then 2) 144 (72 Miles, the Answer.

37. PERMUTATION.

- (1) First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ Days.
 Then 365) 5040 (13 Years, 295 Days, the Answer.
- (2) First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ Rounds. Which $\times 3 = 1437004800$ Seconds. And in a Year there are
 35576,00) 14370048,00 (45 Years.
 A Day = 864,00) 169128,00 (19 Days.
 An Hour = 36,00) 502,00 (13 Hours.
 A Minute = 6,0) 340,0 (56 Men.
 Remain 40 Seconds.

45 Years, 19 Days, 13 Hrs. 56 Min. 40 Secs. the Answer.

- (3) First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ Days.
 And 20 Guineas = 5040 Pence.
 Then, as 362880 Days : 5040 d. :: 365 D. : $5\frac{315}{36}$ d.
 per Year.

38. REDUCTION OF VULGAR FRACTIONS.

CASE I.

- (2) 2832) 12848 (4 Then 16) $\frac{2832}{12848} (= \frac{177}{803}$, the Answer.
 1520) 2832 (1
 1312) 1520 (1
 208) 1312 (6
 64) 288 (3
 16) 64 (4

(3) $144)560(3$ Then $16)\frac{144}{560}(=\frac{2}{35}$, the Answer.

$$\begin{array}{r} 128)144(1 \\ \hline \end{array}$$

$$\begin{array}{r} 16)128(8 \\ \hline \end{array}$$

(4) $192)336(1$ Then $48)\frac{192}{336}(=\frac{4}{7}$, the Answer.

$$\begin{array}{r} 144)192(1 \\ \hline \end{array}$$

$$\begin{array}{r} 48)144(3 \\ \hline \end{array}$$

(5) $1476)1938(1$ Then $6)\frac{1476}{1938}(=\frac{249}{323}$, Answer.

$$\begin{array}{r} 462)1476(3 \\ \hline \end{array}$$

$$\begin{array}{r} 90)462(5 \\ \hline \end{array}$$

$$\begin{array}{r} 12)9(7 \\ \hline \end{array}$$

$$\begin{array}{r} 6)12(2 \\ \hline \end{array}$$

(7) $6)\frac{4980}{5400}(=\frac{7}{9}$, the Answer.

(8) Thus $8)\frac{24}{112}(=\frac{3}{14}$ $12)\frac{96}{324}(=\frac{8}{27}$ $12)\frac{60}{144}(=\frac{5}{12}$

And $12)\frac{120}{360}(=\frac{1}{3}$

Answer $\frac{3}{14}$, $\frac{8}{27}$, $\frac{5}{12}$, and $\frac{1}{3}$, the Fractions required.

C A S E II.

(9) Thus $\frac{2}{3}$ of $\frac{1}{2}=\frac{2}{6}$, which reduced by the last Case $=\frac{1}{3}$ the Answer.

(10) Thus $\frac{4}{9}$ of $\frac{5}{7}$ of $\frac{8}{10}=\frac{18}{63}$, reduced, $=\frac{2}{7}$, the Fraction required.

(11) Thus $\frac{2}{9}$ of $\frac{3}{4}$ of $\frac{4}{5}=\frac{24}{90}$ or $\frac{2}{3}$, the Fraction required.

(12) Thus $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{8}=\frac{21}{168}$ or $\frac{1}{8}$, the Fraction required.

Note. If a Numerator of one Term in a Compound Fraction, be equal to a Denominator in another Term, cancel or reject both, and divide these Numerators and Denominators; which are divisible by each other, or by the same Number, which Quotients multiply into the remaining Numerators and Denominators, reduce the Compound Fraction to a single one in the lowest Terms.

As

As for Example suppose the last.

Thus $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{7}{8} = \frac{1}{8}$ the Fraction as before.

For the 3's and 7's divide off.

C A S E III.

- (16) Thus $\frac{1^2}{1}$, $\frac{2^2}{1}$, and $\frac{1^2}{1}$, the Fractions required.
 (17) Thus $27 \times 12 = 324$. Then $3\frac{2}{12}$, the Fraction required.
 (18) Thus $7 \times 4 + 2 = 30$. Then $3\frac{0}{7}$, the Fraction required.
 (19) Thus $16 \times 2 + 1 = 33$. Then $3\frac{2}{2}$, the Fraction required.
 (20) Thus $142 \times 23 + 17 = 3283$. Then $3\frac{2}{2}\frac{3}{3}$, the Answer.
 (21) Thus $146 \times 37 + 21 = 5423$. Then $5\frac{4}{3}\frac{3}{7}$, the Answer.

C A S E IV.

- (22) Thus $33 \div 2 = 16\frac{1}{2}$, the Answer.
 (23) Thus $3283 \div 23 = 142\frac{1}{2}\frac{7}{3}$, the Answer.
 (24) Thus $5423 \div 37 = 146\frac{2}{3}\frac{1}{7}$, the Answer.

C A S E V.

- (26) $\frac{1}{3} \cdot \frac{7}{8} \cdot \frac{5}{9}$ and $\frac{1}{12}$
 First $1 \times 8 \times 9 \times 12 = 864$
 Also $7 \times 3 \times 9 \times 12 = 2268$
 $5 \times 3 \times 8 \times 12 = 1440$
 $11 \times 9 \times 8 \times 3 = 2376$ } New Numerators.
 And $3 \times 8 \times 9 \times 12 = 2592$ Common Denominator.
 Answer $\frac{864}{2592}$, $\frac{2268}{2592}$, $\frac{1440}{2592}$ and $\frac{2376}{2592}$.

- (7) $\frac{3}{4} \cdot \frac{1}{12}$ and $\frac{7}{10}$
 First $3 \times 12 \times 10 = 360$
 Also $11 \times 4 \times 10 = 440$
 $7 \times 12 \times 4 = 336$ } New Numerators.
 And $4 \times 12 \times 10 = 480$ Common Denominator.
 Answer $\frac{360}{480}$, $\frac{440}{480}$, and $\frac{336}{480}$.

- (28) First $\frac{1}{6}$ of $\frac{7}{8} = \frac{7}{48} \cdot \frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{7}{48}$.
 Then $1 \times 4 \times 5 \times 48 = 960$
 Also $1 \times 3 \times 5 \times 48 = 720$
 $1 \times 3 \times 4 \times 48 = 576$
 $7 \times 5 \times 4 \times 3 = 420$ } New Numerators.
 And $3 \times 4 \times 5 \times 48 = 2880$ Common Denominator.
 Answer $\frac{960}{2880}$, $\frac{720}{2880}$, $\frac{576}{2880}$, and $\frac{420}{2880}$.

$$\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \text{ and } \frac{1}{6}.$$

$$(29) \quad \begin{array}{l} \text{First } 1 \times 3 \times 4 \times 5 \times 6 = 360 \\ \text{Also } 1 \times 2 \times 4 \times 5 \times 6 = 240 \\ \quad 1 \times 2 \times 3 \times 5 \times 6 = 180 \\ \quad 1 \times 2 \times 3 \times 4 \times 6 = 144 \\ \quad 1 \times 2 \times 3 \times 4 \times 5 = 120 \end{array} \left. \vphantom{\begin{array}{l} 1 \times 3 \times 4 \times 5 \times 6 \\ 1 \times 2 \times 4 \times 5 \times 6 \\ 1 \times 2 \times 3 \times 5 \times 6 \\ 1 \times 2 \times 3 \times 4 \times 6 \\ 1 \times 2 \times 3 \times 4 \times 5 \end{array}} \right\} \text{New Numerators.}$$

$$\text{And } 2 \times 3 \times 4 \times 5 \times 6 = 720 \text{ Common Denominator.}$$

$$\text{Answer } \frac{360}{720}, \frac{240}{720}, \frac{180}{720}, \frac{144}{720}, \text{ and } \frac{120}{720}.$$

$$(30) \quad \text{First } \frac{1}{6} \text{ of } \frac{3}{1} = \frac{3}{6} \text{ or } \frac{1}{2} \cdot \frac{5}{6}, \frac{7}{15}, \frac{3}{4} \text{ and } \frac{1}{2}.$$

$$\begin{array}{l} \text{Then } 5 \times 10 \times 4 \times 2 = 400 \\ \text{Also } 7 \times 6 \times 4 \times 2 = 336 \\ \quad 3 \times 6 \times 10 \times 2 = 360 \\ \quad 1 \times 4 \times 10 \times 6 = 240 \end{array} \left. \vphantom{\begin{array}{l} 5 \times 10 \times 4 \times 2 \\ 7 \times 6 \times 4 \times 2 \\ 3 \times 6 \times 10 \times 2 \\ 1 \times 4 \times 10 \times 6 \end{array}} \right\} \text{New Numerators.}$$

$$\text{And } 6 \times 10 \times 4 \times 2 = 480 \text{ Common Denominator.}$$

$$\text{Answer } \frac{400}{480}, \frac{336}{480}, \frac{360}{480}, \text{ and } \frac{240}{480}.$$

C A S E VI.

$$(33) \quad \text{Thus } \frac{3}{4 \times 21} = \frac{3}{84} \text{ or } \frac{1}{28}, \text{ the Fraction required.}$$

$$(34) \quad \text{Thus } \frac{1 \times 20 \times 12}{620} = \frac{240}{620} \text{ or } \frac{12}{31}, \text{ the Fraction req.}$$

$$(35) \quad \text{Thus } \frac{3}{4 \times 4 \times 12 \times 27} = \frac{3}{5184} \text{ or } \frac{1}{1728}, \text{ the Fract. req.}$$

$$(36) \quad \text{Thus } \frac{5}{7 \times 12 \times 20} = \frac{5}{1680} \text{ or } \frac{1}{336}, \text{ the Fraction req.}$$

$$(37) \quad \text{Thus } \frac{1 \times 21 \times 12 \times 4}{1344} = \frac{1008}{1344} \text{ or } \frac{3}{4}, \text{ the Fract. req.}$$

$$(38) \quad \text{Thus } \frac{5}{6 \times 20 \times 12} = \frac{5}{1440} \text{ or } \frac{1}{288}, \text{ the Fraction req.}$$

$$(39) \quad \text{Thus } \frac{3 \times 112}{448} = \frac{336}{448} \text{ or } \frac{3}{4}, \text{ the Fraction required.}$$

$$(40) \quad \text{Thus } \frac{3}{4 \times 16 \times 16 \times 112} = \frac{3}{114088}, \text{ the Fract. req.}$$

$$(41) \quad \text{Thus } \frac{5 \times 12 \times 20}{144} = \frac{1200}{144} \text{ or } \frac{25}{3}, \text{ the Fraction req.}$$

$$(42) \quad \text{Thus } \frac{7 \times 3 \times 8 \times 40}{9} = \frac{6720}{9} \text{ or } \frac{2240}{3}, \text{ the Fract. req.}$$

(43)

(43) Thus $\frac{7}{45} \times \frac{4}{4} \times \frac{4}{4} = \frac{112}{45}$, the Fraction required,

(44) Thus $\frac{11}{12 \times 63} = \frac{11}{756}$, the Fraction required.

(45) Thus $\frac{7}{9} \times \frac{48}{8} \times \frac{8}{8} = \frac{2688}{9}$ or $\frac{896}{3}$, the Fraction req.

(46) Thus $\frac{7}{142} \times \frac{36}{36} = \frac{252}{142}$ or $\frac{126}{71}$, the Fraction required.

(47) Thus $\frac{3}{5 \times 8 \times 36} = \frac{3}{1440}$ or $\frac{1}{480}$, the Fraction req.

(48) Thus $\frac{7}{142} \times \frac{7}{7} \times \frac{24}{24} \times \frac{60}{60} \times \frac{60}{60} = \frac{4233600}{142}$ or $\frac{2116800}{71}$,
the Fraction required.

(49) Thus $\frac{10}{11 \times 60 \times 24} = \frac{10}{15840}$ or $\frac{1}{1584}$, the Fract. req.

C A S E VII.

(51) 367 Moidores.
× 27

1126)9909(7s.

Remains 837s.

× 12

1126)10044(7d.

Remains 972d.

× 4

1126)3888(3 Qrs.

Answer 7s. 7 $\frac{3}{4}$.

(54) £. s.
3 12
× 4

5)14 8

Answer 2 17 7 $\frac{1}{5}$.

(52) 5 Guineas.
× 21

12)105

Answer 8s. 9d.

(53) 5s.
12

8)60

Answer 7 $\frac{1}{2}$ d.

(55) 5£.
20

7)100

Answer 14s. 3 $\frac{1}{4}$ d. $\frac{5}{7}$.

(56)

Reduction of Vulgar Fractions.

(56)

lb.

$$\begin{array}{r} 411 \\ \times 12 \\ \hline \end{array}$$

$$720)4932(6 \text{ oz.}$$

Remains 612

$$\times 20$$

$$72,0)1224,0(17 \text{ dwts.}$$

$$504$$

Answer 6 oz. 17 dwts.

(59)

$$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$$

$$8)56$$

Answer 7 Furlongs.

Ell. Eng.

(60)

$$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$$

$$8)25$$

Answer $3\frac{1}{8}$ Qrs.

(62)

Hhd. W.

$$\begin{array}{r} 11 \\ \times 63 \\ \hline \end{array}$$

$$252)693(2 \text{ Galls.}$$

Remains 189

$$\times 4$$

$$252)756(3 \text{ Qts.}$$

Answer 2 Galls. 3 qts.

(57)

cwt.

$$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$$

$$9)28$$

Answer 3 qrs. 3 lb. 1 oz $12\frac{4}{9}$ dwts.

(58)

Tons.

$$63$$

$$\times 20$$

$$124)1260(10 \text{ cwt.}$$

Remains 20

$$\times 112$$

$$124)2240(18 \text{ lb.}$$

$$1000$$

Remains 8

$$\times 16$$

$$124)128(1 \text{ oz.}$$

Remains 4

Answer 10 C. 18 lb. $1\frac{1}{31}$ oz.

(61)

Acres.

$$7$$

$$\times 4$$

$$8)28$$

Answer 3 Rds. 20 pol.

(63)

Bar. B.

$$\times 36$$

$$9)252$$

Answer 28 Galls.

$$(64) \quad \begin{array}{r} \text{Chal.} \\ 25 \\ \times 36 \\ \hline 72 \overline{) 900} (12 \text{ Bush.} \end{array}$$

Remains 36 Bush.

$$\begin{array}{r} \times 4 \\ \hline 72 \overline{) 144} (2 \text{ Pecks.} \end{array}$$

Answer 12 Bush. 2 Pecks.

$$(65) \quad \begin{array}{r} \text{Months.} \\ 7 \\ \times 4 \\ \hline 8 \overline{) 28} \end{array}$$

Answer Weeks 3. 4. 3. 12

$$(66) \quad \begin{array}{r} 5 \text{ Days.} \\ \times 24 \\ \hline 7 \overline{) 120} \end{array}$$

Answer Hours 17. 8. $34\frac{2}{7}$

CASE VIII.

(68) First $4\frac{1}{2}d. = 9$ Halfpence; and $1s. = 24$. Ans. $\frac{9}{24}$ or $\frac{3}{8}$.

(69) First $2\text{ } \pounds. 17s. 7\frac{1}{2}d. = 3456$ Fifths. Ans. $\frac{3456}{4320}$ or $\frac{4}{5}$.

And $3s. 12d. = 4320$

(70) First $8s. 2d. = 98$ Pence.

And $1\text{ } \pounds. 1s. = 252$

Answer $\frac{98}{252}$ or $\frac{7}{18}$.

(71) First 6 oz. 17 dwts. 21 grs. = 3309 grs.

And 1 lb.

= 5760 Ans. $\frac{3309}{5760}$ or $\frac{1103}{1920}$.

(72) First 3 qrs. 3 lb. 1 oz. $12\frac{5}{9} = 200705$ Ninths.

And 1 Cwt.

= 358048 Answer $\frac{200705}{358048}$.

(73) First 10 Cwt. 18 lb. $1\frac{1}{3}\text{ oz.} = 564480$

And 1 Ton.

= 1111040 or $\frac{63}{124}$ Ans.

(74) First 3 Qrs. $\frac{1}{8} = 25$ Eighths.

And 1 Ell. Eng. = 40

or $\frac{5}{8}$, the Answer.

(75) First 2 Feet 6 inch. = 30 Inches.

And 1 Yard

= 36

or $\frac{5}{6}$, the Answer.

(76) First 4 Fur. 32 p. = 192 Poles.

And 1 Mile

= 330

or $\frac{3}{5}$, the Answer.

(77) First 3 Roods. 2 p. = 122 Poles.

And 1 Acre

= 160

or $\frac{61}{80}$, the Answer.

(78) Thus $\frac{42}{63}$ or $\frac{2}{3}$, the Fraction required.

(79) Thus $\frac{23}{36}$ or $\frac{7}{9}$, the Fraction required.

- (80) First 4 Bush. 2 Pks = $\underline{58}$ Pecks.
 And a Chaldron = $\underline{144}$ or $\frac{29}{72}$, the Answer.
- (81) First 1 W. 1 d. 12 h. = $\underline{252}$ Hours.
 And 1 Month = $\underline{672}$ or $\frac{3}{8}$, the Answer.
- (82) First 243 D. 8 h. = $\underline{5840}$ Hours.
 And 1 Year = $\underline{8760}$ or $\frac{2}{3}$, the Answer.

39. ADDITION OF VULGAR FRACTIONS.

- (2) Thus $\frac{3}{7} + \frac{1}{4} + \frac{5}{6} = \frac{72}{168} + \frac{42}{168} + \frac{140}{168} = \frac{254}{168}$ or $1\frac{43}{84}$ Answer.
- (3) Thus $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = \frac{360}{720} + \frac{480}{720} + \frac{540}{720} + \frac{576}{720} + \frac{600}{720} = \frac{2556}{720}$ or $3\frac{11}{20}$, the Sum required.
- (4) First $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.
 Then $\frac{8}{15} + \frac{13}{15} = \frac{21}{15}$ or $1\frac{2}{5}$, the Sum required.
- (5) First $\frac{3}{7}$ of $\frac{5}{6} = \frac{5}{14}$, and $\frac{5}{9}$ of $\frac{3}{4} = \frac{5}{12}$.
 Then $\frac{5}{14} + \frac{5}{12} = \frac{60}{168} + \frac{70}{168} = \frac{130}{168}$ or $\frac{65}{84}$, the Sum required.
- (6) First $\frac{1}{5}$ of $\frac{5}{6} = \frac{1}{6}$, (see Note Case II.)
 Then $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{72}{144} + \frac{48}{144} + \frac{36}{144} + \frac{24}{144} = \frac{180}{144}$ or $1\frac{1}{4}$, the Sum required.
- (11) First $\frac{1}{8}$ of 1s. = $\frac{1}{160}$ of 1£. per Case VI.
 Then $\frac{201}{400} + \frac{1}{160} = \frac{32160}{73600} + \frac{460}{73600} = \frac{32620}{73600}$ or $\frac{1631}{3680}$ £.
 which by Case VII. = 8s. 10 $\frac{1}{4}$ d. $\frac{11}{13}$.
- (12) First $\frac{3}{8}$ of a oz. = $\frac{13}{96}$ or $\frac{1}{2}$ of a lb.
 Then $\frac{1}{4} + \frac{1}{32} = \frac{32}{128} + \frac{4}{128} = \frac{36}{128} = \frac{9}{32}$ lb. or 3 oz. 7 dwts. 12 grs. the Sum required.
- (13) First $\frac{1}{3}$ of a lb. = $\frac{1}{336}$ of a Cwt.
 Then $\frac{5}{8} + \frac{1}{336} = \frac{1680}{2688} + \frac{8}{2688} = \frac{1688}{2688} = \frac{211}{336}$ Cwt. or 2 qrs. 14 lb. 5 oz. $5\frac{1}{5}$ drs. the Answer.
- (14) First $\frac{2}{5}$ of an Ell Eng. = $\frac{2}{4}$ of a Yard.
 Then $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ yd. or 1 yd. 1 qr. the Sum required.
- (15) First $\frac{2}{3}$ of a Yard. = $\frac{2}{5280}$ or $\frac{1}{2640}$ of a Mile.
 Then $\frac{5}{6} + \frac{1}{2640} = \frac{13200}{15840} + \frac{6}{15840} = \frac{13206}{15840}$ Miles, or 6 Fur. 26 p. 4 yds. 1 ft. the Sum required.
- (16) First $\frac{1}{2}$ of a Peck = $\frac{1}{288}$ of a Chaldron.
 Then $\frac{3}{8} + \frac{1}{288} = \frac{864}{2304} + \frac{8}{2304} = \frac{872}{2304} = \frac{109}{288}$ or 13 Bush. 2 pks. 1 gall. the Sum required.

(17) First $\frac{3}{4}$ of a Week $= \frac{3}{10}$ of a Month.
Then $\frac{3}{10} + \frac{5}{10} = \frac{8}{10} = \frac{4}{5}$ Mon. or 1 m. 14 hrs.
the Sum required.

(18) First $\frac{3}{4}$ of an Hour $= \frac{3}{24}$ or $\frac{1}{8}$ of a Week.
Then $\frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4}$ or 4 D 9 h. 45 m.
the Sum required.

(19) First $\frac{2}{5}$ of 12 £ $= 2\frac{4}{5}$ or $4\frac{4}{5}$ £. and $\frac{1}{5}$ of $\frac{9}{10} = \frac{9}{50}$.
Also $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{1}{20} = \frac{1}{40}$.
Then $\frac{4}{5} + \frac{3}{5} + \frac{9}{50} + \frac{1}{40} = \frac{56000}{70000} + \frac{30000}{70000} + \frac{12600}{70000} + \frac{1750}{70000}$
 $= \frac{100350}{70000}$ or $1\frac{607}{14000}$.
Then $4 + 4 + 1\frac{607}{14000} = 9\frac{607}{14000}$ £. or 9 £. 8s. 8d. $\frac{8}{35}$, the
Sum required.

40. SUBTRACTION OF VULGAR FRACTIONS.

(2) Thus $\frac{11}{12} - \frac{2}{9} = \frac{99}{108} - \frac{24}{108} = \frac{75}{108}$, the Diff. required.

(3) Thus $\frac{19}{64} - \frac{2}{13} = \frac{247}{832} - \frac{128}{832} = \frac{119}{832}$, the Diff. required.

(4) Thus $\frac{14}{17} - \frac{7}{11} = \frac{154}{187} - \frac{119}{187} = \frac{35}{187}$, the Diff. required.

(5) First $\frac{3}{4}$ of $\frac{7}{8} = \frac{21}{32}$.
Then $\frac{21}{32} - \frac{2}{5} = \frac{105}{160} - \frac{64}{160} = \frac{41}{160}$, the Diff. required.

(6) First $\frac{5}{6}$ of $\frac{7}{10} = \frac{7}{12}$, and $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$.
Then $\frac{7}{12} - \frac{1}{3} = \frac{21}{36} - \frac{12}{36} = \frac{9}{36}$ or $\frac{1}{4}$, the Diff. required.

(7) Thus $10\frac{3}{4} - 6\frac{5}{6} = 10\frac{18}{24} - 6\frac{20}{24} = 3\frac{22}{24}$ or $3\frac{11}{12}$, the Diff.

(8) Thus $17\frac{11}{12} - 16\frac{7}{8} = 17\frac{22}{24} - 16\frac{21}{24} = 1\frac{1}{24}$, the Diff.

(9) First $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{4} = \frac{35}{64}$.
Then $12 - \frac{35}{64} = 11\frac{29}{64}$, the Difference.

(10) First $\frac{3}{4}$ of a penny $= \frac{3}{48}$ or $\frac{1}{16}$ of 1s. per Case VI.
Then $\frac{2}{3} - \frac{1}{16} = \frac{22}{48} - \frac{3}{48} = \frac{19}{48}$ s. or $7\frac{1}{2}$ d. per Case VII. Ans.

(11) First $\frac{3}{4}$ of an oz. $= \frac{3}{7168}$ of a Cwt.
Then $\frac{1}{3} - \frac{3}{7168} = \frac{7168}{21504} - \frac{9}{21504} = \frac{7159}{21504}$ C. or 1qr. 19lb.
4oz. $9\frac{1}{3}$ drs. the Difference required.

(12) First $\frac{3}{8}$ of an Inch $= \frac{3}{288}$ or $\frac{1}{96}$ of a Yard.
Then $\frac{3}{4} - \frac{1}{96} = \frac{288}{384} - \frac{4}{384} = \frac{284}{384} = \frac{71}{96}$ yds. or 2 ft. $2\frac{5}{8}$ inc.
the Difference required.

(13) First $\frac{3}{4}$ of a Peck $= \frac{3}{576}$ or $\frac{1}{192}$ of a Chaldron.
Then $\frac{3}{8} - \frac{1}{192} = \frac{576}{1536} - \frac{8}{1536} = \frac{568}{1536} = \frac{71}{192}$ Chal. or 13
bush. 1 pk. 2 galls. the Difference.

(14) First $\frac{5}{6}$ of an Hour $= \frac{5}{96}$ or $\frac{1}{32}$ of a Day.
Then $\frac{5}{6} - \frac{1}{32} = \frac{160}{192} - \frac{6}{192} = \frac{154}{192} = \frac{77}{96}$ Day; or 19 h. 15 min.
the Difference.

41. MULTIPLICATION OF VULGAR FRACTIONS.

- (2) Thus $\frac{6}{11} \times \frac{7}{12} = \frac{7}{22}$ (see Note in Case II.) the Product required.
- (3) Thus $12\frac{1}{2} \times 6 = 75$, the Product required.
- (4) First $17\frac{3}{5} = \frac{88}{5}$.
Then $\frac{88}{5} \times \frac{7}{8} = 7\frac{7}{5}$ or $15\frac{2}{5}$, the Product required.
- (5) First $2\frac{3}{4} = \frac{11}{4}$, and $\frac{3}{4}$ of $\frac{7}{8} = \frac{21}{32}$.
Then $\frac{11}{4} \times \frac{21}{32} = \frac{231}{128}$ or $1\frac{103}{128}$, the Product required.
- (6) First $12\frac{3}{4} = \frac{51}{4}$, and $\frac{7}{8}$ of $1\frac{2}{3} = \frac{84}{8}$ or $2\frac{1}{2}$.
Then $\frac{51}{4} \times 2\frac{1}{2} = 107\frac{1}{8}$ or $133\frac{7}{8}$, the Product required.
- (7) First $\frac{3}{4}$ of $\frac{10}{11} = \frac{15}{22}$.
Then $\frac{15}{22} \times \frac{6}{1} = \frac{45}{11}$ or $4\frac{1}{11}$, the Product required.
- (8) First $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$, and $\frac{2}{5}$ of $\frac{5}{7}$ of $1\frac{4}{7} = 2\frac{8}{7}$.
Then $\frac{21}{32} \times 2\frac{8}{7} = 2\frac{1}{8}$ or $2\frac{5}{8}$, the Product.
- (9) First $3\frac{2}{3} = \frac{11}{3}$.
Then $\frac{11}{3} \times \frac{1}{7} = \frac{11}{21}$. Also $\frac{3}{5}$ of $\frac{3}{4} = \frac{9}{20}$.
Again $\frac{11}{21} \times \frac{9}{20} = \frac{33}{140}$, the Product required.

42. DIVISION OF VULGAR FRACTIONS.

- (2) Thus $\frac{6}{7} \div \frac{3}{5}$ or $\frac{6}{7} \times \frac{5}{3} = \frac{30}{21}$, the Quotient required.
- (3) First $\frac{2}{3}$ of $\frac{7}{8} = \frac{7}{12}$.
Then $\frac{2}{11} \div \frac{7}{12}$ or $\frac{2}{11} \times \frac{12}{7} = \frac{24}{77}$, the Quotient required.
- (4) First $12\frac{1}{2} = \frac{25}{2}$, and $17\frac{2}{3} = \frac{53}{3}$.
Then $\frac{25}{2} \div \frac{53}{3}$ or $\frac{25}{2} \times \frac{3}{53} = \frac{75}{106}$, the Quotient required.
- (5) First $12\frac{3}{4} = \frac{51}{4}$, and $3\frac{7}{8} = \frac{31}{8}$.
Then $\frac{51}{4} \div \frac{31}{8}$ or $\frac{51}{4} \times \frac{8}{31} = \frac{408}{124}$ or $3\frac{9}{31}$, the Quot. req.
- (6) First $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$, and $\frac{2}{5}$ of $1\frac{2}{3} = \frac{24}{5}$.
Then $\frac{21}{32} \div \frac{24}{5}$ or $\frac{21}{32} \times \frac{5}{24} = \frac{105}{768}$, the Quotient required.
- (7) First $7\frac{2}{7} = \frac{51}{7}$, then $\frac{51}{7} \div \frac{9}{1}$ or $\frac{51}{7} \times \frac{1}{9} = \frac{51}{63}$, the Quot.
- (8) Thus $9\frac{6}{1} \div 1\frac{4}{6}$ or $9\frac{6}{1} \times \frac{1}{146} = \frac{96}{146}$ or $\frac{48}{73}$, the Quot. req.
- (9) First $14\frac{3}{7} = \frac{101}{7}$, and $\frac{3}{4}$ of $1\frac{2}{3} = \frac{9}{4}$.
Then $\frac{101}{7} \div \frac{9}{4}$ or $\frac{101}{7} \times \frac{4}{9} = \frac{404}{63}$, the Quotient required.
- (10) First $142\frac{7}{12} = \frac{1711}{12}$, and $12\frac{3}{5} = \frac{63}{5}$.
Then $\frac{1711}{12} \div \frac{63}{5}$ or $\frac{1711}{12} \times \frac{5}{63} = \frac{8555}{756}$ or $11\frac{279}{756}$, the Quotient required.

The Rule of Three Direct in Vulgar Fractions. 205

(11) First $\frac{7}{8}$ of $\frac{6}{1} = \frac{21}{4}$, and $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12} = \frac{33}{56}$.

Then $\frac{21}{4} \div \frac{33}{56}$ or $\frac{21}{4} \times \frac{56}{33} = \frac{1176}{132}$ or $8\frac{10}{11}$, the Quot. req.

1. If the Divisor and Dividend have both the same Denominator, the Quotient may be found by dividing one Numerator by the other.

Thus $\frac{7}{11} \div \frac{4}{11}$; thus $7 \div 4 = \frac{7}{4}$, the Quotient required.

2. If the Divisor and Dividend have each the same Numerator, divide one of the Denominators by the other, which will give the Quotient required.

Divide $\frac{8}{17}$ by $\frac{8}{11}$; thus $17 \div 11 = \frac{17}{11}$, the Quotient.

3. If a Number can be found that will divide both the Numerators, or both the Denominators, (viz. those of the Divisor and Dividend) without a Remainder; use those Quotients instead of the given Numerators and Denominators, which will give the Result in its lowest Terms.

Divide $\frac{8}{15}$ by $\frac{9}{16}$; thus $\frac{8}{15} \div \frac{9}{16} = \frac{1}{5} \div \frac{3}{2} = \frac{2}{15}$, the Quot. req.

For 8 and 16 divide by 8, also 15 and 9 divide by 3.

43. THE RULE OF THREE DIRECT.
IN VULGAR FRACTIONS.

(2) If $2\frac{2}{5}$ yds. : $3\frac{3}{4}$ £. :: $4\frac{4}{5}$ yds. or $\frac{12}{5}$: $\frac{15}{4}$ £. :: $2\frac{4}{5}$ yd.

Then $24 \times 15 \times 5$ } = $\frac{1800}{240}$ £. or 7£. 10s. the Answer.
And $12 \times 4 \times 5$ }

(3) If $\frac{3}{4}$ lb : $5\frac{1}{2}$ s. :: $42\frac{3}{8}$, or as $\frac{3}{4}$: $\frac{11}{2}$ s. : $33\frac{9}{8}$ lb.

Then $339 \times 11 \times 4$ } = $\frac{14916}{960}$ £. or 15£. 10s. 9d. Answ.
And $3 \times 2 \times 8$ }

(4) If $\frac{7}{9}$: $14\frac{2}{3}$ s. :: 8 Cwt. or $\frac{7}{9}$ s. : $4\frac{4}{3}$:: $\frac{8}{1}$.

Then $44 \times 9 \times 8$ } = 3168 s. or 7£. 10s. $10\frac{1}{4}$ d. $\frac{1}{7}$ Answ.
And $7 \times 3 \times 1$ }

(5) If $10\frac{1}{2}$: 100£. :: $2700\frac{1}{2}$, or as $2\frac{1}{2}$: $100\frac{1}{1}$ £. :: $270\frac{1}{2}$,

Then $2701 \times 2 \times 100$ } = 540200 £. or 283£. 11s. $\frac{1}{2}$ d. $\frac{2}{5}$,
And $21 \times 1 \times 2$ } the Answer.

(6) If $112\frac{5}{8}$ £. : 100£. :: 1270£. or as $90\frac{1}{8}$ £. : $100\frac{1}{1}$:: $1270\frac{1}{1}$.

Then $1270 \times 8 \times 100$ } = 1016000 £. or 1127£. 12s.
And $901 \times 1 \times 1$ } $8\frac{1}{2}$ d. $\frac{470}{901}$, the Answer.

(7) First $4\frac{3}{4} = \frac{19}{4}$, and $22\frac{3}{8} = \frac{179}{8}$, $\therefore \frac{179}{8} \times \frac{19}{4} = 34\frac{0\frac{1}{2}}{32}$ yds.
in the $22\frac{3}{8}$ Pieces. Then,

If $1 : 8\frac{3}{4} s. \therefore 34\frac{0\frac{1}{2}}{32}$, or as $\frac{1}{1} : 3\frac{5}{4} s. : 34\frac{0\frac{1}{2}}{32}$ yds.

Then 3401×35 } $= \frac{1190}{128} \text{ £. or } 46\text{ £. } 9s. \text{ } 11\frac{1}{2}d. \frac{1}{8} \text{ Answer.}$
And 32×4 }

(8) If $5s. 6\frac{3}{4}d. : 3 \text{ qrs. } :: 100\text{ £. } 10s. 6d. \text{ or as } \frac{267}{960} \text{ £. } : \frac{3}{5} \text{ Ell.}$
 $:: 40\frac{2\frac{1}{2}}{40} \text{ £.}$

Then $4021 \times 3 \times 960$ } $= \frac{11580480}{33400} \text{ or } 289\frac{512}{1335} \text{ Ells. } \therefore$
And $267 \times 5 \times 40$ }

$\frac{289512}{1335} \div \frac{12}{1} = \frac{289512}{10020}$, or $18\frac{32}{445}$ Ells in each Piece,
the Answer.

44. THE RULE OF THREE INVERSE.

(2) If $25\frac{1}{3} \text{ £. } : 6\frac{3}{4} \text{ mo. } :: 10\frac{3}{4} \text{ £. or as } 7\frac{6}{3} : 2\frac{7}{4} :: 4\frac{3}{4}.$

Then $76 \times 27 \times 4$ } $= \frac{8208}{516}$, or 15 mo. 3 w. $4\frac{51}{129} d.$ Anf.
And $43 \times 3 \times 4$ }

(3) If $4 : 12\frac{3}{4} \text{ ho. } :: 12 \text{ or as } \frac{4}{1} : 5\frac{1}{4} \text{ ho. } :: 1\frac{2}{1} \text{ per.}$

Then $51 \times 4 \times 1$ } $= \frac{204}{48} = 1\frac{7}{4}$, or $4\frac{1}{4}$ Hours, Answer.
And $1 \times 4 \times 12$ }

(4) If $12\frac{2}{3} \text{ oz. } : 5s. :: 8\frac{3}{4} \text{ oz. or as } 3\frac{8}{3} : 5s. :: 3\frac{5}{4}.$

Then $38 \times 5 \times 4$ } $= \frac{760}{105}$, or 7s. $2\frac{3}{4}d. \frac{3}{7}$, the Answer.
And $35 \times 1 \times 3$ }

(5) If $6\frac{2}{3} \text{ m. } : 100\frac{2}{3} \text{ £. } :: 3\frac{5}{6} \text{ yrs. or as } \frac{5}{9} \text{ yr. } : 30\frac{2}{3} \text{ £. } ::$
 $2\frac{3}{6} \text{ yrs.}$

Then $302 \times 5 \times 6$ } $= \frac{9060}{621}$, or 14 £. 11s. $9\frac{1}{4}d. \frac{55}{469}$, Answ.
And $33 \times 9 \times 3$ }

(6) If $5\frac{7}{12} s. : 26\frac{5}{8} \text{ yds. } :: 8\frac{1}{2} s. \text{ or as } \frac{67}{12} : 21\frac{3}{8} :: 1\frac{7}{2}.$

Then $67 \times 213 \times 2$ } $= \frac{28542}{1632}$, or 17 yds. $\frac{133}{272} \text{ qrs. Answ.}$
And $12 \times 8 \times 17$ }

45. THE DOUBLE RULE OF THREE.

(2) First $14\text{ £. } 6s. 8d. = 14\frac{1}{3} \text{ or } 4\frac{2}{3} \text{ £.}$

Then $12. 16. 4\frac{2}{3}$ | Now $12 \times 16 = 192$ Divisor per Sect. 15.

$18. 34.$ | And $4\frac{2}{3} \times 34 \times 18 = 263\frac{16}{3}$ Dividend.

$\therefore 263\frac{16}{3} \div 192 = 26\frac{316}{576}$, or 45 £. 13s. 9d. the Answer.

- (3) First $16\text{£}. 13\text{s}. 4\text{d}. \times 16\frac{2}{3}\text{£}. \text{ or } \frac{50}{3}$, and $6\text{£}. 17\text{s}. 6\text{d}. = 6\frac{7}{8} \text{ or } \frac{55}{8}$.

Cwt. m. £.
 Then $40 \cdot 30 \cdot \frac{50}{3}$ | Now $\frac{50}{3} \times \frac{80}{1} = 4000$ Divisor.
 $\quad \quad \quad \cdot 80 \cdot \frac{55}{8}$ | Also $4\frac{0}{1} \times 3\frac{0}{1} \times \frac{55}{8} = 6600$ Dividend.
 $\therefore 66000 \div 4000 = 16\frac{5}{8} \text{ or } 6 \text{ Cwt. } 21 \text{ lb. the Answer.}$

- (4) First $26\text{£}. 19\text{s}. 4\text{d}. = 26\frac{29}{30} \text{ or } \frac{809}{30}$, and $14\text{£}. 15\text{s}. = 14\frac{3}{4} \text{ or } \frac{59}{4}$.

Per. m. £.
 Then $12 \cdot 3 \cdot \frac{809}{30}$ | Now $\frac{809}{30} \times \frac{36}{1} = 29124$ Divisor.
 $\quad \quad \quad 36 \cdot \quad \cdot \frac{59}{4}$ | Also $\frac{59}{4} \times \frac{12}{1} \times \frac{3}{1} = 2124$ Dividend.
 $\therefore 2124 \div 29124 = \frac{63720}{116496} \text{ m. or } 16\frac{833}{14562} \text{ Days, Answer.}$

M. d. w.

- (5) $30 \cdot 11 \cdot 1$ | Now $\frac{11}{5} \times \frac{1}{1} = \frac{11}{5}$ Divisor.
 $\quad \quad \quad \cdot \frac{11}{5} \cdot 4$ | Also $3\frac{0}{1} \times \frac{11}{1} \times \frac{4}{1} = 1320$ Dividend.
 $\therefore 1320 \div \frac{11}{5} = 600$ Men, the Answer.

- (6) First $2\frac{1}{2} = \frac{5}{2}$, and $3 - \frac{1}{10} = 2\frac{9}{10} \text{ or } \frac{29}{10}$, also $\frac{1}{8}$ of $\frac{3}{5}$ of $2\frac{1}{2} = \frac{63}{400}$ s.

£. m. £.
 Then $\frac{5}{2} \cdot \frac{29}{10} \cdot \frac{63}{400}$ | Now $\frac{29}{10} \times \frac{5}{2} = 2\frac{9}{2}$ Divisor.
 $\quad \quad \quad \frac{1}{20} \cdot 1 \cdot \quad$ | Also $\frac{63}{400} \times \frac{1}{20} \times \frac{1}{1} = \frac{63}{8000}$ Dividend.
 $\therefore \frac{63}{800} \div \frac{29}{2} = \frac{126}{23200} \text{ s. or } \frac{378}{7250} \text{ qr. the Answer.}$

45. QUESTIONS IN VULGAR FRACTIONS.

- (1) Thus $99\frac{9}{10}$ the Answer. For $\frac{9}{10} = 1$, and $99 + 1 = 100$.

- (2) First $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213} = \frac{7614}{14910} \text{ or } \frac{1269}{2485}$.
 $\therefore 1 - \frac{1269}{2485} = \frac{1216}{2485}$ Answ. For $\frac{1269}{2485} + \frac{1216}{2485} = 1$, Proof.

- (3) First $\frac{1}{25}$ of $\frac{7}{8} = \frac{7}{200}$, and $\frac{1}{16}$ of $\frac{47}{19} = \frac{47}{304}$.
 $\therefore 3\frac{7}{200} - \frac{47}{304} = 3\frac{2128}{60800} - \frac{9400}{60800} = 2\frac{53528}{60800} \text{ or } 2\frac{6591}{7600} \text{ the Answer.}$

For $2\frac{53528}{60800} - \frac{2128}{60800} = 2\frac{51400}{60800}$ which added to $\frac{9400}{60800} = 3$, the Proof.

- (4) First $\frac{1}{11}$ of $\frac{12}{1} = \frac{12}{11}$, and $\frac{1}{19}$ of $\frac{27}{1} = \frac{27}{19}$.
 Then $\frac{12}{11} + \frac{27}{19} = \frac{228}{209} + \frac{297}{209} = \frac{525}{209} = 2\frac{107}{209}$.
 Again $7\frac{1}{2} = \frac{15}{2}$, then $\frac{1}{3}$ of $\frac{15}{2} = \frac{5}{2}$; also $1\frac{1}{4} = \frac{5}{4}$, and $\frac{29}{30}$ of $\frac{5}{4} = \frac{29}{24}$; then $\frac{5}{2} - \frac{29}{24} = \frac{31}{24}$ or $1\frac{7}{24}$, which added to $8 = 9\frac{7}{24}$.
 $\therefore 9\frac{7}{24} - 2\frac{107}{209} = 9\frac{1463}{5016} - 2\frac{2568}{5016} = 6\frac{3911}{5016} \text{ the Answer.}$

- (5) First $\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{1}{4} = \frac{231}{128}$.
 $\therefore \frac{1}{1} \div \frac{231}{128} = \frac{128}{231}$ the Number, therefore $\frac{128}{231} \times \frac{128}{231} \times \frac{12}{231} = \frac{2097152}{1233211}$, the Cube of that Number.

(6) First $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8} = 1$, and $9 \frac{2}{3} = 3 \frac{6}{3}$.

$\therefore 3 \frac{6}{3} \times \frac{1}{1} = 3 \frac{6}{3}$, the Number.

Then $3 \frac{6}{3} \times 3 \frac{6}{3} = 13 \frac{0}{3} = 95 \frac{2}{3}$, the Answer.

(7) First $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8} = \frac{3}{10}$; also $\frac{1}{5}$ of $\frac{6}{7}$ of $\frac{1}{1} = \frac{2}{7}$.

Then, as $\frac{3}{10} : \frac{2}{7} :: 12000 : 88000 = 3223 \text{ £. } 8\text{s. } 80\frac{1}{4}\text{d.}$
 $\frac{4}{9}$, Value of the Ship.

$\therefore 12000 \div 3223 \text{ £. } 8\text{s. } 10\frac{1}{4}\text{d. } \frac{4}{9} = 15223 \text{ £. } 8\text{s. } 10\frac{1}{4}\text{d. } \frac{4}{9}$,
 the Answer.

(8) First $\frac{3}{4}$ of $\frac{3}{5} = \frac{9}{20}$ fold. Then, as $\frac{9}{20} : 17 \frac{10}{1} :: \frac{20}{9} :$
 $34200 = 3800 \text{ £.}$ the Answer.

(9) His whole Estate $\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$. Then $\frac{3}{9}$ of $\frac{4}{9} = \frac{16}{81}$
 youngest Son's Part; and $\frac{3}{9} = \frac{28}{81}$ eldest.

$\therefore \frac{28}{81} - \frac{16}{81} = \frac{12}{81} = 57 \text{ £. } 3\text{s. } 4\text{d.}$ or $154\frac{3}{6}$.

Also $\frac{28}{81} + \frac{16}{81} = \frac{44}{81}$ both the Sons Part.

So that $\frac{5}{9} - \frac{44}{81} = \frac{24}{81}$, Widow's Part.

Then by rejecting the Common Denominator we shall
 have the following Proportion

As $1156 : 154\frac{3}{6} :: 2401 : 370 \frac{4743}{6936} = 534 \text{ £. } 2 \text{ s. } 8 \frac{1616}{3468}$
 Widow had the Use of.

As $1156 : 154\frac{3}{6} :: 2822 : 217 \frac{7170}{3468} = 627 \text{ £. } 15 \text{ s. } 9\frac{1}{4} \frac{2852}{3468}$
 eldest Son had.

As $1156 : 154\frac{3}{6} :: 1666 : 128 \frac{5219}{3468} = 370 \text{ £. } 19 \text{ s. } 5\frac{1}{4} \frac{2244}{2468}$

His whole Estate 1622 10 10 $\frac{3}{4}$ $\frac{3224}{3468}$

(10) First $\frac{1}{3} - \frac{1}{9} = \frac{2}{9} = \frac{741}{1170} - \frac{339}{1170} = \frac{411}{1170}$.

Then, as $\frac{411}{1170} : 108 \frac{1}{2} \text{ £. } :: \frac{1170}{411} : 1264 \frac{770}{822} = 1538 \text{ £.}$
 $12\text{s. } 11\frac{3}{4}\text{d. } \frac{257}{411}$, the Answer.

(11) First 4000 his whole Stock.

Then $\frac{5}{8}$ of $\frac{2}{3}$ of 4000 = $1666\frac{2}{3}$ Mad Tom took.

2333 $\frac{1}{3}$ = 7000 left.

$\frac{2}{5}$ of $\frac{5}{8}$ of 7000 = 583 $\frac{1}{3}$ Raving Ned took.

1750 left.

$\frac{7}{10}$ of $\frac{1}{2}$ of 1750 = 1041 $\frac{1}{4}$ Positive Jack took.

708 $\frac{3}{4}$ = 708 $\frac{48}{64}$ left.

$\frac{3}{4}$ of $\frac{1}{4}$ of 708 $\frac{48}{64}$ = 132 $\frac{57}{64}$ Dolly had.

Answer 575 $\frac{55}{64}$ left.

- (12) As $\frac{5}{12} : 2200 :: \frac{12}{12} : 26400 = 5280\text{£}$. Brother's Fortune.
 And $5280 \times 3\frac{1}{8} = 16500$; Also $1\frac{1}{2} = \frac{3}{2}$.
 $\therefore 16500 \div \frac{3}{2} = 33000 \div 3 = 11000\text{£}$. Father's Part.

| | | | |
|--|---|-------|----|
| (13) | | £. | s. |
| First 17 Hhds. at 34 £. each | $= 34 \times 17 =$ | 578 | 0 |
| As $\frac{4}{7} : 57\frac{8}{1} :: \frac{7}{7} : 404\frac{6}{4} =$ | | 1011 | 10 |
| 13 Guns at 18 £. 10s. each | $= 18\text{£}. 10\text{s.} \times 13 =$ | 240 | 10 |
| | | <hr/> | |
| Value of what was cast overboard | | 1830 | 0 |
| | | <hr/> | |

Then $\frac{3}{7}$ of $\frac{9}{13} = \frac{27}{91} = 1830\text{£}$. and $\frac{27}{91} + \frac{64}{91} = \frac{91}{91}$.
 $\therefore$ as $\frac{27}{91} : 1830\text{£} :: \frac{64}{91} : 1171\frac{20}{27} = 4337\text{£}. 15\text{s. } 6\frac{2}{3}\text{d.}$ the
 Value which came into Port.

- (14) First $16131\frac{7}{10} = 16131\frac{7}{10}$.
 Then $\frac{7}{8}$ of $\frac{2}{5}$ of $\frac{1}{2}$ of $16131\frac{7}{10} = 3387\frac{657}{800} = 4234\text{£}. 11\text{s. } 5\frac{1}{10}\text{d.}$
 A had at first.
 And $\frac{3}{5}$ of $\frac{4}{5}$ of $3387\frac{657}{800} = 1016\frac{2971}{5000} = 2032\text{£}. 11\text{s. } 10\frac{1}{2}\text{d.}$
 $\frac{54}{525}$ sold B.
 $\therefore 16131\frac{7}{10} - 3387\frac{657}{800} = 12905\frac{3600}{8000} - 3387\frac{657}{800} = 9417\frac{603}{800}$.
 Also $\frac{9}{11}$ of $9417\frac{603}{800} = 8565\frac{9327}{8800} = 9734\text{£}. 3\frac{1}{4}\text{d. } \frac{47}{55}$ Cousin
 P paid.

- (15) First $\frac{1}{24} = \frac{12}{408}$ X, $\frac{1}{24} = \frac{17}{408}$.
 Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$ performed in one Day by X, and Z.
 And $\frac{1}{12} = \frac{34}{408}$ performed in one Day by all three working
 together.
 $\therefore \frac{34}{408} - \frac{29}{408} = \frac{5}{408}$ done in one Day by Y.
 Therefore as $\frac{5}{408} : 1\text{ Day} :: 1\text{ Work} : 40\frac{8}{5} = 81\frac{3}{5}$ Days, the
 Answer.

- (16) First $\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$ left at 6 Months End.
 Then $\frac{2}{3}$ of $\frac{13}{16} = \frac{26}{48}$; and $\frac{13}{16} - \frac{26}{48} = \frac{624}{768} - \frac{416}{768} = \frac{208}{768} =$
 348 left.
 $\therefore$ as $\frac{208}{768} : 348 :: \frac{768}{768} : 267\frac{264}{208} = 1284\text{£}. 18\text{s. } 5\frac{1}{2}\text{d. } \frac{2}{3}$ Ans.

- (17) As $\frac{3}{5} : 1931\frac{12}{1} :: \frac{5}{5} : 96560 = 32186\text{£}. 13\text{s. } 4\text{d.}$ two
 Years Rent; which $\div$ by 2 = $16093\text{£}. 6\text{s. } 8\text{d.}$ the yearly
 income required.

- (18) First 880 Guineas = 924£ .
 Then as $\frac{3}{20} : 924 :: \frac{20}{20} : 18480 = 6160\text{£}$.
 $\therefore 2200 + 6160 = 8366 = \frac{4}{5}$ of his whole Fortune.
 Now as $\frac{4}{5} : 8366 :: \frac{5}{5} : 41830 = 10450\text{£}$. the Answer.

- (19) Here as the Son was to have twice as much as the Mother, and the Mother twice as much as the Daughter, so we will suppose the Estate to be divided as follows, viz. $4 + 2 + 1 = 7$ the whole Estate; then as she had both a Son and a Daughter, the Mother must have but $\frac{2}{7}$ of the Estate; whereas had it been only a Daughter, she would have had $\frac{2}{3}$.

Then $\frac{\frac{2}{3}}{\frac{2}{7}} = \frac{14}{12} = \frac{7}{6} = \frac{8}{6} = 2000\text{£}.$

$\therefore$ as $\frac{8}{21} : 2000\frac{0}{1} :: \frac{1}{3} : 420\frac{0}{4} = 1750\text{£}.$ the Answer.

- (20) First Cock runs off 3 Galls. $= \frac{3}{103}$ of the Cistern in a Minute; and $1\frac{1}{4}$ min. $= \frac{5}{4}.$

Then, as $\frac{5}{4} : \frac{3}{103} :: 1 : \frac{12}{515}$ runs off by the second in a Minute.

And $\frac{3}{103} + \frac{12}{515} = \frac{15}{515} + \frac{12}{515} = \frac{27}{515}$ runs off by both in a Minute.

$\therefore$ as $\frac{27}{515} : 1 :: 1 : 5\frac{15}{27} = 19$ min. $4\frac{4}{9}$ sec. the Answer.

- (21) First $\frac{1}{4} + \frac{1}{3} + \frac{1}{6} = \frac{18}{72} + \frac{24}{72} + \frac{12}{72} = \frac{54}{72} = \frac{3}{4}.$

Then as $\frac{3}{4} : 45\frac{5}{1} :: \frac{36}{36} : 162\frac{0}{27} = 60$ Crowns, the Answer.

- (22) Here for one twelfth, read 50.

First $\frac{1}{2} = \frac{6}{12}$ Apples; $\frac{1}{4} = \frac{3}{12}$ Pears; $\frac{1}{6} = \frac{2}{12}$ Plums.

Then $\frac{6}{12} + \frac{3}{12} + \frac{2}{12} = \frac{11}{12}$; also $\frac{12}{12} - \frac{11}{12} = \frac{1}{12} = 50$ Cherries.

$\therefore 50 \times 6 = 300$ Apples.

$50 \times 3 = 150$ Pears.

$50 \times 2 = 100$ Plums.

50 Cherries.

Answer in all 600 Trees.

- (23) First $1 = \frac{4}{4}$, and $\frac{1}{2} = \frac{2}{4}.$

Then $\frac{4}{4} + \frac{4}{4} + \frac{2}{4} + \frac{1}{4} = \frac{11}{4} = 99$ by the Question.

$\therefore$ As $\frac{11}{4} : 99 :: 1 : 36\frac{6}{11} = 36$ Scholars.

- (24) First $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{15}{30} + \frac{10}{30} + \frac{6}{30} = \frac{31}{30}.$

Then $\frac{31}{30} + \frac{20}{30} = \frac{51}{30}$; and $74 - \frac{4}{5} = 73\frac{1}{5} = 36\frac{6}{5} = 21\frac{96}{30}.$

$\therefore \frac{51}{30} : 21\frac{96}{30} :: 1 : 21\frac{96}{61} = 36$ Years, the Answer.

- (25) First $\frac{3}{8}$ of $\frac{2}{3} = \frac{1}{4}$ B's. } First Acquisition, their Sum
And $\frac{3}{10}$ of $\frac{2}{3} = \frac{1}{5}$ C's. } $= \frac{9}{20}.$

Then $\frac{2}{3} - \frac{9}{20} = \frac{13}{60} = \frac{91}{420}$ left.

$\therefore \frac{1}{8}$ of $\frac{13}{60} = \frac{13}{480} =$ E's first Acquisition.

Also $\frac{91}{420} - \frac{13}{420} = \frac{78}{420} =$ D's, thus ended the first Heat.

Again

Addition of Decimals.

211

Again $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$ B's
 Retained $\frac{1}{5}$ C's
 And $\frac{13}{70} + \frac{1}{10} = \frac{139}{560}$ D's
 Also $\frac{13}{420} + \frac{1}{16} = \frac{157}{1680}$ E's

Part at the End of the second Scuffle.

Proceeding $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{20}$ A's.

$\frac{1}{3}$ of $\frac{1}{5} + \frac{1}{8} = \frac{233}{1200}$ B's.

$\frac{2}{7}$ of $\frac{1}{5} + \frac{139}{560} = \frac{171}{560}$ D's.

Then $\frac{1}{20} + \frac{1}{15} + \frac{2}{35} = \frac{73}{420}$.

$\frac{1}{5} - \frac{73}{420} = \frac{11}{420}$, and $\frac{1}{2}$ of $\frac{11}{420} = \frac{11}{840}$ C's
 $\frac{157}{1680} + \frac{11}{840} = \frac{179}{1680}$ E's

Part of the third Smuss.

Further $\frac{1}{20} + \frac{1}{15} = \frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60} = \frac{7}{80}$, lost by A and B.

Then $\frac{5}{10}$ of $\frac{7}{80} + \frac{1}{4}$ of $\frac{1}{20} = \frac{51}{1280}$ A's.

Also $\frac{5}{10}$ of $\frac{7}{80} + \frac{1}{4}$ of $\frac{1}{25}$ of $\frac{1}{8} = \frac{649}{3840}$ B's
 $\frac{1}{8}$ of $\frac{7}{80} + \frac{11}{840} = \frac{323}{13440}$ E's

Part after the last Smuss.

$\frac{1}{8}$ of $\frac{7}{80} + \frac{171}{840} = \frac{1417}{4480}$ D's
 $\frac{1}{8}$ of $\frac{7}{80} + \frac{179}{1680} = \frac{1579}{13440}$ E's

Part after the last Smuss.

Then $\frac{51}{1280} + \frac{1}{15} = \frac{2863}{26880}$ A's

And $\frac{649}{3840} + \frac{1}{15} = \frac{6335}{26880}$ B's

Also $\frac{323}{13440} + \frac{1}{15} = \frac{2438}{26880}$ C's

$\frac{1417}{4480} + \frac{1}{15} = \frac{10294}{26880}$ D's

$\frac{1579}{13440} + \frac{1}{15} = \frac{4950}{26880}$ E's

Share carried off at the last.

So that if the Number of Sugar-Plumbs were 26880.

| | | |
|-------|-------|---------------------|
| A got | 2863 | } = 26880, the Sum. |
| B | 6335 | |
| C | 2438 | |
| D | 10294 | |
| E | 4950 | |

47. ADDITION OF DECIMALS.

(1) $\begin{array}{r} .0476 \\ 21,476 \\ .0067 \\ .64 \\ 17,6 \\ .20764 \\ \hline \text{Sum } 39,97794 \end{array}$

(2) $\begin{array}{r} .427 \\ 64,075 \\ 27,6421 \\ 10,8 \\ .0074 \\ 104,046842 \\ \hline 206,998342 \end{array}$

(3) $\begin{array}{r} .274 \\ .076 \\ .64762 \\ .0706 \\ .47 \\ .007 \\ 968,42 \\ \hline 969,96522 \end{array}$

To

1. To add Decimals, wherein there are single Repetends.

R U L E.

Make every Line end at the same Place, filling up the Vacancies by the repeating Digits, and annexing a Cypher or Cyphers to the finite Terms; then add as before; only increase the Sum of the Right-hand Row, with as many Units as it contains Nines, and the Figure in the Sum under that place will be a Repetend.

E X A M P L E S.

| | | | | | |
|-----|---------|-----|------------|-----|-----------|
| (1) | 47,674 | (2) | 11,4 | (3) | 14,276421 |
| | 4,0264 | | 6,14274 | | 7,4 |
| | 32,6 | | 91,8 | | 21,646 |
| | 6,14 | | 37,671 | | 9,27 |
| | 27,0646 | | 146,476741 | | 31,1474 |
| | <hr/> | | <hr/> | | <hr/> |

In each of the above Examples there are single recurring Figures, which before they are added must be made to end together, and then they will stand as follows.

| | | | | | |
|-----|-----------|-----|------------|-----|-----------|
| (1) | 47,67444 | (2) | 11,444444 | (3) | 14,276421 |
| | 4,02642 | | 6,142740 | | 7,444444 |
| | 32,66666 | | 91,788888 | | 21,646666 |
| | 6,14444 | | 37,671111 | | 9,277777 |
| | 27,06466 | | 146,476741 | | 31,147444 |
| | <hr/> | | <hr/> | | <hr/> |
| | 117,57664 | | 293,522925 | | 83,792754 |
| | <hr/> | | <hr/> | | <hr/> |

2. To add Decimals having Compound Repetends.

R U L E.

Make the Repetends similar and conterminous; then add as before, only increase the Right-hand Figure by as many Units as are carried from the Column of Figures, wherein all the Repetends begin together: lastly, dash off for a Repetend as many Places as were so in the Numbers added together.

E X A M P L E S.

| | | | |
|-----|----------|-----|------------|
| (1) | 14,1472 | (2) | ,246 |
| | ,7684246 | | 3,67 |
| | 7,664 | | 27,0427694 |
| | 26,0662 | | 9,9431 |
| | <hr/> | | <hr/> |

Now

Now the two last Examples being made similar and conterminous, will become as follows,

| | | | |
|-----|--|-----|--|
| (1) | $ \begin{array}{r} 14,1472472 \\ ,7684246 \\ 7,0640640 \\ 26,0062062 \\ \hline 47,9859428 \end{array} $ | (2) | $ \begin{array}{r} ,2462462 \\ 3,6767676 \\ 27,0427694 \\ 9,9431431 \\ \hline 40,909264 \end{array} $ |
|-----|--|-----|--|

18. SUBTRACTION OF DECIMALS.

| | | | | | |
|-----|---|-----|-----------------------------------|-----|---------------------------------|
| (1) | From 176,
Take 10,764
<hr/> Diff. 165,236 | (2) | 647,
,00746
<hr/> 646,99254 | (3) | 74,6407
69,5
<hr/> 5,1407 |
|-----|---|-----|-----------------------------------|-----|---------------------------------|

To subtract Decimals that have a single Repetend.

R U L E.

Make both end together as in Addition: and if the Repetend of the Number to be subtracted, be greater than the Repetend of the Number it is to be taken from, then the Right-hand Figure of the Remainder must be less by Unity, than it would be; or instead of borrowing 10, as in whole Numbers or Infinites, borrow in this Place 9, the rest as usual; and the Right-hand place or Figure will be a Repetend.

E X A M P L E S.

| | | | | | |
|-----|--------------------------------------|-----|--------------------------|-----|------------------------------|
| (1) | From 41,74
Take 21,94648
<hr/> | (2) | 24,1466
19,9
<hr/> | (3) | 16,126
4,1942764
<hr/> |
|-----|--------------------------------------|-----|--------------------------|-----|------------------------------|

These Examples being made to end together as before directed in Addition, will stand as follows,

| | | | | | |
|-----|--|-----|---------------------------------------|-----|--|
| (1) | From 41,74444
Take 21,94648
<hr/> Diff. 19,79795 | (2) | 24,14466
19,99999
<hr/> 4,14466 | (3) | 16,1262626
4,1942764
<hr/> 1,9389862 |
|-----|--|-----|---------------------------------------|-----|--|

49. MULTIPLICATION OF DECIMALS.

| | | | | | |
|-----|---|-----|---|-----|--|
| (1) | $ \begin{array}{r} ,17506 \\ \times ,76 \\ \hline \text{Prod. } ,1331304 \end{array} $ | (2) | $ \begin{array}{r} 8,04704 \\ \times ,2575 \\ \hline 1,1869384 \end{array} $ | (3) | $ \begin{array}{r} 27,42 \\ \times 3,56 \\ \hline 97,6152 \end{array} $ |
|-----|---|-----|---|-----|--|

| | | | | | | | |
|-------|--|-----|--|-----|--|-----|---|
| (4) | $\begin{array}{r} 5745 \\ \times 0675 \\ \hline \end{array}$ | (5) | $\begin{array}{r} .4 \\ \times .2 \\ \hline \end{array}$ | (6) | $\begin{array}{r} .047 \\ \times .046 \\ \hline \end{array}$ | (7) | $\begin{array}{r} .000476 \\ \times .00078 \\ \hline \end{array}$ |
| Prod. | $\begin{array}{r} .03877875 \\ \hline \end{array}$ | | $\begin{array}{r} .08 \\ \hline \end{array}$ | | $\begin{array}{r} .002042 \\ \hline \end{array}$ | | $\begin{array}{r} .00000037122 \\ \hline \end{array}$ |

(8) $.47 \times$ by $.0008 = .000376$, the Product.

CONTRACTIONS.

(9) $2.74 \times$ by $10 = 27.4$. (10) $.2746 \times 100 = 27.46$.
 (11) $1.076 \times 1000 = 1076$. (12) $.42768 \times 10000 = 4276.8$.

(14) 3.47678
 $\times 287672$ Mul. inverted.

$$\begin{array}{r} 69536 \\ 24338 \\ 2086 \\ 243 \\ 28 \\ 1 \\ \hline 96,232 \\ \hline \end{array}$$

(15) 47.689464
 $\times 49671.62$

$$\begin{array}{r} 95378928 \\ 28613678 \\ 476895 \\ 333826 \\ 28613 \\ 4292 \\ 191 \\ \hline 1248,36423 \\ \hline \end{array}$$

(1) If the Right-hand Figure of the Multiplicand be a Circulate,

RULE.

Multiply the Multiplicand as before, by every Figure in the Multiplier; observing to increase the Right-hand of each resulting Line, by as many Units as there are Nines in the Product of the first Figure in that Line, and the Right-hand Figure of each Line will be a Circulate; and before you add them together, make them all end at the same Place as shewn in Addition.

EXAMPLES.

(1)
$$\begin{array}{r} 147,64 \\ \times 7 \\ \hline \end{array}$$

Prod. $103,352$

(2)
$$\begin{array}{r} 42,644 \\ \times 276 \\ \hline \end{array}$$

$$\begin{array}{r} 255853 \\ 2986955 \\ 8528444 \\ \hline 117,71253 \\ \hline \end{array}$$

- (2) When the Right-hand Figure of a Multiplier be a Circulate.

R U L E.

Multiply by, as by a finite Digit, setting the Product one Place extraordinary towards the Left-hand, then divide the Product by 9, continuing the Quotient if needful till it arrives at a Circulate; then beginning at the Place under the Right-hand Figure of the Multiplicand, cut off for Decimal Parts.

E X A M P L E S.

$$\begin{array}{r} (3) \quad 46,276\dot{2} \\ \quad \quad 8 \\ \hline 9)370,2096 \\ \hline \text{Prod. } 41,1344 \end{array}$$

$$\begin{array}{r} (4) \quad 261,27\dot{6} \\ \quad \quad ,4\dot{7} \\ \hline 9)18,28932 \\ \hline 2,03214\dot{6} \\ 104,5104 \\ \hline \text{Prod. } 106,54254\dot{6} \end{array}$$

- (3) When the Multiplicand and Multiplier are each a single Circulate.

R U L E.

The first Line (or that produced by multiplying the Circulate in the Multiplier) must be managed as in Note 2. only the Right-hand Figure must be increased by as many Units as there are Nines in the Product of the first Figure of that Line; the Products of the rest must be managed as directed in Note 1.

$$\begin{array}{r} (6) \quad 141,1\dot{4} \\ \quad \quad 8,4\dot{7} \\ \hline 9)98801 \\ \hline 109667 \\ 564577 \\ 11291555 \\ \hline 1196,5800\dot{7} \end{array}$$

$$\begin{array}{r} (5) \quad 24,607\dot{2} \\ \quad \quad ,49\dot{6} \\ \hline 9)1476432 \\ \hline 164048 \\ 22146500 \\ 98428888 \\ \hline 12,183943\dot{7} \end{array}$$

50. DIVISION OF DECIMALS.

*Divis. Divid. Quot.**Divis. Divid. Quot.*

(1) $6,5 \overline{) 1735,5} (267$

(2) $,0084 \overline{) 8332} (99,19047+$

(3) $,0574 \overline{) 49,3066} (859$

(4) $,47 \overline{) 17,46} (37,14893+$

(5) $7,476 \overline{) 186,900} (25$

(6) $,7875 \overline{) 14,4100} (18,2984+$

(7) $,04 \overline{) 104,00} (2600$

(8) $,008 \overline{) 6,000} (750$

(9) $34 \overline{) 1229,42112} (36,15944$

(10) $4,7 \overline{) 754,4578} (160,5293$

(11) $604,25 \overline{) 246,1476} (,40736+$

(12) $119 \overline{) 7,268401} (,061079$

(13) $647 \overline{) 6,7258} (,010395+$

(14) $678 \overline{) 0,008136} (,0000012$

(15) $12 \overline{) 0,0072} (,006$

(16) $2,46 \overline{) 0,016728} (,0068$

CONTRACTIONS.

(17) $1,0 \overline{) 24,6} (2,46$

(18) $1,00 \overline{) 4076} (,00476$

(19) $1000 \overline{) 487,67} (,48767$

(20) $1,000 \overline{) 474,6} (,4746$

(22) $2137,2 \overline{) 913,08} (,426$

85488

(23) $240,649 \overline{) 6109,2674} (25,3866$

481298

5820

129628

4274

120325

1436

9303

1282

7220

154

2083

1925

(24) $52,7438 \overline{) 165,9923} (3,14$

1582314

77609

52744

24865

21097

3768

158

144

14

14

(1) If the Dividend be a Repetend.

R U L E.

If it be a single Repetend, bring down the circulating Figure, until the Quotient either repeats, or is as exact as required; but if the Repetend in the Dividend be a compound one, then bring down the circulating Figures in the same Order they stand in; and when you have got through them all, bring down the first Figure in the Repetend over again, and so proceed until your Quotient either repeats, or be as exact as necessary.

E X A M P L E S.

$$6,84)14,41(2,11176+$$

$$\begin{array}{r} \underline{764} \\ 804 \\ \underline{1204} \\ 5204 \\ \underline{4164} \\ 60 \\ \underline{} \end{array}$$

$$46,849)1694847(36,170412$$

$$\begin{array}{r} \underline{289177} \\ 80836 \\ \underline{339874} \\ 193176 \\ \underline{57804} \\ 109557 \\ \underline{15859} \end{array}$$

(2) If the Divisor be a single Repetend.

R U L E.

Place the Dividend under itself, but removed one Place towards the Right hand, which subtract, and the Remainder will be a new Dividend; which divide by the Divisor in the same Manner as if it was a terminate Number.

Divide 42,86 by 8.

$$\begin{array}{r} 428 \\ \underline{} \\ ,8)3859 \\ \underline{} \\ ,48225 \end{array}$$

Divide 6,426 by 68.

$$\begin{array}{r} 642 \\ \underline{} \\ ,68)5,784(8,505832+ \end{array}$$

- (3) If the Divisor and Dividend consist of terminate Numbers joined to the Repetend.

R U L E.

Place the Divisor and Dividend under each other, but removed one Place towards the Right; then subtract the lower Line from the upper one, the Remainder will be a new Divisor and Dividend, which proceed with as before directed.

E X A M P L E S.

$$\begin{array}{r} (1) \quad 7414,864 \overline{) 81869,8694} \\ \underline{741486} \quad \underline{8186,9869} \\ 763,378 \quad 73682,8825(9.6024+ \end{array}$$

$$\begin{array}{r} (2) \quad 64,842 \overline{) 9,46946} \\ \underline{6,484} \quad \underline{94694} \\ 58,358 \quad 8,52252(14603+ \end{array}$$

- (4) If a compound Repetend is found in your Divisor, or in both your Divisor and Dividend,

R U L E.

Proceed as in the last Case with your Divisor and Dividend, only remove them each so many Places towards the Right-hand, as they are Places in the Repetend of the Divisor; but if the Divisor is a compound Repetend without any terminate Figures, divide by it as a terminate Number; first subtracting the Dividend from itself, as above directed.

E X A M P L E S.

$$\begin{array}{r} (1) \quad \text{Divide } 147,42683 \text{ by } 8,467 \\ \quad \quad 8,467 \overline{) 147,42683268} \\ \quad \quad \quad 8 \quad \quad 14742683 \\ \quad \quad \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \\ \quad \quad 8,459 \overline{) 147,27940585} (17,41085+ \end{array}$$

Divide

(3) Divide 4176,4266268 by 37,2694.

$$\begin{array}{r} 37,2694 \overline{) 4176,4266268} \\ \underline{37} \\ 4176426 \end{array}$$

$$37,2657 \overline{) 4176,0089842} (112,06393+$$

(4) Divide 47,69642 by 476.

$$\begin{array}{r} 4749 \\ \hline \end{array}$$

$$476 \overline{) 47,64873} (100,1+$$

51. REDUCTION OF DECIMALS.

CASE I.

- (2) 2) 1,0',5 (3) 4) 3,0(,75 8) 1,0(,125, and 16) 100(,625
 (4) 24) 5,0(,2083 (5) First $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; then 16) 5,000(,3125
 (6) 7) 4,0(,57142+ (7) 3) 2,0(,6
 (8) First $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{7}{8} = \frac{21}{64}$; then 64) 21,0(,328125
 (9) First $\frac{1}{4}$ of $\frac{5}{12} = \frac{5}{48}$; then 48) 5,00000(,10416.

CASE II.

First 6s. 9d. = $\frac{27}{8}$ £, per Case VIII. Sect. 38.

- (2) Thus 6s. 9d. = 27 Three-pences, and 1 £. = 80.
 Then 80) 27,0(,3375, the Decimal required.
 (3) Thus 21) 9,00(,42857+, the Decimal required.
 (4) First 14s. 6½d. = 349 Half-pence, and a Moidore = 648 ditto.
 Then 648) 349,0(,53858+, the Decimal required.
 (5) First 18s. 4½d. = 441 Half-pence, and 1 £. = 480 ditto.
 Then 480) 441,0(,91875, the Decimal required.
 (6) First $\frac{3}{4}$ of a Penny = $\frac{1}{320}$ £. per Case VI. Sect. 38.
 Then 320) 1,000(,003125, the Decimal required.
 (7) First 1lb. = 240 dwts. 240) 11,00(,04583, the Dec. req.
 (8) First 1lb. = 256 drs. then 256) 10,00(,03906+, the Dec.
 (9) First $\frac{3}{4}$ qrs. 14lb. = 98lb. then 112) 98,0(,8741, the Dec.
 (10) A Yd. = 36 inches, then 36) 6,0(,16, the Decimal req.
 (11) First a League = 24 fur. then 24) 6,0(,25, the Dec.
 (12) First 18 gall. 2 qts. = 74 qts. and a hhd. = 252 ditto.
 Then 252) 74,0(,29365+, the Decimal required.

- (13) First 3 qts. 1 pt. = 7 pts, and a bar. = 256 pts.
Then $256)7,00(.02734+$, the Decimal required.
- (14) First an Acre = 160 Perches, then $160)8,00(.05$, the Decimal required.
- (15) First 4 bush. 2 pks. = 18 pks. and a chald. = 144 pks.
Then $144)18,0(.125$, the Decimal required.
- (16) Thus $60)12,0(.2$ the Decimal required.
- (17) First 12 Days = 288 hrs. and 365 d. 6 h = 8766 hrs.
Then $8766)288,00(.03285+$, the Decimal required.

C A S E III.

| <i>£.</i> | <i>Guinea.</i> | <i>Moidore.</i> |
|---|---|---|
| (2) $\begin{array}{r} ,3375 \\ \times 20 \\ \hline s. 6,7500 \\ \times 12 \\ \hline d. 9,00 \end{array}$ | (3) $\begin{array}{r} ,45 \\ \times 7 \times 3 = 21 \\ \hline 3,15 \\ 3 \\ \hline s. 9,45 \\ 12 \\ \hline d. 5,40 \\ 4 \end{array}$ | (4) $\begin{array}{r} ,72708 \\ \times 9 \times 3 = 27 \\ \hline 6,54372 \\ \times 3 \\ \hline s. 19,63116 \\ 12 \\ \hline d. 7,57392 \\ 4 \end{array}$ |
| Answer 6 s. 9 d. | Ans. 9 s. 5 d. $\frac{1}{2}$, 6. | Ans. 19 s. 7 d. $\frac{1}{2}$, 29 + |
| <i>Lb.</i> | <i>Ton.</i> | <i>Lb.</i> |
| (5) $\begin{array}{r} ,00243 \\ 12 \\ \hline ,02916 \\ 20 \\ \hline ,58320 \\ 4 \times 6 = 24 \\ \hline 2,3328 \\ 6 \\ \hline 13,99680 \end{array}$ | (6) $\begin{array}{r} ,3375 \\ 20 \\ \hline cwt. 6,7500 \\ 4 \\ \hline qrs. 3,00 \end{array}$ | (7) $\begin{array}{r} ,0306 \\ 16 \\ \hline 6336 \\ 16 \\ \hline drs. 10,1376 \end{array}$ |
| Answer 14 Grs. | Ans. 6 Cwt. 3 Qrs. | Ans. 10 drs. 1376. |

| <i>Cwt.</i> | <i>Yard.</i> | <i>League.</i> |
|---|--|---|
| (8) $\begin{array}{r} ,875 \\ 4 \\ \hline \end{array}$ | (9) $\begin{array}{r} ,16669 \\ 3 \\ \hline \end{array}$ | (10) $\begin{array}{r} ,259 \\ 3 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{Qrs. } 3,500 \\ 28 \\ \hline \end{array}$ | $\begin{array}{r} ,50007 \\ \times 12 \\ \hline \end{array}$ | $\begin{array}{r} ,777 \\ 8 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{lb. } 14,0 \\ \hline \end{array}$ | $\begin{array}{r} 6,00084 \\ \hline \end{array}$ | $\begin{array}{r} \text{Fur. } 6,216 \\ 40 \\ \hline \end{array}$ |
| Answer 3 Qrs. 14 lb. | Answ. 6 Inches + | $\begin{array}{r} 8,640 \\ \hline \end{array}$ |
| | | Answer 6 Fur. 8,64 Poles. |

| <i>Hbd.</i> | <i>Bar.</i> | <i>Acre.</i> |
|---|---|--|
| (11) $\begin{array}{r} ,29365 \\ \times 63 \\ \hline \end{array}$ | (12) $\begin{array}{r} ,875 \\ \times 32 \\ \hline \end{array}$ | (13) $\begin{array}{r} ,05 \\ 4 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{Galls. } 18,5 \text{ nearly.} \\ 4 \\ \hline \end{array}$ | $\begin{array}{r} 28,000 \\ \hline \end{array}$ | $\begin{array}{r} ,20 \\ 40 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{Qts. } 2,0 \\ \hline \end{array}$ | Answer 28 Galls. | $\begin{array}{r} 8,00 \\ \hline \end{array}$ |
| Answer 18 Galls. 2 Qts. | | Answer 6 Fur. 8,64 Poles. |

| <i>Chaldron.</i> | <i>Day.</i> |
|---|---|
| (14) $\begin{array}{r} ,125 \\ \times 136 \\ \hline \end{array}$ | (15) $\begin{array}{r} ,4765 \\ \times 24 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{Bush. } 4,500 \\ 4 \\ \hline \end{array}$ | $\begin{array}{r} \text{Hrs. } 11,4360 \\ 60 \\ \hline \end{array}$ |
| $\begin{array}{r} \text{Pks. } 2,0 \\ \hline \end{array}$ | $\begin{array}{r} \text{Min. } 26,1600 \\ 60 \\ \hline \end{array}$ |
| Answer 4 Bush. 2 Pks. | $\begin{array}{r} \text{Sec. } 9,60 \\ 60 \\ \hline \end{array}$ |
| | $\begin{array}{r} 36,00 \\ \hline \end{array}$ |
| | Answer 11 Hrs. 26 Min. 9 Sec. 36 M. |
| U 3 | E X. |

53. EXTRACTION OF THE SQUARE ROOT.

(2) 60516 (246 the Root required.)

4

44) 205

176 = 44 × 4

486) 2916

2916 = 486 × 6

Now $246 \times 2,46 = 60516$, the Proof.

(3) 765 (or 87,5099, or rather 87.34 + the Root

167) 1258

1745) 8900

175009) 1750000

1750189) 17491900

Rem. 1740199

(4) 39342864 (6272,389 + the Root

122) 334

1247) 9028

12542) 29964

125443) 488000

1254468) 11167100

12544769) 113135600

Rem. 232679

(5) 8209667940,5290(90607,21792 + the Root.

1806)10966

181207)1307940

1812142)3949152

18121441)32486890

181214427)1436544900

1812144349)16804391100

18121443582)49509195900

Rem. 13266308736

(6) ,000729(,027 the Root. (7) 2)1,41421 + the Rt.

4

47)329

1

24)100

281)400

(8) 2,27109570(1,50701 +

1

25)127

3007)21095

301401(467000

Rem. 165599

2824)11900

28282)60400

282841)383600

Rem. 100759

(9) 36,00000625(6,0000005 + the Root.

120000005)625000000

Rem. 2499997500

54. VULGAR FRACTIONS.

- (12) Thus $\sqrt{\frac{25}{81}} = \frac{5}{9}$, the Root required.
 (13) Thus $\sqrt{\frac{225}{144}} = \frac{15}{12}$, or $\frac{5}{4}$, the Root.
 (14) First $10\frac{39}{49} = 5\frac{29}{49}$. Then $\sqrt{5\frac{29}{49}} = 2\frac{3}{7}$, or $3\frac{2}{7}$, the Root.
 (15) First $\frac{4608}{6272} = \frac{36}{49}$. Then $\sqrt{\frac{36}{49}} = \frac{6}{7}$, the Root.
 (16) First $\frac{726}{864} = \frac{121}{144}$. Then $\sqrt{\frac{121}{144}} = \frac{11}{12}$, the Root.
 (17) First $27\frac{9}{16} = 44\frac{1}{16}$. Then $\sqrt{44\frac{1}{16}} = 2\frac{1}{4}$, or $5\frac{1}{4}$ the Root.
 (18) First $8\frac{3}{7} = 8,428571428$. Then $\sqrt{8,428571428} = 2,9032+$, the Root.
 (19) First $\frac{597}{676} = ,7352071005$. Then $\sqrt{,7352071005} = ,85744+$, the Root.
 (20) First $76\frac{14}{17} = 76,8235941176$. Then $\sqrt{76,8235941176} = 8,7649+$, the Root.

55. THE USE OF THE SQUARE ROOT.

- (2) First $124 \times 67 = 8308$. Then $\sqrt{8308} = 91,148225+$ the mean Proportion nearly.

For $67 : 91,148225 :: 91,148225 : 124$ nearly.

- (4) First 9 A. 2 R. 15 P. = 1535 Perches. And $5.5 \times 5.5 = 30.25$ Yards in 1 Perch.

$\therefore 1535 \times 30.25 = 46433.75$ Yds. the superficial Content.

Then $\sqrt{46433.75} = 215.484918$ Yds. + = 215 Yds. 1 Ft. 5,467 Inc. the Side required.

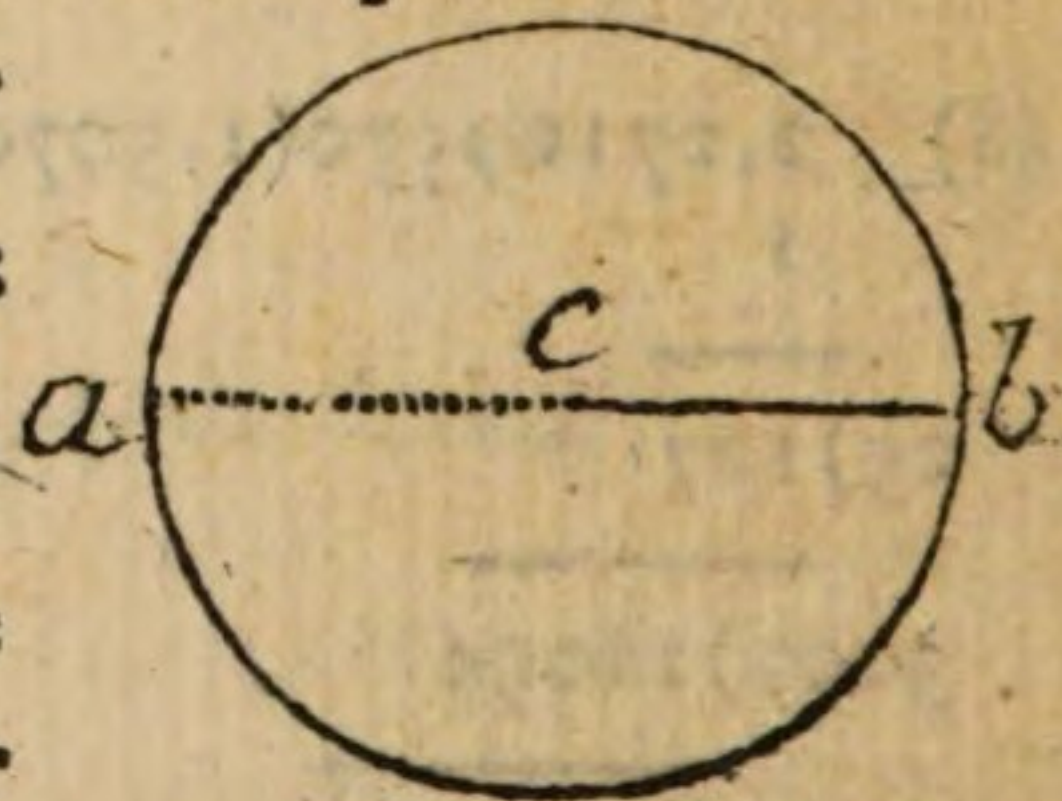
- (5) Thus $\sqrt{160} = 12.649+$, the Side required.

- (7) First $4840 \times 2 = 9680$ Yards, Content of the two Acres.

Then as $355 : 452 :: 9680 : 12324$ Yds. = $2ab$.

$\therefore \sqrt{12324} = 111 = ab$.

Therefore $111 \div 2 = 55\frac{1}{2}$ Yds. = cb , the Length of the Cord req.



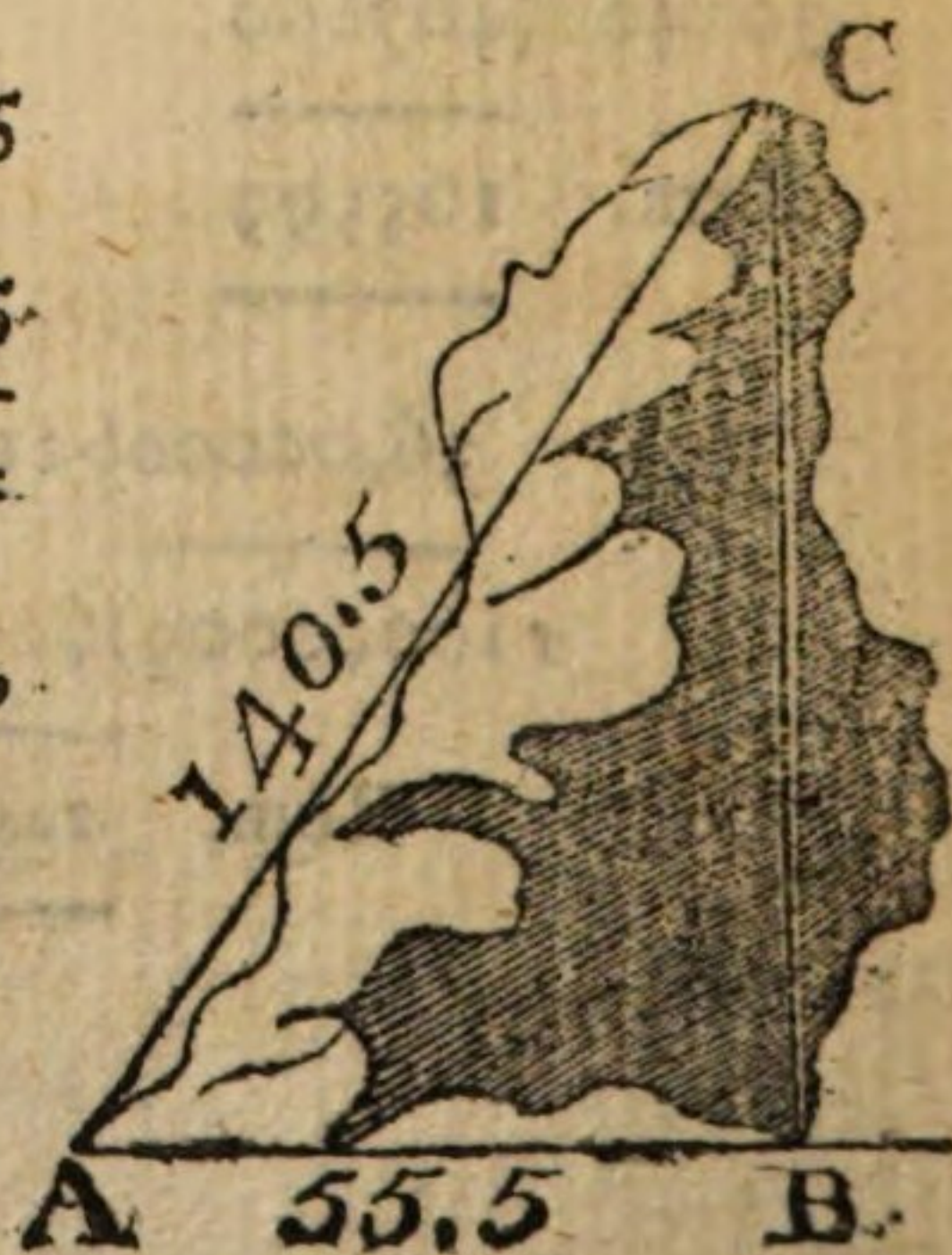
- (8) First $140,5 \times 140,5 = 19740.25$ = $2AC$.

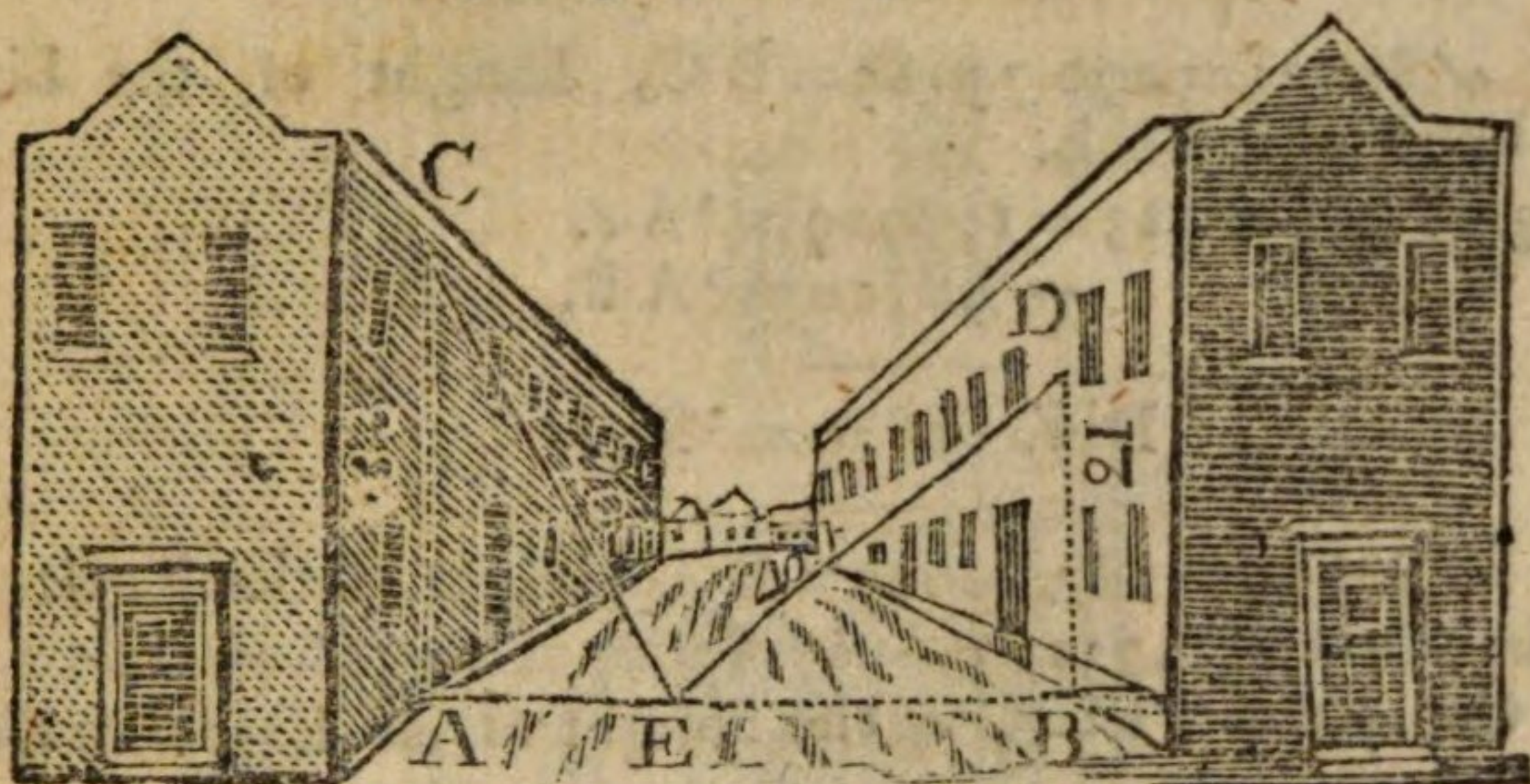
And $55,5 \times 55,5 = 3080.25$ = $2AB$.

$16660 =$

$2BC$.

$\therefore \sqrt{16660} = 129,07$ Yds. = BC , the Height required.





(9) First $40 \times 40 = 1600 = {}^2CE$, or 2DE .
And $33 \times 33 = 1089 = {}^2AC$.

$$\therefore \sqrt{511} = 22.6 = AE.$$

Again $21 \times 21 = 441 = {}^2BD$.

Then $1600 - 441 = 1159 = {}^2EB$.

$$\therefore \sqrt{1159} = 34.04 = EB.$$

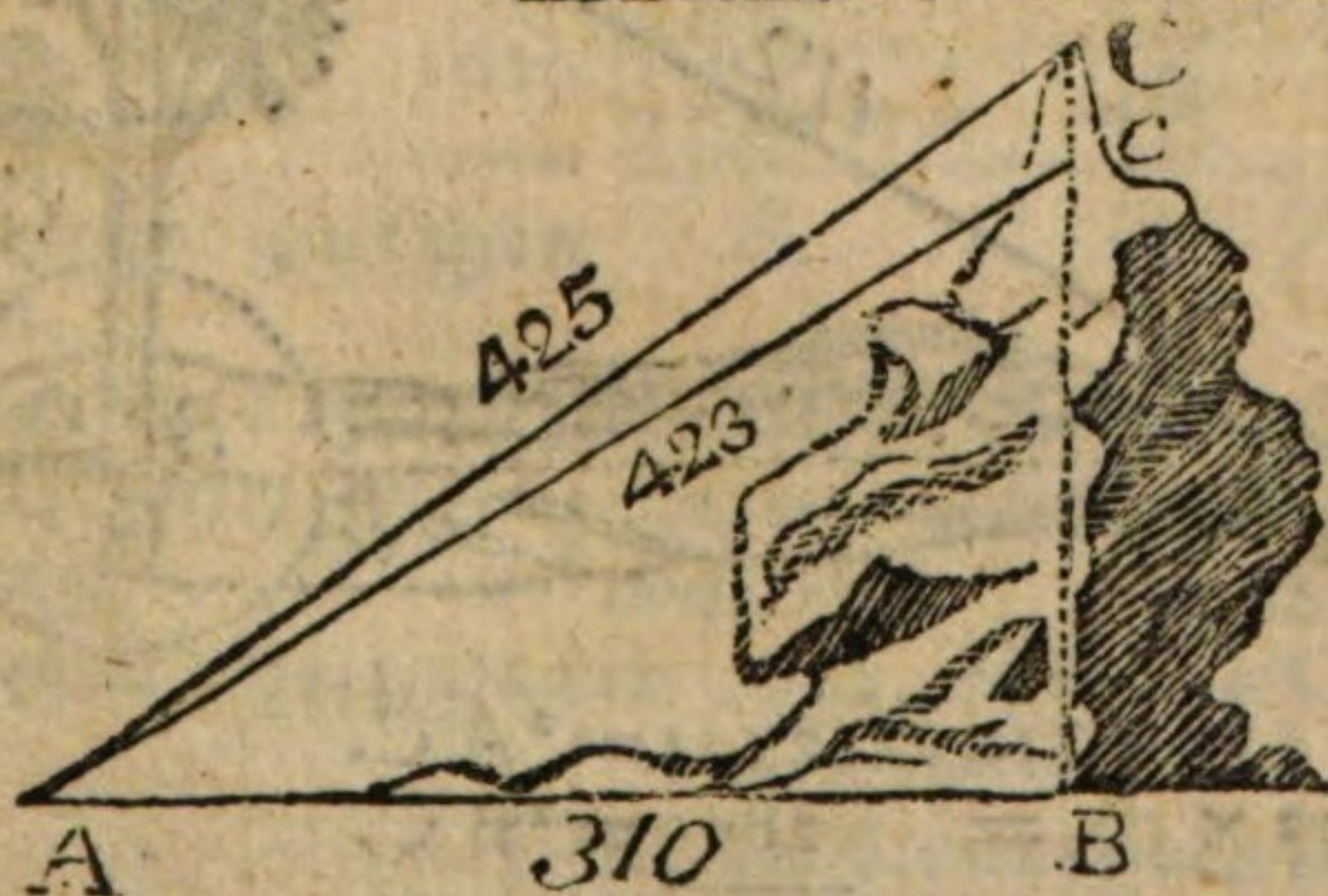
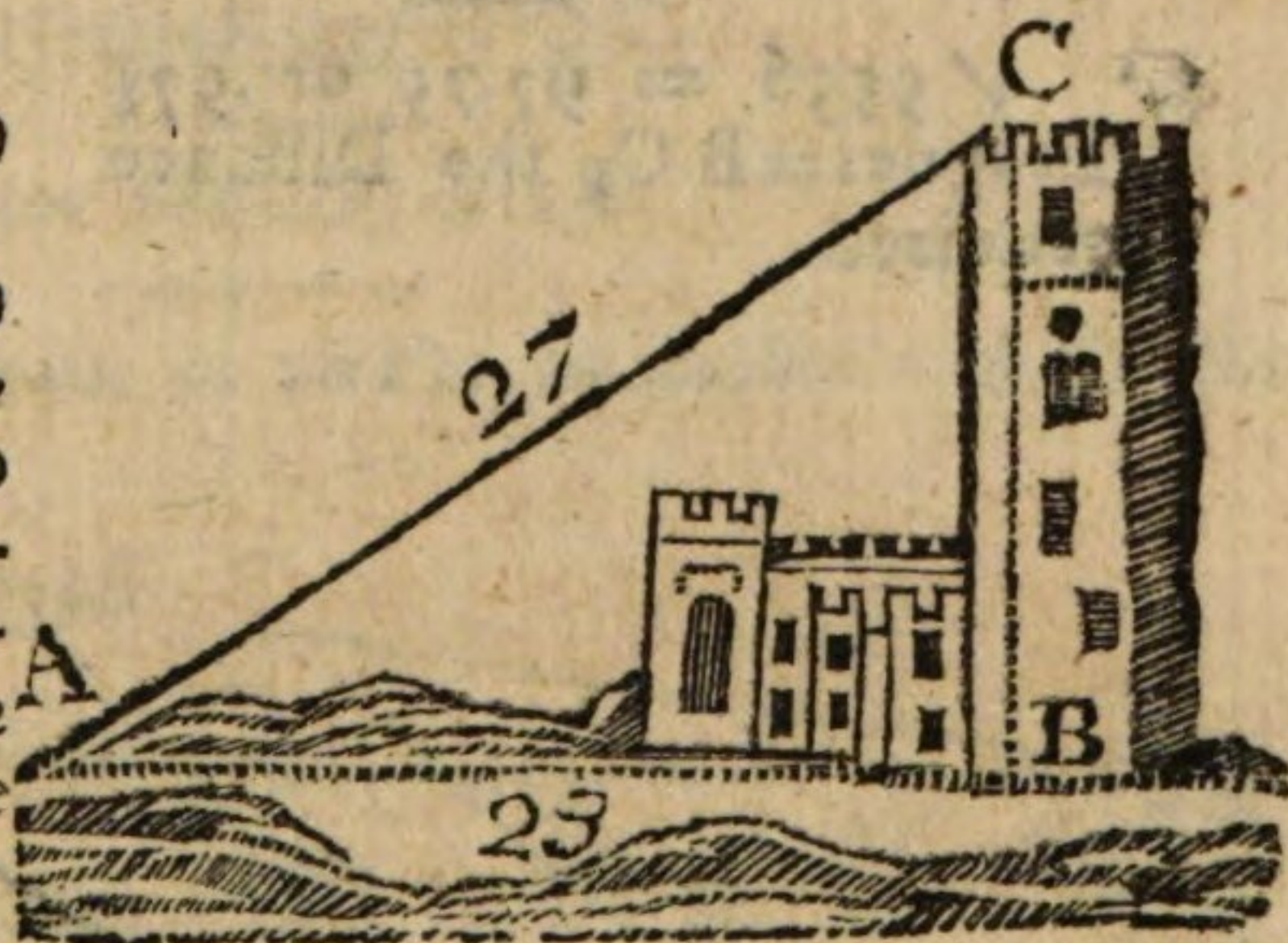
Therefore $22.6 + 34.04 = 56.64 + \text{Ft.} = AE + EB = AB$,
the Answer.

(10) First $27 \times 27 = 729$
 $= {}^2AC$.

And $23 \times 23 = 529$
 $= {}^2AB$.

Diff. 200
 $= {}^2BC$.

$\therefore \sqrt{200} = 14.142 +$
or 14 Yds. 5.112
Inches $= BC$, the
Height required.



(11) First $425 \times 425 = 180625 = {}^2AC$.

And $310 \times 310 = 96100 = {}^2AB$.

$$\text{Diff. } 84525 = {}^2BC.$$

$\therefore \sqrt{84525} = 290.73183 = BC$, Height of the Light-house and Rock.

$$\text{Again } 423 \times 423 = 178929 = {}^2A c.$$

$$\quad \quad \quad - 96100 = {}^2AB.$$

$$\text{Diff. } 82829 = {}^2B c.$$

$\therefore \sqrt{82829} = 287.80027 = B c$, Height of the Rock.

Therefore $290.73183 - 287.80027 = 2.9356$ Fathoms.

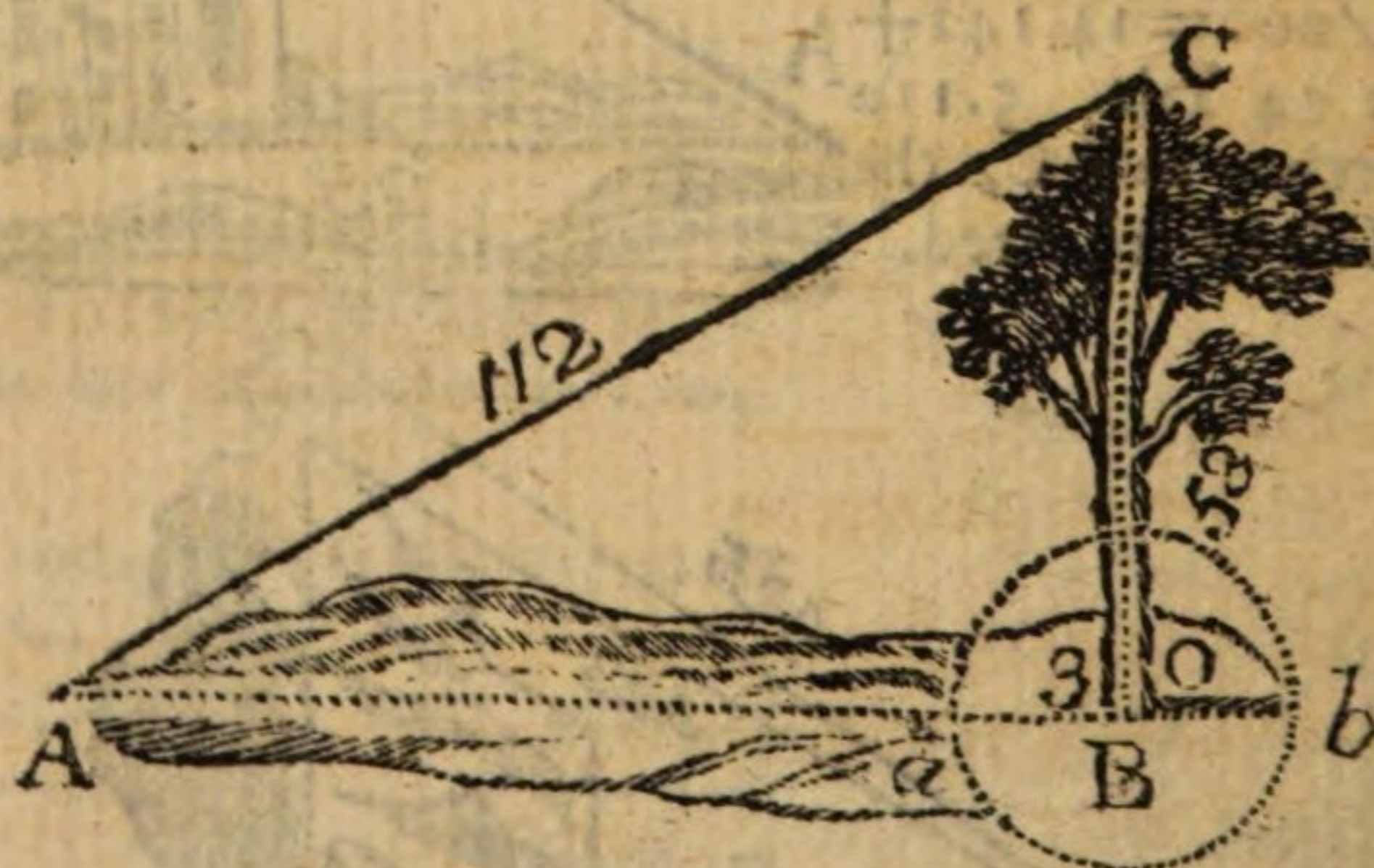
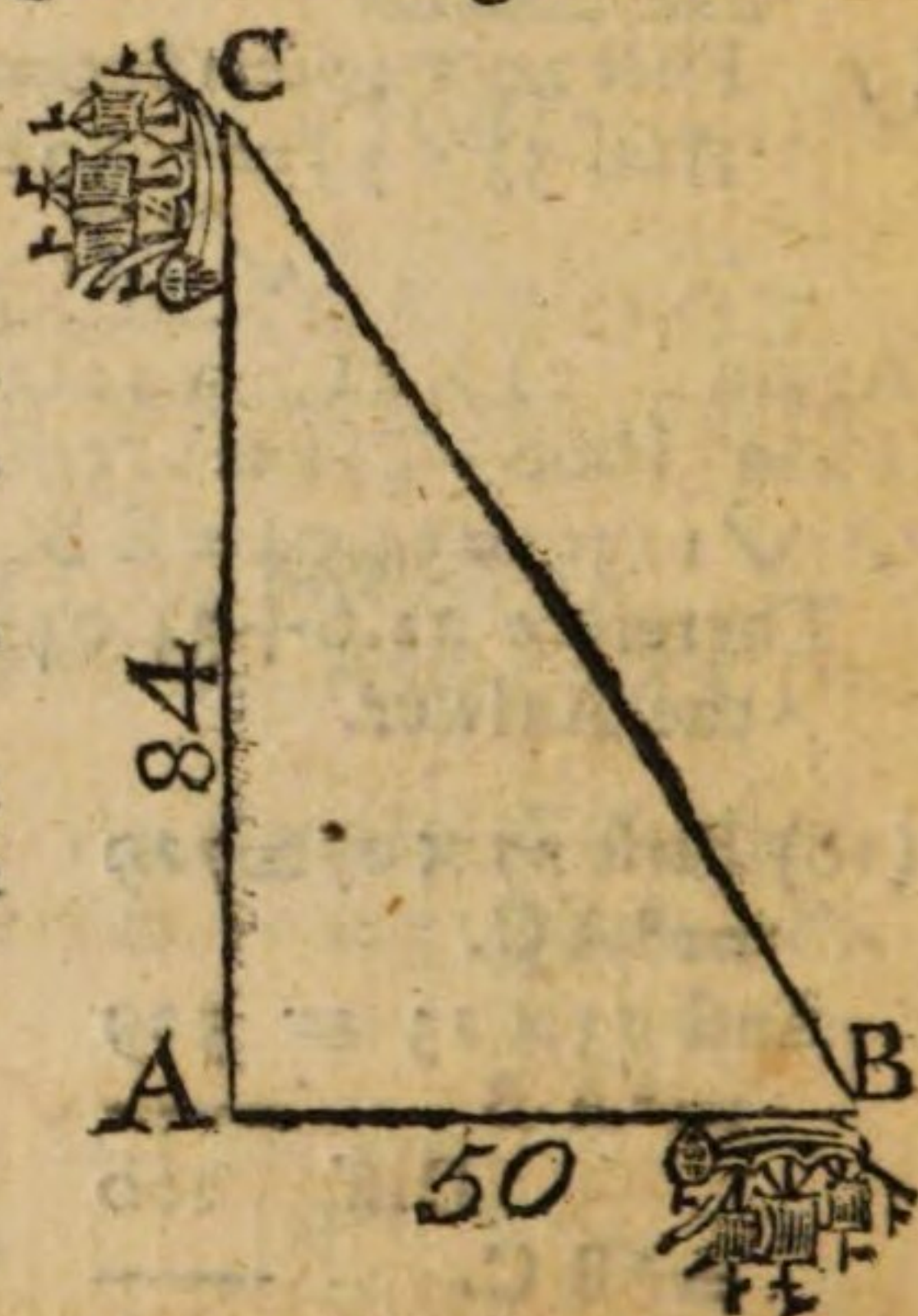
Or $17.6 + \text{Ft.} = C c$, the Height of the Light-house.

(12) First $50 \times 50 = 2500 = {}^2AB$.

And $84 \times 84 = 7056 = {}^2AC$.

Sum $9556 = {}^2BC$.

$\therefore \sqrt{9556} = 97.75$ or $.97\frac{3}{4}$ Leagues $= BC$, the Distance required.



(13) First $112 \times 112 = 12544 = {}^2AC$.

And $53 \times 53 = 2809 = {}^2BC$.

Diff. $9735 = {}^2AB$.

$\therefore \sqrt{9735} = 98.6 = AB$. Now $ab = 30$. Which $\div 2 = 15 = aB$. And $98.6 - 15 = 83\frac{2}{3} = Aa$. Brdth of the Mt. req.

$\therefore \sqrt{6466.02213} = 80.4121 = a E$, or $A F$.

Now $76 + 80.4121 = 156.4121 = A B$, or $c D$, Distance of the Columns.

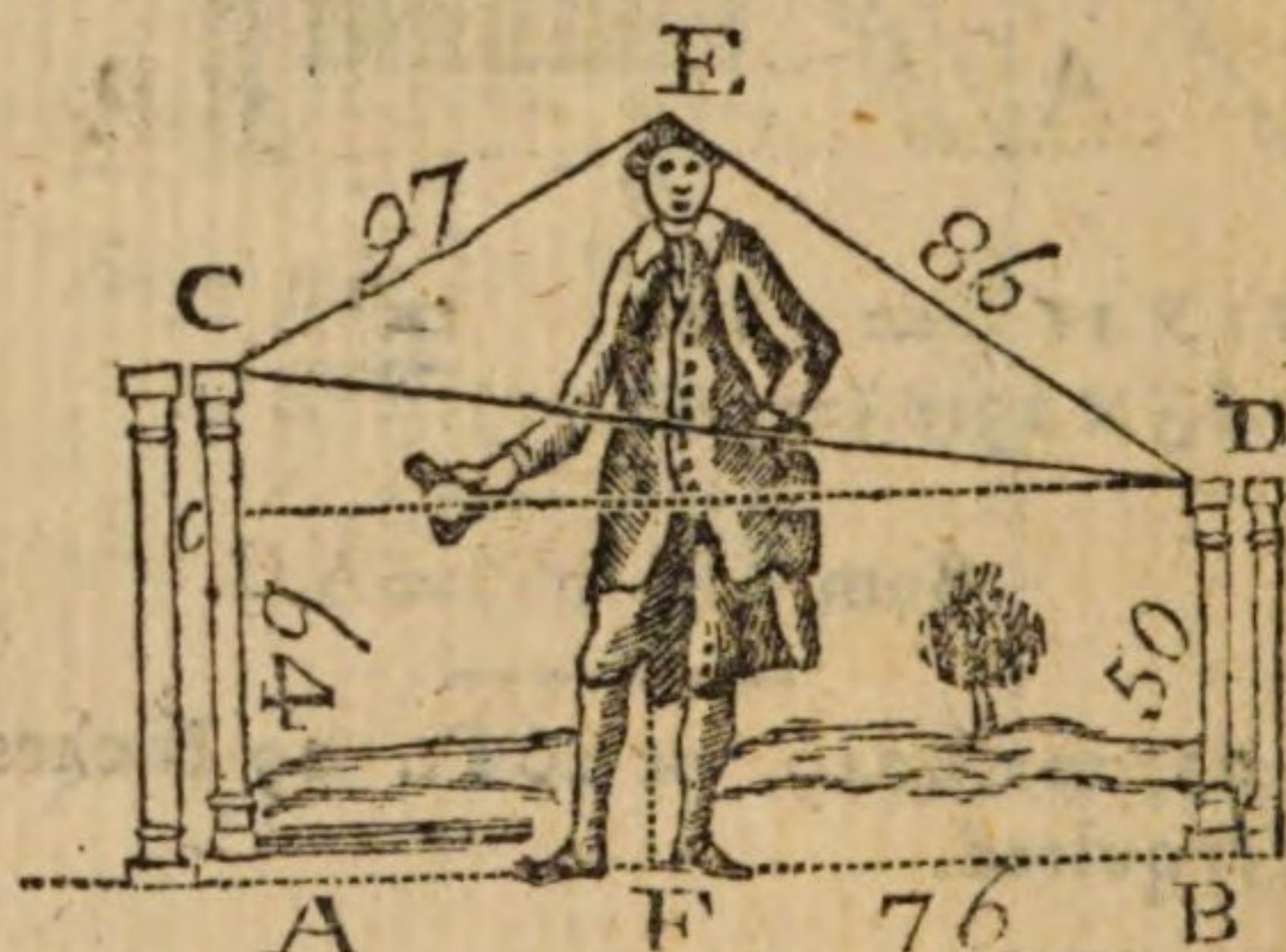
And $64 - 50 = 14 = c C$ Diff. of their Heights.

Therefore $156.4121 \times 156.4121 = 24464.745 = {}^2 C D$.
 $14 \times 14 = 196 = {}^2 C c$.

Sum $24660.745 = {}^2 C D$.

$\therefore \sqrt{24668.745} = 157.06 + = C D$, Distance of the Top of the Columns.

But if the Statue be higher than the Columns, then by working as before we shall find the Statue to be 40.24922 Feet higher than the lower Column.



Therefore $50 + 40.24922 = 90.24922 = E F$, Height of the Statue.

Also $90.24922 - 64 = 26.24922$ higher than highest Column,

Then $97 \times 97 = 9409 = {}^2 C E$.

And $26.24922 \times 26.24922 = 689.02135$

Diff. 8719.97845

$\therefore \sqrt{8719.97845} = 93.3808 A F$.

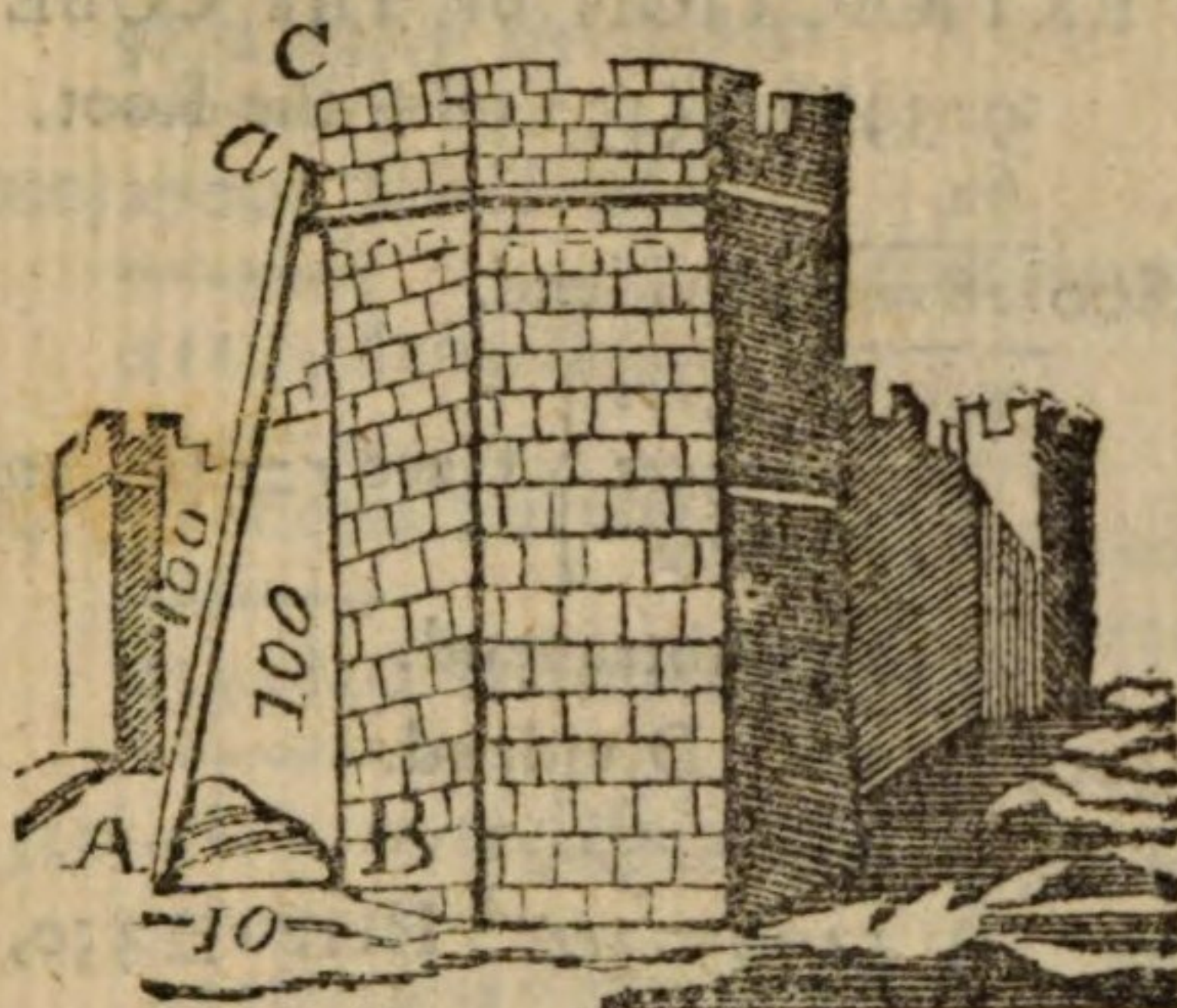
So that $76 + 93.3808 = 169.3808 = A F + F B$, Distance of the Columns.

Again $169.3808 \times 169.3808 = 28689.85540864 = {}^2 c D$.

And $14 \times 14 = 196 = {}^2 c C$.

Sum $28885.85540864 = {}^2 C D$.

$\therefore \sqrt{28885.85540864} = 169.95839$ Ft. = $C D$. Distance required.



(16) First $100 \times 100 = 10000 = {}^2A a.$

And $10 \times 10 = 100 = {}^2A B.$

Diff. $9900 = {}^2B a.$

Then $\sqrt{9900} = 99.49874 = B a.$

$\therefore 100 - 99.49874 = .50125 = a C,$ which is 6 Inches nearly. Q. E. F.

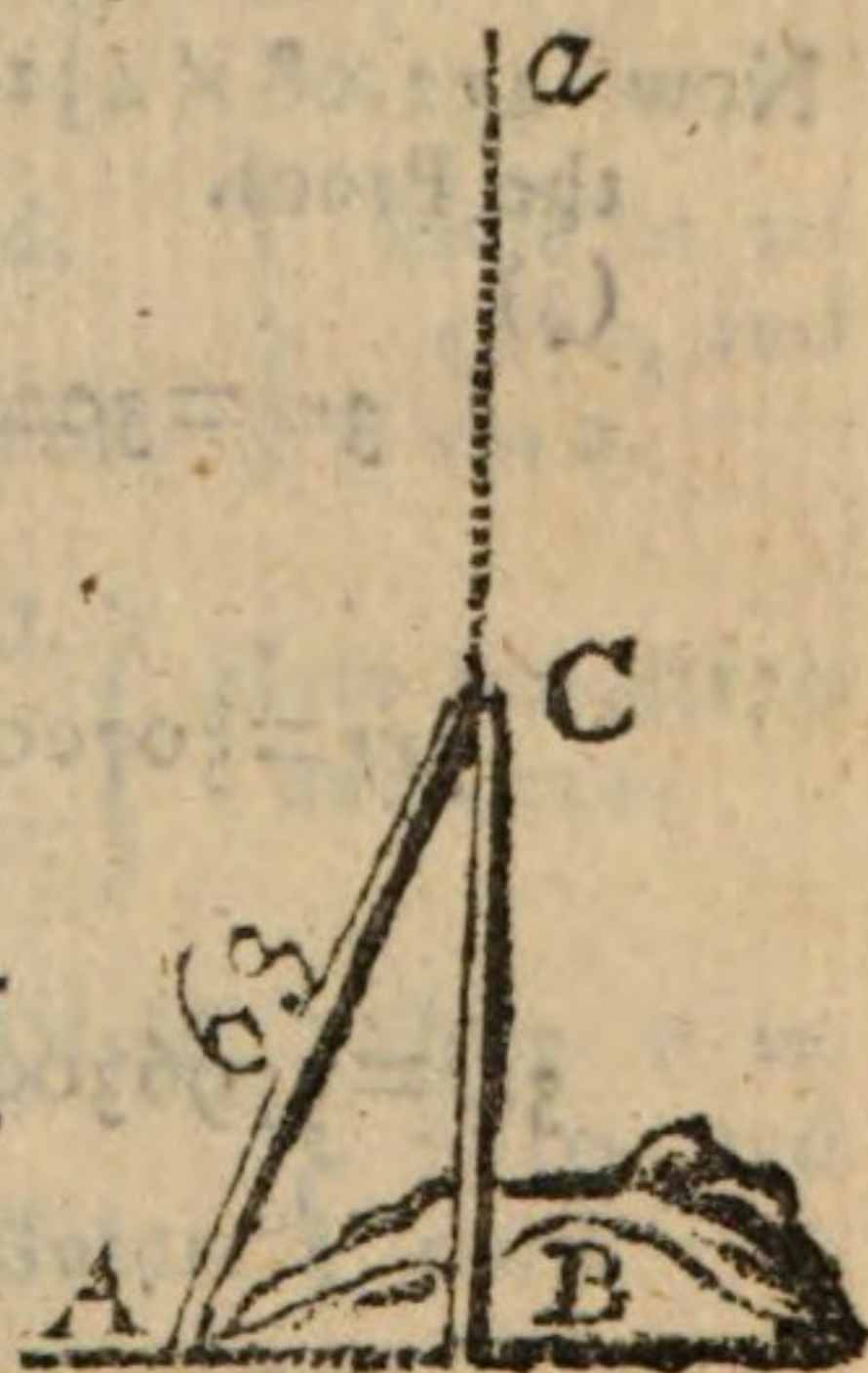
(17) First $63 \times 63 = 3969 = {}^2A C,$ or
C D.

And $30 \times 30 = 900 = {}^2A B.$

Diff. $3069 = {}^2B C.$

Then $\sqrt{3069} = 55.3985$ Yds. = B C,
Piece standing.

$\therefore 63 + 55.3985 = 118.3985$ Yds. or
118 Yds. 1 Foot $2\frac{1}{4}$ Inches = B a,
Height of the Pole.



(18) Thus $\sqrt{23716} = 154$ Men, the Answer.

56. THE EXTRACTION OF THE CUBE ROOT.

(2) $92398647(452.08 + \text{the Root.}$

64

 $3rr = 4800)28398 \text{ Dividend.}$

$$\left. \begin{array}{l} 24000 = 3rre \\ 2000 = 3ree \\ 125 = eee \end{array} \right\} \text{Here } r = 40, \text{ and } e = 5.$$

 $27125 \text{ Subtrahend.}$ $3rr = 607500)1273647 \text{ Dividend.}$

$$\left. \begin{array}{l} 1215000 = 3rre \\ 5400 = 3ree \\ 8 = eee \end{array} \right\} \text{Here } r = 450, \text{ and } e = 2.$$

 1220408 $3rr = 6129120000)53239000000 \text{ Dividend.}$

$$\left. \begin{array}{l} 49032960000 = 3rre \\ 8678400 = 3ree \\ 512 = eee \end{array} \right\} \text{Here } r = 45200, \text{ and } e = 8.$$

 $49041638912 \text{ Subtrahend.}$ 4197361088 Rem. Now $452.08 \times 452.08 \times 452.08 + 4197361088 = 92398647$,
the Proof.(3) $2716243264(1395.24 + \text{the Root.}$

$$\left. \begin{array}{l} 3rr = 300)1716 \text{ Dividend.} \\ 1197 \text{ Subtrahend.} \end{array} \right\} \text{Here } r = 10, \text{ and } e = 3.$$

$$\left. \begin{array}{l} 3rr = 50700)519243 \text{ Dividend.} \\ 488619 \text{ Subtrahend.} \end{array} \right\} \text{Here } r = 130, \text{ and } e = 9.$$

$$\left. \begin{array}{l} 3rr = 5796300)30624264 \text{ Dividend.} \\ 29085875 \text{ Subtrahend.} \end{array} \right\} \text{Here } r = 1390, \text{ and } e = 5.$$

$$\left. \begin{array}{l} 3rr = 583807500)1538389000 \text{ Dividend.} \\ 1167782408 \text{ Subtrahend.} \end{array} \right\} \text{Here } r = 13950, \text{ and } e = 2.$$

$$\left. \begin{array}{l} 3rr = 58397491200)370606592000 \text{ Dividend.} \\ 233596661824 \text{ Subtrahend.} \end{array} \right\} \text{Here } r = 13950, \text{ and } e = 2.$$

 $37009936976 \text{ Remains.}$

(4)

(4) $91)4,497+$ the Root.

$$\begin{array}{r} 4800)27000 \text{ Dividend.} \\ \underline{21184} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 4800)27000 \\ \underline{21184} \end{array}} \right\} \text{Here } r=40, \text{ and } e=4.$$

$$\begin{array}{r} 580800)5816000 \text{ Dividend.} \\ \underline{5334849} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 580800)5816000 \\ \underline{5334849} \end{array}} \right\} \text{Here } r=440, \\ e=9.$$

$$\begin{array}{r} 60480300)481151000 \text{ Dividend.} \\ \underline{424622473} \text{ Subtrahend.} \\ \underline{57128527} \text{ Remains.} \end{array} \left. \vphantom{\begin{array}{r} 60480300)481151000 \\ \underline{424622473} \end{array}} \right\} \text{Here } r=4490, \\ \text{and } e=7.$$

(5) $67527834239(4072.18+$ the Root.

$$\begin{array}{r} 3rr=480000)3527834 \text{ Dividend.} \\ \underline{3419143} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=480000)3527834 \\ \underline{3419143} \end{array}} \right\} \text{Here } r=40, \text{ and } e=7.$$

$$\begin{array}{r} 3rr=49694700)108691239 \text{ Dividend.} \\ \underline{99438248} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=49694700)108691239 \\ \underline{99438248} \end{array}} \right\} \text{Here } r=4070, \text{ and } e=2.$$

$$\begin{array}{r} 3rr=4974355200)9252991000 \text{ Divid.} \\ \underline{4974477361} \text{ Subtr.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=4974355200)9252991000 \\ \underline{4974477361} \end{array}} \right\} \text{Here } r=40270 \\ \text{and } e=1.$$

$$\begin{array}{r} 3rr=497459952300)4278513639000 \text{ Divid.} \\ \underline{3979757803232} \text{ Subtr.} \\ \underline{290755835768} \text{ Remains.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=497459952300)4278513639000 \\ \underline{3979757803232} \end{array}} \right\} \text{Here } r=7210, \text{ and } e=8.$$

(6) $4764,750)16,827 + \text{the Root.}$ $3rr=300)3764$ Dividend. 3096 Subtrahend.Here $r=10$, and $e=6$. $3rr=76800)668750$ Dividend. 645632 Subtrahend.Here $r=160$, and $e=8$. $3rr=8467200)23118000$ Dividend. 16954568 Subtrahend.Here $r=1680$, and $e=2$. $3rr=848737200)6163432000$ Dividend. 5943633293 Subtrahend.Here $r=16820$, and $e=7$. 219798717 (7) $36155.027576(33\ 0665$ $3rr=2700)9155$ 8937 $3rr=32670000)218027576$ Dividend. 196376616 Subtrahend.Here $r=3300$, and $e=6$. $3rr=3278890800)21650960000$ Dividend. 19676915496 Subtrahend.Here $r=33060$, and $e=6$. $3rr=32800806800)1974044504000$ Divid. 1640065333625 Subtr.Here $r=330660$, & $e=5$. 333979170375 Remains.

(8) 67667.921875 (40.75 the Root.

$$\begin{array}{r} 3rr=480000)3667.921 \text{ Dividend.} \\ \underline{3419143} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=480000)3667.921 \text{ Dividend.} \\ \underline{3419143} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=408, \text{ and} \\ e=7. \end{array}$$

$$\begin{array}{r} 3rr=49694700)248778875 \text{ Dividend.} \\ \underline{248778875} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=49694700)248778875 \text{ Dividend.} \\ \underline{248778875} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=4070, \\ \text{and } e=5. \end{array}$$

(9) 219365329 (603.1+ the Root.

$$\begin{array}{r} 3rr=10800000)3365329 \text{ Dividend.} \\ \underline{3256227} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=10800000)3365329 \text{ Dividend.} \\ \underline{3256227} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=600, \\ \text{and } e=3. \end{array}$$

$$\begin{array}{r} 3rr=109082700)109102000 \text{ Divid.} \\ \underline{109100791} \text{ Subtr.} \\ 1209 \text{ Remains.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=109082700)109102000 \text{ Divid.} \\ \underline{109100791} \text{ Subtr.} \end{array}} \right\} \begin{array}{l} \text{Here } r=6030, \text{ and} \\ e=1. \end{array}$$

(10) 3105926.917 (145.9+ the Root.

$$\begin{array}{r} 3rr=300)2105 \text{ Dividend.} \\ \underline{1744} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=300)2105 \text{ Dividend.} \\ \underline{1744} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=10, \text{ and } e=4. \end{array}$$

$$\begin{array}{r} 3rr=58800)361926 \text{ Dividend.} \\ \underline{304625} \text{ Subtrahend.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=58800)361926 \text{ Dividend.} \\ \underline{304625} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=140, \text{ and } e=5. \end{array}$$

$$\begin{array}{r} 3rr=6307500)57301917 \text{ Dividend.} \\ \underline{57120579} \text{ Subtrahend.} \\ 181338 \text{ Remains.} \end{array} \left. \vphantom{\begin{array}{r} 3rr=6307500)57301917 \text{ Dividend.} \\ \underline{57120579} \text{ Subtrahend.} \end{array}} \right\} \begin{array}{l} \text{Here } r=1450, \text{ and} \\ e=9. \end{array}$$

234 To Extract the Cube Root of Vulgar Fractions,

(11) ,000421875 (,075 the Root.

3rr=14700)78375 Dividend.

78875 Subtrahend.

} Here $r=70$, and $e=5$.

(12) 28022810.39625 (303.7472+ the Root.

3rr=270000)1022810 Dividend.

818127 Subtrahend.

} Here $r=300$,
and $e=3$.

3rr=27542700)204683390 Dividend.

193244653 Subtrahend.

} Here $r=$
3030, and
 $e=7$.

3rr=2767010700)11438737625 Divid.

11082607350 Subtr.

} Here $r=$
30370, and
 $e=4$.

3rr=276773962800)356130275000 Divid.

276774874021 Subtr.

} Here $r=$
303740, &
 $e=7$.

3rr=27677578524300)79355400979050 Div.

553551934975284 Su.

} Here $r=$
3037470,
and $e=2$.

24000207481472 Remains.

57. TO EXTRACT THE CUBE ROOT OF VULGAR FRACTIONS.

(13) First $\frac{324}{1500} = \frac{27}{125}$, then $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$, the Root req.

(14) First $\frac{352}{1188} = \frac{8}{27}$, then $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$, the Root.

(15) First $5 \times 125 + 104 = 729$, then $\sqrt[3]{\frac{729}{125}} = \frac{9}{5}$, or $1\frac{4}{5}$, Root.

(16) First $405 \times 125 + 104 = 50653$, then $\sqrt[3]{\frac{50653}{125}} = \frac{37}{5}$ or $7\frac{2}{5}$, the Root.

(17)

- (17) First $5\frac{3}{5}=5.6$, then $\sqrt[3]{5,600}=1,775 + \text{the Root.}$
- (18) First $7\frac{5}{7}=7,7142857$, then $\sqrt[3]{7,7142857}=1,775$,
the Root required.

58. THE USE OF THE CUBE ROOT.

- (1) First $\sqrt[3]{21952} = 28$, Side of the Cube.
Then $28 \times 28 = 784$, the Content required.
- (2) First $125 \times 125 \times 125$ }
And $25 \times 25 \times 25$ } $\times 3 = \left\{ \begin{array}{l} 5859375 \\ 46875 \\ 10125 \end{array} \right.$
Also $15 \times 15 \times 15$ }
- $\therefore \sqrt[3]{5859375} = 180,05$ Keel.
And $\sqrt{46875} = 36,05$ Midship-beam.
Also $\sqrt{10125} = 21,6$ Depth in the Hold.

- (3)
$$\left. \begin{array}{l} \text{First } \frac{125 \times 125 \times 125}{25 \times 25 \times 25} \\ \text{And } \frac{25 \times 25 \times 25}{15 \times 15 \times 15} \\ \text{Also } \end{array} \right\} \div 2 = \begin{cases} 976562,5 \\ 7812,5 \\ 1687,5 \end{cases}$$

Which Numbers extracted will give 99.202; 19.84 and 11.906 the Dimensions required.

- (4) First $75 \times 75 \times 75 = 421875$, and $100 \times 100 \times 100 = 1000000$.

Then, as 421875 C. K. : 300 Tons, :: 1000000 C. K. :
711.111 + Tons, or 711 Tons. 2 cwt. 24,64 lb. the
Burthen required.

- (5) First $4 \times 4 \times 4 = 64$, Cube of the Diameter.
Then, as 18 lb. : 64 inc. :: 114 lb. : 405,3 inc. Cube of
the Diameter.

$\therefore \sqrt{405,3} = 7,4+$, the Diameter required.

- (6) First $11,5 \times 11,5 \times 11,5 = 1520,875$, and $20,83 \times 20,83 \times 20,83 = 9037,905787$.

Then, as 1520,875 : 1000 lb. :: 9037,905787 :
5942,5697lb. +, the Weight required.

- (7) First $189 \div 7 = 27$, whose Cube Root is 3.
Then $3 \times 7 = 21$, the lesser Mean, and $21 \times 3 = 63$, the greater Mean.

For, as $7 : 21 :: 63 : 189$, the Proof.

236 *To Extract the First Sur-solid Root.*

(8) First $256 \div 4 = 64$, then $\sqrt[3]{64} = 4$, which $\times$ by $4 = 16$, the lesser Mean, and $16 \times 4 = 64$ the greater.
For, as $4 : 16 :: 64 : 256$, the Proof.

59. TO EXTRACT THE FIRST SURSOLID ROOT.

R U L E.

1. Having pointed the given Resolvend into Periods of five Figures, seek such a Sur-solid Number in the Table, or otherwise, as comes nearest to the first Period of the Resolvend, whether greater or less; and call the respective Root, either more than just, or less than just, as it falls out; annexing so many Cyphers to it, as there are remaining Periods of whole Numbers in the Resolvend.

2. Find the Difference between the Resolvend and the Sur-solid Number, so taken, by subtracting the lesser from the greater.

3. Find the Cube of the foresaid Sur-solid Root, with its annexed Cyphers, which also may be done by the Table; and multiply that Cube into five, the Index of the Sur-solid, and divide the Difference between the Resolvend and the Sur-solid Number by that Product; by which it will be depressed to a Square, and when pointed into Periods of two Figures each, call it the new Resolvend.

4. Make the first Root without Cyphers a Divisor, enquiring how often it may be found in the first Period of the new Resolvend; with this Consideration, if the Root, now a Divisor, be less than just, annex twice the Quotient Figure to it; but if more than just, subtract twice the Quotient Figure from a Cypher, either annexed or supposed to be annexed, to that Divisor or Root, multiplying it so increased or diminished, with the said Quotient Figure; setting down the Units Place of the Product under the pointed Figure of that Period, subtracting it as in Division.

T A B L E.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|----|-----|------|------|------|-------|-------|-------|
| 1 | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 |

E X A M-

E X A M P L E S.

- (1) Extract the Sur-solid Root of 12309502009375 , the Resolvend pointed.

The nearest Sur-solid Number to 1230, the first Period of the Resolvend is 1024, whose Root is 4 (by the Table) less than just.

$$\begin{array}{r} \text{Therefore } 12309502009375 \\ - 1024 \end{array}$$

2069502009375 their Difference.

Now, as there are two Periods remaining in the Resolvend, place two 0's to 4, the Root of the first Period.

Next the Cube of $400 = 64000000$.

And $64000000 \times 5 = 320000000$ the Divisor.

Then $320000000 \overline{) 2069502009375}$ (6497, &c.

First Root = 400

$+ 2 \times 10 = 20$

1st. Divisor = $420 \overline{) 6467}$ (15 last Root.

$+ 20 \times 2 \times 5 = 30 \ 42 \ 400$ First.

450 2267 415 Root (true) required.
2250

Rem. 17 to be rejected.

Now $415 \times 415 \times 415 \times 415 \times 415 = 12309502009375$, the Proof.

- (2) Extract the Sur-solid Root of 2327834559873 .

Now the nearest Sur-solid Number to 232 is 243 (per Table) whose Root is 3, being more than just.

Therefore 2430000000000

$- 2327834559873$

Rem. 102165440127 for a Dividend.

300 Cubed = 27000000 , which $\times 5 = 135000000$ Divisor.

Then $135000000 \overline{) 102165440127}$ (756, 7810 new Resolvend.

First

238 To Extract the Root of the Second Sur-solid.

| | | |
|--------------------|--------------|--|
| First Root | 300 | |
| -2×2 | $= -4$ | |
| 1 Divisor | $= 296$ | 300 first Root. |
| $-4 - 2 \times 5$ | $= -5,0$ | 2,566 last Ditto, too much. |
| 2 Divisor | $= 291,0$ | |
| $-1 - 2 \times 06$ | $= -1,12$ | 2 97,434 the Root only too little by 2 in the lowest Figure. |
| 3 Divisor | $= 289,88$ | |
| | $= -19,2810$ | |
| | &c. | &c. |

60. To EXTRACT THE ROOT OF THE SECOND SURSOLID, OR SEVENTH POWER.

R U L E.

Having pointed the Resolvend into Periods of seven Figures, seek out such a Number by the Table, as comes nearest to the first Period of the Resolvend, whether greater or lesser, calling its Root more than just, or less than just, annexing a proper Number of Cyphers.

2. Find the Difference between the Resolvend and that Number of the seventh Power, by subtracting the lesser from the greater.

3. Find the Sur-solid or fifth Power of that Root, with its annexed Cyphers, by the Table; and multiply that Sur-solid Number into seven, the Index of the Resolvend.

4. Make that Product a Divisor, by which the foresaid Difference must be divided; so that it may be depressed to a Square, and pointed as such.

5. Make the first Root without a Cypher, a Divisor working with it and the new Resolvend, as in Sur-solid, only here you must increase or diminish the Divisor with three quotient Figures.

T A B L E.

| 1 | 2 | 3 | 4 | 5 | 6 | .7 | .8 | 9 |
|---|-----|------|-------|-------|--------|--------|---------|---------|
| 1 | 128 | 2187 | 16384 | 78125 | 279936 | 823543 | 2097152 | 4782969 |

E X A M.

EXAMPLES.

What is the second Surfsolid Root of

382986553955078125 Resolvend pointed.
—2187 the nearest of the seventh Power.

164286553955078125 their Difference.

The first Root is 300, being less than just, and the fifth Power of 300 = 2430000000000, which being multiplied by 7 = 17010000000000 for a Divisor, by which the aforesaid Difference must be divided, which contracted must stand thus 1701)16428655(9658,23 &c.

First Root = 300

+3 × 20 = +60

1 Divisor = 360)9658(³⁰⁰25

60 + 3 × 05 = +75 72 325 the true Root.

2 Divisor = 435)2458
2175

Rem. 283 to be rejected.

70. THE RULE OF THREE IN DECIMALS.

(2) First $2\frac{1}{2}$ lb. = 2,5. And 1£. 5s. = 1,25. Also $14\frac{1}{4}$ = 14,75.

Then, as 2,5 lb. : 1,25£. :: 14,75 lb. : 8,85£. = 8£. 17s. the Answer.

(3) First 1 lb. = ,00892858 C. $11\frac{3}{4}$ d. = ,0489583£.

And 4hhds. each, 4C. 2 qrs. 14 lb. = 4,625 × 4 = 18,5 C.

Then, as, 00892858 : ,0489583£. :: 18,5 C. : 101,4417£. = 101£. 8s. 10d. the Answer.

(4) First 4 Chests each, : 2C. 3 qrs. 14 lb. = 2,875 C. × 4 = 11,5 C. And 906£. 10s. = 906,5£.

Then, as 11,5 : 906,5 :: 00892857 : 7038 = 14s. $\frac{3}{4}$ d. .6 the Answer.

(5) First 4 Tuns, $201\frac{1}{2}$ galls. = $63 \times 4 + 201,5 = 453,5$ gal.

And 240£. 16s. 6d. = 240,825£. Also $2\frac{1}{2}$ = 24,5.

Then 453,5 - 24,5 = 429 galls. the Remainder.

As 429 galls. : 240,825£. :: 1 gall. : ,561 34£. = 11s. $2\frac{1}{2}$ d. 8 + the Answer.

(6) First $4\frac{7}{8} = 4,875$, which $\div 12 = ,3471883$ ft. this added to 10 ft. = 10,3471883 ft. = to 6,5 Cubits.

Then, as 6,5 cub. : 10,3471883 :: 1 cub. : 1,591875 + = 1 ft. 7.1025 Inches, the Answer.

(7) First $78,4 \div 8 = 9,8$, the second Number.

Then, as 5 : 9,8 :: 8 : 15,68 the fourth Number.

$\therefore 15,68 - 9,8 = 5,88$, the Answer.

(8) First $13\frac{1}{3} = 13,3$. And $13,3 \times 282 = 3760$ cubic Inches, the Content of the Cask.

Then $3760 \times ,52835 = 1986,596$ oz. Weight of the Oil.

Which $\div 16 = 124,16225$ lb. Also $7\frac{1}{2} = 7,5$ lb. Weight of 1 Gall.

Therefore, as 7,5 lb. : 1 gall. :: 124,16225 lb. : 16,555 = 16 gall 4.4 pts. the Answer.

(9) First $4,63 - 1,5688 = 3,0612$ oz. Difference.

And a solid Foot and a half = 1728 + 864 = 2592 solid Inc.

Then as 1 inc. : 2,0612 oz. :: 2592 inc. : 7934,6304 oz. = 4,4278 cwt. or 4 cwt. 1 qr. 19 lb. 14 oz. 9,88 drs. Answ.

(10) Now, as 5 : 8 :: 75 : 120, the greater Number.

Then $75 + 120 = 195$, their Sum.

And $120 - 75 = 45$, their Difference.

Also $195 \times 45 = 8775$, Prod. of their Sum and Diff.

$120 \times 120 = 14400$, Square of the gr. and $75 \times 75 = 5625$ of the lesser.

$14400 - 5625 = 8775$, Diff. of those Squares.

$14400 + 5625 = 20025$, Sum of ditto.

$120 \div 75 = 1,6$, Quot. of the gr. divided by the lesser.

$75 \div 120 = ,625$, Quot. of the lesser div. by the greater.

$1,6 \times 1,6 = 2,56$ Square of the greater Quote.

$,625 \times ,625 = ,390625$, Sq. of the lesser Quote.

Answ. 2,950625, Sum of those Squares.

(11) Now, as 8 : 7 :: 224 : 196, the lesser Number.

Then $224 + 196 = 420$, their Sum.

And $420 \times 420 = 176400$, Square of their Sum.

Also $240 - 196 = 28$, Diff.

$28 \times 28 = 784$, Square of that Diff.

$224 \div 196 = 1,142856$, Quot. of greater $\div$ lesser.

$196 \div 224 = ,875$, Quot. of the lesser $\div$ greater.

$1,142856 \times 1,142856 = 1,30612 +$ } Square of those
 $,875 \times ,875 = ,765625$ } Quotes.

$420 \times 28 + 28 = 11788$, Prod. of their Sum and Difference, and the Diff. added.

Lastly

Lastly, $11788 \times 11788 = 138956944$, Square of the Sum of their Diff. added to the Prod. of their Sum and Diff.

| | | |
|--|---------------|--|
| | <i>Miles.</i> | |
| (12) He that rows towards }
And he that rows from } | London goes | $\left\{ \begin{array}{l} 7,5 \\ 2,5 \end{array} \right\}$ per Ho. |
| | Sum | <u>10</u> |

$\therefore 34 \div 10 = 3,4$ Hours, before they will meet.

Then, as 1 h. : 2,5 m. :: 3,4 h. : 8,5, or $8\frac{1}{2}$ Miles from London.

And $34 - 8\frac{1}{2} = 25\frac{1}{2}$ Miles from Chertsey, the Answer.

(13) First $13,5 - 7,5 = 6$ Inches Diff.

Then, as 4 : 9 :: 6 : 13,5 lb. the Answer.

(14) First 18200£. at 3 per cent. = 546£. (per Sect. XIX.) spent on the Funeral and Monument.

Then $18200 - 546 = 17654$ £. left.

Again 17654£. at 9£. per cent. = 1583,86 gave to his Cousins.

Then $17654 - 1583,86 = 16065,14$ £. what he had then left.

Again $\frac{2}{7}$ of 16065,14 = 4590,04£. paid for the Seat.

Then $16065,14 - 4590,04 = 11475,1$ left.

Again $11475,1 \div 8 = 1434,3875$ paid for Horses.

Then $11475,1 - 1434,3875 = 10040,7125$ left.

Also $10040,7125 - 550$ £. = 9490,7125 left, after he had spent 550£. on his Mistress.

Now, as 12 : 2000 :: 19 : 3166,6£. spent in riotous living.

$\therefore 9490,7125 - 3166,6 = 6324,04583$ £. or 6324£. 11d. the Answer.

(25) First $6 \times 6 = 36$. And $3 \times 3 = 9$ Square of their Distances.

Then, Recip. as 36 : 1 :: 9 : 4, so that A's place is four times as hot as B's.

(16) First $81000000 \times 81000000 = 6561000000000000$.

Then, Recip. as

1 : 6561000000000000 :: 2 : 3280500000000000.

$\therefore \sqrt{3280500000000000} = 57275649$ Miles, the Answer.

(17) First $81 \times 81 = 6561$. And $424 \times 424 = 179776$, Squares of their Distances; the Cyphers being omitted.

Then, Recip. as $179776 : 1 :: 6561 : 27,4$; so that the Sun's influence on the Earth to that on the Planet Jupiter, is as $27,4$, to 1 .

(18) First $32 \times 32 = 1024$. And $777 \times 777 = 603729$, the Squares of the Distances, Cyphers omitted.

Then, Recip. as $603729 \text{ Sat.} : 1 :: 1024 : 589,584 \text{ Merc.}$ nearly.

$\therefore$ The solar Influence on Mercury to that of Saturn, is as $589,584$ to 1 , nearly.

(19) First $115 \times 115 = 13225$ Square.

As $13225 : 1 :: 1 : 13225$ Degrees hotter.

(20) The square Roots of the Distances being as the Times, viz. as the $\sqrt{1} : \sqrt{2} ::$ is the Time of falling through the whole required Height.

Now the $\sqrt{1} = 1$, and $\sqrt{2} = 1,4142$, from which take 1 , Remains $,4142$.

$\therefore$ as, $4142 : 1,4142 :: 1 : 3,414$ Secs. the Descent; the Square of which is $11,6574$.

Then $A^2 1 : 16,083 :: 11,6574 : 187,48 \text{ ft.}$ the Tower's Height.

(21) First $7970 \times 7970 \times 7970 = 506261573000$.

And $2170 \times 2170 \times 2170 = 1021831313000$.

Then, as $1021831313000 : 506261573000 :: 1 : 49,5445$, Times bigger than the Moon.

$\therefore$ as $32,5 : 100 :: 49,5445 : 40,117$, Q. E. F.

That is, the Earth contains $40,117$ times more Matter than the Moon.

(22) First $81000000 + 240000 = 81240000$, Sun from a full Moon.

And $81000000 - 240000 = 80760000$, Sun from a new Moon.

Now $\left\{ \begin{array}{l} 8124 \times 8124 = 65999376 \\ 8076 \times 8076 = 65221776 \end{array} \right\}$ Squares of their Distances, Cphs. omitted.

Recip. $65221776 : 1 :: 65999376 : ,9882$.

So that the Proportion of Light and Heat a new Moon has to that of a full one, is

As 1 to $,9882$, or as 458329 to 452929 in whole Numbers.

(23) First $11 \times 11 = 121$, and the Square of 1 is 1 .

Then, as $1 : 16,083 :: 121 : 1946,083$, Q. E. F.

Or

| | | | | |
|-----------------|--|--------|---|----------------|
| $16,083 \times$ | $\left\{ \begin{array}{l} 1 = 16,083 \\ 3 = 48,25 \\ 5 = 80,416 \\ 7 = 112,583 \\ 9 = 144,75 \\ 11 = 176,916 \\ 13 = 219,083 \\ 15 = 241,25 \\ 17 = 273,416 \\ 19 = 305,583 \\ 21 = 337,75 \end{array} \right\}$ | in the | $\left\{ \begin{array}{l} 1\text{st.} \\ 2\text{d.} \\ 3\text{d.} \\ 4\text{th.} \\ 5\text{th.} \\ 6\text{th.} \\ 7\text{th.} \\ 8\text{th.} \\ 9\text{th.} \\ 10\text{th.} \\ 11\text{th.} \end{array} \right\}$ | Secs. of Time. |
|-----------------|--|--------|---|----------------|

Ans. 1946,083 as before.

(24) First $10 \times 10 = 100$, and $6 \times 6 = 36$, Square of their Descents.

Then, as $1 : 16,083 :: \left\{ \begin{array}{l} 100 : 1608,3 \\ 36 : 579, \end{array} \right\}$ their Depths.

Answer 1029,3 Difference.

(25) First $19,5 \times 19,5 = 380,25$ sq. of the Descent.

Then $A^2 1 : 16,083 :: 380,25 : 6115,6875$ Feet.

Which $\div 6 = 1019$ faths. 1 ft. $8\frac{1}{4}$ inc. Q. E. F.

(26) Thus, as $16,083 : 1^2 :: 400 : 24,8702$.

$\therefore \sqrt{24,8702} = 4,987+$, or 5 secs. nearly, the Answer.

(27) The Semi-diameter of the Earth = 3980 Miles, or 21014400 ft.

Then, as $16,083 : 1^2 :: 21014400 : 1306594,82$.

$\therefore \sqrt{1306594,82} = 1135,554$ sec. = 18 min. 55" 33".
Q. E. F.

62. THE DOUBLE RULE OF THREE IN DECIMALS.

(1) First, as $1,1\text{£} : 12 \text{ Per.} :: 1,6\text{£} : 17,45 \text{ Persons.}$

Then, Recip. $25,2\text{£} : 17,45 :: 18,9\text{£} : 23 \text{ Persons,}$
nearly, the Answer.

(2) As $19 \text{ lb.} : 4,25 \text{ lb.} :: 13 \text{ lb.} : 2,908 \text{ lb.}$

Also $2,908 : 6 :: 10 : 20,6334 \text{ lb.}$ worth 63 lb. at 5d. per lb. = $63 \times 5 = 315\text{d.}$ or $1,3125\text{£}.$

Then, as $20,6334 : 1,3125\text{£} :: 112 \text{ lb.} : 7,12437\text{£} =$
 $7\text{£} . 2\text{s.} . 5\text{d.} . \frac{3}{4} . 9$ (nearly) the Answer.

244 *The Double Rule of Three in Decimals.*

(3)
$$\begin{array}{rcl} \text{£. P. yr.} & \text{I.} & \\ 100 . 1 & . 3,5 & \\ - 1,25 . 38,5 & & \end{array} \quad \left| \quad \begin{array}{l} \text{Then } \frac{10 \times 1 \times 38,5}{3,5 \times 1,25} = \frac{3850}{4,375} \text{ £.} \\ = 880 \text{ £. the Answer.} \end{array} \right.$$

(4) First from Decemb. 11th. to May 10th. = 150 Days.

And from Sept. 3d. to Christmas day = 113.

Also 91 Guin. = 95,55 £. and 100 Mar. = 66,6 £.

Then, as 95,55 £. : 150 Days, :: 66,6 £. : 215 Days.

∴ 215 — 113 = 102 Days.

Recip. as 66,6 : 102 :: 40 £. : 169,83 Days, or rather 170 Days, the Answer.

(5) *Days.*

As $\left\{ \begin{array}{l} 12 \\ 14 \\ 15 \\ 18 \end{array} \right\} \text{Work. Day.} : 1 : : 1 : \left\{ \begin{array}{l} \frac{1}{12} = ,0833333 = \text{A.B.C.} \\ \frac{1}{14} = ,0714285 = \text{B.C.D.} \\ \frac{1}{15} = ,0666666 = \text{A.C.D.} \\ \frac{1}{18} = ,0555555 = \text{A.B.D.} \end{array} \right\} \begin{array}{l} \text{Will do} \\ \text{part of} \\ \text{the whole} \\ \text{work.} \end{array}$

All working three Days
will do

,276984 Part of the Work.

Then, as ,276984 : 3 :: 1 : 10,8309505 Days, all working
14 = B. C. D.

— 10,83095 A. B. C. D.

As 3,16905 : 10,8395 :: 14 : 47,848 Days by A.

15 = A. C. D.

— 10,83095

As 4,16905 : 10,8395 :: 15 : 38,969 Days by B.

18 = A. B. D.

— 10,83095

As 7,16905 : 10,8395 :: 18 : 27,194 Days by C.

12 = A. B. C.

— 10,83095

As 1,16905 : 10,8395 :: 12 : 111,1766 Days by D.

alone.

- (6) First $34'' = ,009444$ ho. and 5 Rods, $= ,015625$ Mile.
 Then, as 1 ho. : 12 Miles. $\therefore ,009444$ ho. : $,113328$ m.
 $\therefore ,113328 + ,015625 = ,128953$ m. or 680,9 ft. the
 Hare had started.
 Now $20 - 12 = 8$, Dog gained in running 20.
 Again, as 8 : 20 $\therefore ,128953$: $2,57906$ fur. $= 1702\frac{1}{4}$ ft. run
 by the Greyhound.
 $\therefore 8 : 1$ ho. $\therefore ,128953$ m. : $,016119$ ho. $= 58'' ,0284$, run
 by the Greyhound.

63. VIBRATION OF PENDULUMS.

- (8) Recip. as $39,2 : 3600 : \begin{cases} 12 : 11760. \\ 6 : 23520. \end{cases}$
 $\therefore \sqrt{11760} = 108,444$, and $\sqrt{23520} = 153,362$.
 Then $153,362 \times 60 = 9201,72$
 And $108,444 \times 60 = 6506,64$

Answer 2695,08 the Difference.

- (9) A Pendulum which vibrates 60 Times in a Minute,
 will vibrate $60 \times 60 = 3600$ Times, in an Hour; the
 Square of which $= 12960000$, and the Square of 30 $=$
 900; also of 100 $= 10000$. Then Recip.

$$12960000 : 39,2 \text{ inc.} :: \begin{cases} 900 : 564480 \\ 10000 : 50803,2 \end{cases}$$

12) 513676,8 Inches.

Answer, Diff. 42806,4 Feet.

- (10) First a Pendulum which vibrates Thirds, will vi-
 brate 60 Times in a Second.

$\therefore$ As $1^2 : 39,2 :: 60^2 (3600) : ,0108$ inc. the Length of
 that which vibrates Thirds.

Secondly, a Pendulum which vibrates Seconds, will make
 60 Vibrations in a Minute.

$\therefore$ As $60^2 (3600) : 39,2 :: 1^2 : 11760$ inc. $= 2\frac{5}{2}$ mls.
 the Length of a Pendulum which vibrates once in a
 Minute.

Thirdly, a Pendulum which vibrates Seconds, will make
 $(60 \times 60) = 3600$ Vibrations in an Hour.

∴ As 3600^2 (12960000) : 39,2 : : $^2 1$: 508032000 inc.
 $= 8018\frac{2}{11}$ mls. the Length of the Pendulum, which vi-
 brates once in an Hour.

Lastly, a Pendulum which vibrates Seconds will make
 $(60 \times 60 \times 24) = 86400$ Vibrations in a Day, or 24 Hrs.

∴ As 86400^2 (7464960000) : 39,2 : : $^2 1$: 292626432000
 inc. $= 4618188\frac{1}{7}\frac{5}{6}\frac{3}{8}$ mls. the Length of a Pendulum
 which vibrates once in a Day.

(11) First $60 \times 60 = 3600$.

Then, as 39,2 inc. : 3600 sec. : : 18 inc. : 7840 sec.

∴ $\sqrt{7840} = 88,54378$, Vibrations in one Minute.

Then $60 \div 88,54378 = 1,475729$ Vibrations in a Second.

Also $1,475729 \times 8,000000 = 5,421$ sec. in eight Vibrations.

Now $5,421 \times 5,421 = 29,387$.

Then, as $^2 1$: 16,083 ft. : : 29,387 : 472,640916 ft. Sound
 was returning.

Again, as 1150 ft. : 1 sec. : : 472,640916 : ,41099",
 Time Sound was returning.

∴ $5,421 - ,411 = 5,01$ Seconds, Time of the Body's
 Descent.

And $5,01 \times 5,01 = 25,1$. Also $25,1 \times 16,083 = 403,69$ ft.
 the Depth of the Well.

64. FELLOWSHIP.

(59) First $229\text{£}. 13\text{s}. 4\text{d}. = 229,6$; $5\frac{1}{2} = 5,5$; $4\frac{3}{7} = 4,42857$
 $\times ,4\frac{1}{5} = 4,1$; and $3\frac{2}{3} = 3,6$.

Then Nell takes up 3,6, as often as Anne takes up 4,1;
 then as often as Moll takes up 5,5, Anne takes up
 $4,42857$; and Nell must take

$$\frac{3\ 6 \times 4,42857}{4,1} = 3,949809,$$

For, as 4,1 : 3,6 : : 4,42857 : 3,949809.

Hence Moll takes 5,555555
 Anne 4,428571
 And Nell 3,949809 } Ratio of their Shares.

Sum 13,933935

∴ As $13,933935$: $\left\{ \begin{array}{l} 5,555555 : 91,569 = 91\ 11\ 4\frac{3}{4}\text{ M.} \\ 4,428571 : 72,995 = 72\ 19\ 10\frac{3}{4}\text{ A.} \\ 3,949809 : 65,102 = 65\ 2\ \frac{1}{2}\text{ N.} \end{array} \right.$
 $229,6\text{£} : :$

65. S I M.

65. SIMPLE INTEREST.

Theorem I. $P \times r = I$. the Interest.

- (2) Here is given $P = 260,875$, $t = 5,5$, and $r = ,045$.
Then per Theo. $260,875 \times 5,5 \times ,045 = 64,5665625 =$
 $64\text{£}. 11\text{s}. 3\frac{3}{4}\text{d}. ,9$, the Interest required.
- (3) Here is given $P = 85$, $t = 4,5$, and $r = ,05$.
Then per Theo. $85 \times 4,5 \times ,05 = 19,125\text{£}. = 19,26\text{£}.$
the Answer.
- (4) Here $P = 1000$, $t = 4,6$, and $r = ,055$.
Then by Theo. $1000 \times 4,6 \times ,055 = 256,6 = 256\text{£}. 13\text{s}. 4\text{d}.$
the Interest.
- (5) First, from May 12, 1764, to Nov. 24, 1769, is 5
yrs. 196 ds. or $5,536986$ yrs. $= t$, $r = ,0375$, and $P =$
 500 .
Then per Theo. $5,53696 \times ,0375 \times 500 = 103,8184875 =$
 $103\text{£}. 16\text{s}. 4\frac{1}{4}\text{d}. ,748$, the Answer.

When the Interest is for Days.

- (7) Here $r = ,00012328767$, $t = 220$, and $P = 370,5$.
Then per Rule $,00012328767 \times 370,5 \times 220 = 10,049 +$
 $= 10\text{£}. 11\frac{3}{4}\text{d}.$ the Interest required.
- (8) First, from July 1, to Feb. 24, following, is 238 Days,
 $= t$, $P = 600$, and $r = ,00016438356$.
Then $00016438356 \times 238 \times 600 = 23,4739723628 = 23\text{£}.$
 $9\text{s}. 5\frac{3}{4}\text{d}.$ the Interest.

Theorem II. $P \times r + P = A$, the Amount.

- (9) Here $P = 284,5$, $t = 7$, and $r = ,035$.
Then per Theo. $284,5 \times 7 \times ,035 + 284,5 = 354,2025 =$
 $354\text{£}. 4\text{s}. \frac{1}{2}\text{d}. ,4$, the Amount required.
- (10) Here $P = 672,25$, $t = 5,5$, and $r = ,045$.
Then per Theorem, $672,25 \times 5,5 \times ,045 + 672,25 =$
 $838,631875 = 838\text{£}. 12\text{s}. 7\frac{1}{2}\text{d}. ,6$, the Answer.
- (11) Here $P = 500$, $t = 6,328767$, and $r = ,0475$.
Then per Theorem VI. $328767 \times ,0475 \times 500 + 500 =$
 $650,30821625 = 650\text{£}. 6\text{s}. 1\frac{3}{4}\text{d}. 8 +$, the Amount.

Theorem III. $\frac{I}{tr} = P$, the Principal.

- (12) Here $I = 69,675$, $t = 3$, and $r = ,05$.
Then per Theo. $\frac{69,675}{3 \times ,05} = \frac{69,675}{,15} = 464,5 = 464\text{£}. 10\text{s}.$
the Principal.

(13) Here $I=64,35$, $t=5,5$, and $r=,045$.

Then per Theo. $\frac{64,35}{5,5 \times ,045} = \frac{64,3500}{,2475} = 260 \text{ £. the Answ.}$

(14) Here $I=67,790625$, $t=4$, and $r=,04$.

Then per Theo. $\frac{67,790625}{4 \times ,04} = \frac{67,790625}{,16} = 423 \text{ £. } 13 \text{ s.}$

$9\frac{3}{4}d. ,75 +$, the Principal required.

Theorem IV. $\frac{a}{tr+1} = P$, the Principal.

(15) Here $a=354,22083$, $t=7$, and $r=,035$.

Then per Theo. $\frac{354,22083}{,035 \times 7 + 1} = \frac{334,22083}{1,245} = 284,5147 =$

$284 \text{ £. } 10 \text{ s. } 3\frac{1}{2}d. ,112$, the Answer.

(16) Here $A=500,460416$, $t=6,416$, and $r=,05$.

Then per Theo. $\frac{500,460416}{6,416 \times ,05 + 1} = \frac{500,460416}{1,32083} =$

$378,89826 + = 378 \text{ £. } 17 \text{ s. } 11\frac{1}{2}d. ,2296$, the Answer.

(17) Here $A=100$, $t=7,6027397$, and $r=,0475$.

Then per Theo. $\frac{100}{7,6027397 \times ,0475 + 1} = \frac{100}{1,36113} =$

$73,4675 + = 73 \text{ £. } 9 \text{ s. } 4d. ,2$, the Principal.

Theorem V. $\frac{I}{pr} = t$, the Time.

(18) Here $I=69,675$, $P=464,5$, and $r=,05$.

Then per Theo. $\frac{69,675}{464,5 \times ,05} = \frac{69,675}{23,225} = 3 \text{ yrs. the Time}$
required.

(19) Here $I=64,35$, $P=260$, and $r=,045$.

Then per Theo. $\frac{64,35}{260 \times ,045} = \frac{64,35}{11,7} = 5\frac{1}{2} \text{ yrs. the Answer.}$

(20) Here $I=130,47916$, $P=500$, and $r=,065$.

Then per Theo. $\frac{130,47916}{,065 \times 500} = \frac{130,47916}{32,5} = 4,0144 + = 4$

Years, $5,25$ Days, the Answer.

Theorem VI. $\frac{a-P}{Pr} = t$, the Time.

(21) Here $a=354,22083$, $P=284,5$, and $r=,0325$.

Then per Theo. $\frac{354,22083-284,5}{284,5 \times ,0325} = \frac{70,72083}{9,24625} = 7,65405$
 $\dagger = 7$ Years, 238,7 Days, the Answer.

(22) Here $a=847,875$, $P=672,25$, and $r=,0475$.

Then per Theo. $\frac{847,875-672,25}{672,25 \times ,0475} = \frac{175,625000}{31,931875} = 5\frac{1}{2}$ yrs.
 the Answer.

(23) Here $a=500,464583$, $P=378,9$, and $r=,05$.

Then per Theo. $\frac{500,464583-378,9}{378,9 \times ,05} = \frac{121,564583}{18,945} =$
 $6,41467 = 6$ Years, 5 Months, (nearly) the Answer.

Theorem VII. $\frac{I}{Pr} = r$, the Rate.

(24) Here $I=69,675$, $P=464,5$, and $t=3$.

Then per Theo. $\frac{69,675}{464,5 \times 3} = \frac{69,675}{1393,5} = ,05$, or 5 per Ct.
 the Rate required.

(25) Here $I=64,35$, $P=260$, and $t=5,5$.

Then per Theo. $\frac{64,35}{260 \times 5,5} = \frac{64,35}{1430} = ,045$, or $4\frac{1}{2}$ per Cent.
 the Answer.

(26) Here $I=235,46$, $P=560,635416$, and $t=7$.

Then per Theo. $\frac{235,46}{560,635416 \times 7} = \frac{235,46666666}{3924,447916} = ,06$, or
 6 per Cent the Answer.

Theorem VIII. $\frac{a-P}{Pt} = r$ the Rate.

(27) Here $a=354,22083$, $P=284,5$, and $t=7$.

Then per Theo. $\frac{354,22083-284,5}{284,5 \times 7} = \frac{69,72083}{1991,5} = ,035$, or
 $3\frac{1}{2}$ per per Cent. the Answer.

(29) Here $a=500,4635416$, $P=378,9$, and $t=6$.

Then per Theo. $\frac{500,4635416 - 378,9}{378,9 \times 6} = \frac{121,5635416}{2273,4} =$
 $,053472+$, or $5\text{£. } 6\text{s. } 11\frac{1}{4}\text{d. } ,312$, per Cent. the Answ.

(30) Here $a=847,875$, $P=672,25$, and $t=5,5$.

Then per Theo. $\frac{847,875 - 672,25}{672,25 \times 5,5} = \frac{175,62500}{3697,375} = ,0475$,
 or $4\frac{3}{4}$ per Cent. the Answer.

66. OF ANNUITIES, PENSIONS, &c. IN ARREARS, AT SIMPLE INTEREST.

Theorem IX. $\frac{t t u - t u}{2} \times r : + t u = A$, the Amount.

(2) Here $u=250$, $t=6$, and $r=,03$.

Then per Theo. $\frac{250 \times 6 \times 6 - 250 \times 6}{2} \times ,03 : + 250 \times 6 =$
 $\frac{9000 - 1500}{2} \times ,03 : + 1500 = 3750 \times ,03 + 1500 = 1125,$
 $+ 1500 = 1612,5$, or $1612\text{£. } 10\text{s.}$ the Answer.

(3) Here $u=125$, $t=12$, and $r=,015$, per Note.

Then per Theo. $\frac{125 \times 12 \times 12 - 125 \times 12}{2} \times ,015 : + 125 \times$
 $12 = \frac{18000 - 1500}{2} \times ,015 : + 1500 = 8250 \times ,015 +$
 $1500 = 123,75 + 1500 = 1623,75 = 1623\text{£. } 15\text{s.}$ the Ans.

(4) Here $u=62,5$, $t=24$, and $r=,0075$, per Note.

Then per Theo. $\frac{62,5 \times 24 \times 24 - 62,5 \times 24}{2} \times ,0075 : +$
 $62,5 \times 24 = \frac{36000 - 1500}{2} \times ,0075 : + 1500 =$
 $17250 \times ,0075 + 1500 = 129,375 + 1500 = 1629,375 =$
 $1629\text{£. } 7\text{s. } 6\text{d.}$ the Answer.

Theorem X. $\frac{2 a}{t t r - t r + 2 t} = U$, the Annuity.

(5) Here $a=627,2$, $t=7$, and $r=,04$.

Then per Theorem,
$$\frac{627,2 \times 2}{7 \times 7 \times ,04 - 7 \times ,04 + 7 \times 2} = \frac{1254,4}{1,96 - ,28 + 14} = \frac{1254,4}{15,68} = 80\text{£. the Answer.}$$

(6) Here $a=1612,5$, $t=6$, and $r=,03$.

Then per Theo.
$$\frac{1612,5 \times 2}{6 \times 6 \times ,03 - 6 \times ,03 + 6 \times 2} = \frac{3225,0}{12,9} = 250\text{£. the Answer.}$$

Theo. for Half Yearly Payment,
$$\frac{4a}{ttr - tr + 2t} = t.$$

(7) Here $a=1623,75$, $t=12$, and $r=,015$.

Then per Theo.
$$\frac{1623,75 \times 4}{12 \times 12 \times ,015 - 12 \times ,015 + 12 \times 2} = \frac{6495}{1,98 + 24} = \frac{6495,00}{25,98} = 250\text{£. the Answer.}$$

Theo. for Quarterly Payment,
$$\frac{8a}{ttr - tr + 2t} = t.$$

(8) Here $a=1629,375$, $t=24$, and $r=,0075$.

Then per Theo.
$$\frac{1629,375 \times 8}{24 \times 24 \times ,0075 - 24 \times ,0075 + 24 \times 2} = \frac{13035}{4,14 + 48} = \frac{13035,00}{52,14} = 250\text{£. the Answer.}$$

Theo. XI.
$$\frac{2a - 2ut}{utt - ut} = r, \text{ the Rate.}$$

(9) Here $a=627,2$, $u=80$, and $t=7$.

Then per Theo.
$$\frac{627,2 \times 2 - 80 \times 2 \times 7}{80 \times 7 \times 7 - 80 \times 7} = \frac{1254,4 - 1120}{3920 - 560} = \frac{134,40}{3360} = ,04, \text{ or 4 per Cent. the Answer.}$$

(10) Here $a=1612,5$, $u=250$, and $t=6$.

Then per Theo.
$$\frac{1612,5 \times 2 - 250 \times 2 \times 6}{250 \times 6 \times 6 - 250 \times 6} = \frac{3225 - 3000}{9000 - 1500} = \frac{225,00}{7500} = ,03, \text{ or 3 per Cent. the Answer.}$$

Theo. for Half Yearly Payments, $\frac{4a - 4ut}{ut t - ut} = r.$

(11) Here $a = 1623,75$, $u = 125$, and $t = 12$, per Note.

$$\text{Then per Theo. } \frac{1623,75 \times 4 - 125 \times 4 \times 12}{125 \times 12 \times 12 - 125 \times 12} = \frac{6495 - 6000}{18000 - 1500} \\ = \frac{495,00}{16500} = ,03, \text{ or 3 per Cent. the Answer.}$$

Theo. for Quarterly Payments, $\frac{8a - 8ut}{ut t - ut} = r.$

(12) Here $a = 1629,375$, $u = 62,5$, and $t = 24$.

$$\text{Then per Theo. } \frac{1629,375 \times 8 - 62,5 \times 8 \times 24}{62,5 \times 24 \times 24 - 62,5 \times 24} = \\ \frac{13035 - 12000}{36000 - 1500} = \frac{1035,00}{34500} = ,03, \text{ or 3 per Cent. Answ.}$$

Theorem XII. First $\frac{2}{r} - 1 = X$. Then $\sqrt{\frac{2}{ur} + \frac{xx}{4}} : -\frac{x}{2} = T$.

(13) Here $a = 627,2$, $u = 80$, and $r = ,04$.

$$\text{Then per Theo. I. } \frac{2}{,04} - 1 = 50 - 1 = 49 = X.$$

$$\text{Then } \sqrt{\frac{627,2 \times 2}{80 \times ,04} + \frac{49 \times 49}{4}} : -\frac{49}{2} = \sqrt{\frac{1254,4}{3,2} + \frac{2401}{4}} \\ - 24,5 = \sqrt{392 + 600,25} - 24,5 = \sqrt{992,25} - 24,5 = \\ 31,5 - 24,5 = 7 \text{ Years, the Time required.}$$

(14) Here $a = 1612,5$, $u = 250$, and $r = ,03$.

$$\text{Then per Theorem I. } \frac{2}{,03} - 1 = 65,6 = X.$$

$$\text{Then } \sqrt{\frac{1612,5 \times 2}{250 \times ,03} + \frac{65,6 \times 65,6}{4}} : -\frac{65,6}{2} = \\ \sqrt{\frac{3225}{7,5} + \frac{4307,73}{4}} : -32,8 = \sqrt{1506,93} - 32,8 = 38,8 \\ - 32,8 = 6 \text{ Years, the Answer.}$$

(15) Here $a = 1623,75$, $u = 125$, and $r = ,015$.

$$\text{Then per Theorem I. } \frac{2}{,015} - 1 = 132,8 = X.$$

Then

$$\begin{aligned} \text{Then } \sqrt{\frac{1623,75 \times 2}{125 \times ,015} + \frac{132,3 \times 132,3}{4}} : - \frac{132,3}{2} = \\ \sqrt{\frac{3247,5}{1,875} + \frac{17507,68}{4}} : - 66,1 = \sqrt{1732 + 4376,92} \\ : - 66,1 = \sqrt{6108,92} - 66,1 = 78. \quad 1 - 66,1 = 12 \\ \text{Half Years or 6 Years, the Time required.} \end{aligned}$$

(16) Here $a=1629,375$, $u=62,5$, and $r=,0075$.

Then per Theorem I. $\frac{2}{,007} - 1 = 265,6 = X$.

$$\begin{aligned} \text{Then } \sqrt{\frac{1629,375 \times 2}{62,5 \times ,0075} + \frac{265,6 \times 65,6}{4}} : - \frac{265,6}{2} = \\ \sqrt{\frac{3258,75}{,46875} + \frac{70761,06}{4}} : - 132,8 = \sqrt{6952 + 17690,26} \\ - 132,8 = \sqrt{24642,26} - 132,8 = 156,9 - 132,8 = 24 \\ \text{quarterly Payments, or 6 Years, the Answer.} \end{aligned}$$

75. PRESENT WORTH OF ANNUITIES, &c.

Theorem XIII. $\frac{ttr - tr + 2t}{2tr + 2} : \times u = P$. The present Worth.

(18) Here $t=6$, $r=,03$, and $u=250$.

$$\begin{aligned} \text{Then per Theo. } \frac{6 \times 6 \times ,03 - 6 \times ,03 + 6 \times 2}{2 \times 6 \times ,03 + 2} : \times 250 = \\ \frac{1,08 - ,18 + 12}{,36 \times 2} \times 250 = \frac{12,9}{2,36} \times 250 = 5,4661017 \times \\ 250 = 1366,52675, \text{ or } 1366\text{£. } 10\text{s. } 6\text{d. } ,42, \text{ the present} \\ \text{Worth required.} \end{aligned}$$

(19) Here $t=12$, $r=,015$, and $u=125$.

$$\begin{aligned} \text{Then per Theo. } \frac{12 \times 12 \times ,015 - 12 \times ,015 + 12 \times 2}{12 \times 2 \times ,015 + 2} : \times 125 \\ = \frac{2,16 - ,18 + 24}{,36 + 2} : \times 125 = \frac{1,98 + 24}{2,36} : \times 125 = \\ \frac{25,98}{2,36} \times 125 = 11,008474 \times 125 = 1376,05925 = 1376\text{£.} \\ 1\text{s. } 2\text{d. } ,22, \text{ the present Worth.} \end{aligned}$$

254 *Present Worth of Annuities, etc.*

(20) Here is given $t=24$, $u=62,5$, and $r=.0075$.

Then per Theo. $\frac{24 \times 24 \times .0075 - 24 \times .0075 + 24 \times 2}{24 \times 2 \times .0075 + 2} : \times$

$$62,5 = \frac{4,32 - .18 + 48}{2,36} : \times 62,5 = \frac{4,14 + 48}{2,36} : \times 62,5$$

$$= \frac{52,14}{2,36} : \times 62,5 = 22,09322 \times 62,5 = 1380,82625 =$$

1380£. 16s. 6d. ,3, the Answer.

Theorem, XIV. $\frac{tr+1}{11r-1r+2t} : \times 2p = U$, the Annuity.

(21) Here is given $t=7$, $r=.04$, and $p=490$. To find U.

Then per Theo. $\frac{7 \times .04 + 1}{7 \times 7 \times .04 - 7 \times .04 + 7 \times 2} : \times 490 \times 2$

$$= \frac{1,28}{1,96 - .28 + 14} : \times 980 = \frac{1,28}{15,68} : \times 980 = ,0816326$$

$\times 980 = 80$ £. the yearly Rent.

(22) Here is given $t=6$, $r=.03$, and $p=1366,525$. To find U.

Then per Theo. $\frac{6 \times .03 + 1}{6 \times 6 \times .03 - 6 \times .03 + 6 \times 2} : \times 1366,525 \times 2$

$$= \frac{1,18}{12,9} \times 2733,05 = ,0914728 \times 2733,05 = 250$$
£. the Annuity required.

(23) Here is given $t=12$, $r=.015$, and $p=1376,25$. To find U.

Then per Theo. $\frac{12 \times .015 + 1}{12 \times 12 \times .015 - 12 \times .015 + 12 \times 2} : \times$

$$1376,25 \times 4 = \frac{1,18}{25,98} \times 5505 = ,0454195 \times 5505 =$$

250£. 8½d. the Annuity required.

(24) Here is given $t=24$, $r=.0075$, and $p=1380,875$. To find U.

Then per Theo. $\frac{24 \times .0075 + 1}{24 \times 24 \times .0075 \times 12 \times .0075 + 24 \times 2} \times$

$$1380,875 \times 8 = \frac{1,18}{52,14} \times 11047 = ,022631 \times 11047 =$$

250£. the Answer.

Theo.

Theorem XV. $\frac{ut - p \times 2}{2pt + ut - utt} = R$, the Rate.

(25) Here is given $u=80$, $t=7$, and $p=490$. To find R ,

$$\text{Then per Theorem, } \frac{80 \times 7 - 490 \times 2}{490 \times 2 \times 7 + 80 \times 7 - 80 \times 7 \times 7} = \frac{70 \times 2}{7420 - 3920} = \frac{140}{3500} = ,04, \text{ or 4 per Cent. Answ.}$$

(26) Here is given, $u=250$, $t=6$, and $p=1366,525$. To find R .

$$\text{Then per Theo. } \frac{250 \times 6 - 1366,525 \times 2}{1366,525 \times 2 \times 6 + 250 \times 6 - 250 \times 6 \times 6} = \frac{133,475 \times 2}{17898,3 - 900} = \frac{266,95}{8898,3} = ,03 = 3 \text{ per Cent. the Answ.}$$

(27) Here is given $u=125$, $t=12$, and $p=1376,25$. To find R .

$$\text{Then per Theo. } \frac{125 \times 12 - 1376,25 \times 2}{1376,25 \times 2 \times 12 + 125 \times 12 - 125 \times 12 \times 12} = \frac{1500 - 1376,25 \times 2}{33030 + 1500 - 18000} = \frac{123,75 \times 2}{34520 - 18000} = \frac{2475}{16530} = ,015 \text{ half the Rate, which } \times 2 = ,03 \text{ or 3, per Cent. the Answer.}$$

(28) Here is given, $u=62,5$, $t=24$, and $p=1380,875$. Then per Theorem,

$$\frac{62,5 \times 24 - 1380,875 \times 2}{1380,875 \times 2 \times 24 + 62,5 \times 24 - 62,5 \times 24 \times 24} = \frac{238,25}{31782} = ,0075, \text{ which } \times 4 = ,03, \text{ or 3 per Cent. the Answer.}$$

Theorem XVI. $\frac{2}{r} - \frac{2p}{u} - 1 = X$. Then $\sqrt{\frac{2p}{ur} + \frac{xx}{4}} - \frac{x}{2} = t$.
the Time.

(29) Here is given $u=80$, $p=490$, and $r=,04$. To find t .

$$\text{Theo, first } \frac{2}{,04} - \frac{490 \times 2}{80} - 1 = 50 - 13,25 = 36,75 = X.$$

Z 2

Then

$$\begin{aligned} \text{Then } \sqrt{\frac{490 \times 2}{80 \times .04} + \frac{36,75 \times 36,75}{4}} &: - \frac{36,75}{2} = \\ \sqrt{\frac{980}{3,2} + \frac{1350,5625}{4}} &: - 18,375 = \sqrt{306,25 + 307,640625} \\ - 18,375 &= \sqrt{613,890625} - 18,375 = 25,375 - 18,375 \\ &= 7 \text{ Years, the Time required.} \end{aligned}$$

(30) Here is given $u=250$, $p=1366,525$, and $r=.03$.

$$\begin{aligned} \text{Then per Theo. first } \frac{2}{.03} - \frac{1366,525 \times 2}{250} - 1 &= 66,6 - \\ 11,9322 &= 54,73446 = X. \end{aligned}$$

$$\begin{aligned} \text{Then } \sqrt{\frac{1366,525 \times 2}{250 \times .03} + \frac{54,73446 \times 54,73446}{4}} &- \\ \frac{54,73446}{2} &= \sqrt{\frac{2733,05}{7,5} + \frac{2995,86}{4}} - 27,3 = \sqrt{1113,36} - \\ 27,3 &= 33,3 - 27,3 = 6 \text{ Years, the Answer.} \end{aligned}$$

(31) Here is given $u=125$, $p=1376,25$, and $r=.015$.

$$\begin{aligned} \text{Then per Theo. first } \frac{2}{.015} - \frac{1376,25 \times 2}{125} - 1 &= 133,3 - \\ 23,02 &= 110,313 = X. \end{aligned}$$

$$\begin{aligned} \text{Then } \sqrt{\frac{1376,25 \times 2}{125 \times .015} + \frac{110,313 \times 110,313}{4}} &- \frac{110,313}{2} = \\ \sqrt{4510,2486} - 55,15 &= 67,15 - 55,15 = 12 \text{ half} \\ \text{yearly Payments, or 6 Years, the Time required.} \end{aligned}$$

(32) Here is given $u=62,5$, $p=1380,875$, and $r=.0075$.

$$\begin{aligned} \text{Then per Theo. first } \frac{2}{.0075} - \frac{1380,875 \times 2}{62,5} - 1 &= 266,6 - \\ - 45,188 &= 221,4786 = X. \end{aligned}$$

$$\begin{aligned} \text{Then } \sqrt{\frac{1380,875 \times 2}{62,5 \times .0075} + \frac{221,4786 \times 221,4786}{4}} &- \\ \frac{221,4786}{2} &= \sqrt{5891,733 + 12263,1962} - 110,7393 = \\ \sqrt{18154,92} - 110,7 &= 134,7 - 110,7 = 24 \text{ quarterly} \\ \text{Payments, or 6 Years, the Time required.} \end{aligned}$$

76. ANNUITIES, &c. TAKEN IN REVERSION.

Theo. XVII. $\frac{ttr - tr + 2t}{2tr + 2} : \times u = p$, which change to a .

Then $\frac{a}{tr + 1} = p$, the present Worth.

(34) Here is given, first $r = .045$, $t = 7.5$, and $u = 8$.

Then per Theo. I. $\frac{7.5 \times 7.5 \times .045 - 7.5 \times .045 + 7.5 \times 2}{7.5 \times 2 \times .045 + 2}$

$$: \times 80 = \frac{2,53125 - .3375 + 15}{2,675} \times 80 = \frac{17,19375}{2,675} \times 80$$

$$= 64,2756 \times 80 = 514,2048 = p.$$

Now per Theo. II. $p = a$, and r as before $= .045$.

$$\text{Then } \frac{514,2048}{5 \times .045 + 1} = \frac{514,2048}{1,225} = 419,95759 = 419 \text{ £.}$$

19s. 1 $\frac{3}{4}$ d. the Answer.

(35) Here is given $u = 40$, $t = 10$, and $r = .05$.

Then per Theo. I. $\frac{10 \times 10 \times .05 - 10 \times .05 + 10 \times 2}{10 \times 2 \times .05 + 2} \times 40$

$$\frac{4,5 + 20}{1 + 2} \times 40 = \frac{24,5}{3} \times 40 = 80,16 \times 40 = 326,6 = p.$$

Now per Theo. II. $a = 326,6$, and $t = 10$.

$$\text{Then } \frac{326,6}{7 \times .05 + 1} = \frac{326,6}{1,35} = 241,9755 = 241 \text{ £. 19s. 6d.}$$

,12, the present Worth.

Theorem XVIII. $ptr + p = A$ the Amount. Change A , and call it p , and t here will = the Time of its Continuance.

Then $\frac{tr + 1}{ttr - tr + 2t} : \times 2p = U$, the Annuity, &c.

(36) Here is given $p = 1220,114583$, $t = 4$, and $r = .03$.

$$\text{Then per Theorem I. } \frac{1220,114583 \times 4 \times .03 + 1220,114583}{1366,52483} = A.$$

Now A changed, it becomes p , and $t=6$.

Then per Theo. II. $\frac{6 \times .03 + 1}{6 \times 6 \times .03 - 6 \times .03 + 6 \times 2} : \times$
 $1366,52483 \times 2 = \frac{1,18}{12,9} \times 2733,049666 = 2723,049666$
 $\times .0914728 = 250\text{£. the Annuity.}$

(37) Here is given $p=419,755208$, $t=5$, and $r=.045$.

Then per Theorem I. $419,755208 \times 5 \times .045 + 419,755208$
 $= 94,4449218 + 419,755208 = 514,2001298 = A.$

Now A changed becomes p , and t here $= 7,5$.

Then per Theo. II. $\frac{7,5 \times .045 + 1}{7,5 \times 7,5 \times .045 - 7,5 \times .045 + 7,5 \times 2}$
 $\times 514,2001298 \times 2 = \frac{1,3375}{17,19375} \times 1028,4002596 =$
 $2077789 \times 1028,4002596 = 80\text{£. the Answer.}$

(8) Here is given $p=241,975308$, $t=7$, and $r=.05$.

Then per Theorem I. $241,975308 \times 7 \times .05 + 241,975308$
 $= 326,6666658 = A.$

Now A changed becomes $p=326,6666658$, and $t=10$.

Then per Theo. II. $\frac{10 \times .05 + 1}{10 \times 10 \times .05 - 10 \times .05 + 10 \times 2}$
 $\times 326,6666658 \times 2 = \frac{1,5}{24,5} \times 653,3333316 =$
 $653,3333316 \times .06122448 = 80\text{£. nearly the Answer.}$

77. REBATE OR DISCOUNT.

Theorem XIX. $\frac{s}{t+1} = P$, the present Worth.

(40) Here is given $s=1000$, $t=5\text{ mo.} = .416\text{ yr.}$ and $r=.045$.

Then per Theo. $\frac{1000}{.416 \times .045 + 1} = \frac{1000}{1,01875} = 981,595 =$
 $981\text{£. } 11\text{s. } 10\frac{3}{4}\text{d. the present Worth required.}$

(11) Here is given $s=9342$, $r=.04$, and $t=10\text{ mo.} = .83\text{ yr.}$

Then per Theo. $\frac{9342}{.83 \times .04 + 1} = \frac{9342}{1,03} = 9038,9$ the pre-
 sent Worth.

$9342 - 9038,9 = 303,1 = 303\text{£. } 2\text{s. the Discount.}$

Theo.

Theorem XX. $P + r + p = S$, the Sum due.

(41) Here is given $p = 144,578125$, $t = ,75$, and $r = ,05$.

Then per Theo. $144,578125 \times ,75 \times ,05 + 144,578125 = 5,421679 + 144,578125 = 149,9998$, or rather 150 £ . the Answer.

(42) Here is given $p = 981,52083$, $t = ,416$, and $r = ,045$.

Then per Theorem, $981,52083 \times ,416 \times ,045 + 981,52 = 18,403518 + 981,52083 = 999,9243485 = 999 \text{ £}$. 18 s . $5 \frac{3}{4} \text{ d}$. the Sum due.

(43) Here is given $p = 9111,185416$, $t = ,83$, and $r = ,04$.

Then per Theo. $9111,185416 \times ,83 \times ,04 + 9111,185416 = 303,70618 + 9111,185416 = 9414,891326 = 9414 \text{ £}$. 17 s . $9 \frac{3}{4} \text{ d}$. $,6$, the Debt.

Theorem XXI. $\frac{s-p}{pr} = T$, the Time.

(44) Here is given $s = 150$, $p = 144,578125$, and $r = ,05$.

Then per Theo. $\frac{150 - 144,578125}{144,578125 \times ,05} = \frac{5,42187500}{7,22890625} = ,75 \text{ yr. or } 9 \text{ Months}$, the Time required.

(45) Here is given $s = 1000$, $p = 981,52083$, and $r = ,045$.

Then per Theo. $\frac{1000 - 981,52083}{981,52083 \times ,045} = \frac{18,47916}{44,16843749} = ,416$, or 5 Months, the Answer.

(46) Here is given $s = 9342$, $p = 9111,185416$, and $r = ,04$.

Then per Theo. $\frac{9342 - 9111,185416}{9111,185416 \times ,04} = \frac{230,814584}{364,447416} = ,63332 = 7 \text{ mo. or } 18 \text{ Days}$, the Answer.

Theorem XXII. $\frac{s-p}{pt} = R$, the Rate.

(47) Here is given $s = 150$, $p = 144,578125$, and $t = ,75$.

Then per Theo. $\frac{150 - 144,578125}{144,578125 \times ,75} = \frac{5,421875}{108,43359375} = ,055$ or per Cent. the Rate required.

(48) Here is given $s = 1000$, $p = 981,52083$, and $t = ,416$.

Then per Theo. $\frac{1000 - 981,52083}{981,52083 \times ,416} = \frac{18,47916}{40,89570138} = ,045 = 4 \frac{1}{2} \text{ per Cent.}$ the Answer.

(49)

(49) Here is given $s=9342$, $p=9111,185416$, and $t=.83$.

Then per Theo. $\frac{9342-9111,185416}{9111,185416 \times .83} = \frac{230,814584}{769,26989582} = .03 = 3$ per Cent. the Answer.

78. EQUATION OF PAYMENTS.

Theorem XXIII. $\frac{s}{tr+1} = P$, the present Worth.

(51) Here is given $r=.035$, the first $t=.5$, the second β , and the third $=.83$.

Then per Theo. the present Worth will be as follows, viz.

| | $\pounds.$ |
|--|-----------------------|
| Paid down | 400 |
| 1. $\frac{500}{.5 \times .035 + 1} = \frac{500}{1,075}$ | = 491,40049 |
| 2. $\frac{250}{.6 \times .035 + 1} = \frac{250}{1,0213}$ | = 244,78605 |
| 3. $\frac{250}{.83 \times .035 + 1} = \frac{250}{1,025}$ | = 243,902439 |
| the Sum | = 1380,089179 = p . |
| | 1400 = s . |
| | ----- |
| $s - p = \text{Diff.}$ | = 19,910821 = d . |

Now per Theo. II. $\frac{d}{pr} = \frac{19,910821}{1380,8179 \times .035} = \frac{19,910821}{48,303121265} = .41013 = 5$ Months, nearly the true equate Time required.

QUESTIONS FOR EXERCISE.

(52) As 12,3 $\pounds.$: 47 Days :: 949,5 $\pounds.$: 18,45 Days, the Answer.

(53) Here is given $a=542,4$, $p=384$, and $t=8,25$.

Then per Theo. VIII. $\frac{a-p}{pt} = \frac{542,4-384}{384 \times 8,25} = \frac{158,4}{3168} = .05$ or 5 per Cent. the Answer.

79. COMPOUND INTEREST.

Theorem I. $pr^t = A$, the Amount.

- (2) Here is given
- $p = 246,5$
- ,
- $t = 7$
- , and
- $r = 1,05$
- .

Then per Theorem,

$$246,5 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \\ = 246,5 \times 1,4071004 = 346,8502486 = 346\text{£. } 17\text{s. the} \\ \text{Amount required.}$$

- (3) Here is given
- $p = 500$
- ,
- $t = 30$
- , and
- $r = 1,045$
- , which raised to the 30th Power, will give
- $3,7453181 = r^t$
- . (see Table I.)

Then $3,7453181 \times 500 = 1772,65905 = 1772\text{£. } 13\text{s. } 2\text{d. } 172$, the Amount for 30 Years.

Again, suppose the Amount was required only for 30 d.

Then $r = 1,0001206$, which raised to the 30th Power, $= 1,0036243$ (see Table II.) the Amount of 1£. for 30 Days. $\therefore 1,0036243 \times 500 = 501,81215 = 501\text{£. } 16\text{s. } 3\text{d. } 156$, the Amount.

- (4) Here is given
- $p = 523$
- ,
- $t = 5$
- yrs. 194 d. and
- $r = 1,05$
- . Now the Amount of 1£. for 5 yrs. is
- $1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 = 1,2762816$
- . (see Table I.)

Then, per Table II. the Amount of 1£. for 100 d. $= 1,0134569$; for 90 $= 1,0121031$; and for 4 $= 1,0005348$. $\therefore 1,2762816 \times 1,0134569 \times 1,0121031 \times 1,0005348 = 1,310976072$, the Amount of 1£. for 5 yrs. 194 d.Therefore $1,310976072 \times 523 = 685,640485656 = 685\text{£. } 12\text{s. } 9\frac{1}{2}\text{d. } 8$, the Answer.Theorem II. $\frac{a}{r^t} = P$.

- (5) Here is given
- $a = 243,10126$
- ,
- $t = 4$
- , and
- $r = 1,05$
- .

Then per Theo. $\frac{243,10126}{1,05 \times 1,05 \times 1,05 \times 1,05} = \frac{243,10126}{1,2155063} = 200\text{£. the Answer.}$

- (6) Here is given
- $a = 346,85$
- ,
- $t = 7$
- , and
- $r = 1,05$
- .

Then per Theorem,

$$\frac{346,85}{1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05} = \frac{346,85}{1,4071004} \\ = 246,5 = 246\text{£. } 10\text{s. the Answer.}$$

(7)

- (7) Here is given $a=1872,65905$, $t=30$, and $r=1,045$.
Which raised to 30th Power $=3,7453181$. (see Table I.)

Then per Theo. $\frac{1872,65905}{3,7453181}=500\text{£. the Answer.}$

- (8) Here is given $a=685,64$, $t=5\text{ yrs. }194\text{ d.}$ and $r=1,05$,
which raised to the Power of $t=1,310976$.

Then per Theo. $\frac{685,64}{1,310976}=523\text{£. the Answer.}$

Theorem III. $\frac{a}{p}=R^t$.

- (9) Here is given $a=243,10126$, $p=200$, and $r=1,05$.

Then per Theo. $\frac{243,10126}{200}=1,2155063=R^t$.

Now 1st. $1,05)1,2155063(1,157625$

2d. $1,05)1,157625(=1,1025$

3d. $1,05)1,1025(=1,05$

4th. $1,05)1,05(=1$, and nothing remains.

Therefore the Number of the Divisions are 4, which are
4 Years, the Answer.

- (10) Here is given $a=346,85$, $p=246,5$, and $r=1,05$.

Then per Theo. $\frac{346,85}{246,5}=1,4701004=R^t$.

Which $\div 1,05$ as in the last Example, till nothing remains, and the Number of Divisions will be 7, or 7 Yrs.
the Answer.

- (11) Here is given $a=1872,65905$, $p=500$, and $r=1,045$.

Then per Theo. $\frac{1872,65905}{500}=3,7453181=R^t$.

Which Number look for in Table I. under $4\frac{1}{2}$ per Cent.
and you will find it to be even with 30 Yrs. the Answ.

- (12) Here is given $a=685,64$, $p=523$, and $r=1,05$.

Then per Theo. $\frac{685,64}{523}=1,310976=R^t$. the nearest

Number (less) to which under 5 per Cent. is even with
5 yrs. viz. $1,2762816$.

Then $1,310976 \div 1,2762816=1,027184025$ the nearest
Number to which, in Table II. is even with 100 Days,
viz. $1,0134569$.

Therefore $1,027184025 \div 1,0134569=1,01264437$ the
nearest Number (less) to which in Table II. is even
with 90 Days, viz. $1,0121031$.

So

So $1,0126437 \div 1,0121031 = 1,0005348$, which is even with 4 Days, in Table II.

Therefore the Answer is 5 Years, 194 Days.

$$\text{Theorem IV. } \frac{a}{p} = R^t.$$

(13) Here is given $a = 243,10126$, $p = 200$, and $t = 4$.

Then per Theo. $\frac{243,10126}{200} = 1,2155013$, which is the fourth Power of r .

$\therefore \sqrt[4]{1,2155013} = 1,025$, and $\sqrt[4]{1,025} = 1,05$ or 5 per Cent. the Rate required.

(14) Here is given $a = 346,85$, $p = 246,5$, and $t = 7$.

Then per Theo. $\frac{346,85}{246,5} = 1,4071004 = r$, raised to the 7th Power.

Then per Rule in Page 238 $r = \sqrt[7]{1,4071004} = 1,05$ or 5 per Cent. the Answer.

(15) Here is given $a = 1872,65905$, $p = 500$, and $t = 30$.

Then per Theo. $\frac{1872,65905}{500} = 3,7453181 = r^t$, which

Number look for in Table I. and even with 30 yrs. you will find it to be under $4\frac{1}{2}$ per Cent. the Answer.

(16) Here is given $a = 685,64$, $p = 523$, and $t = 5$ yrs. 194 d.

Then per Theo $\frac{685,64}{523} = 1,310976 = r^t$. Now the nearest

Number to which, even with 5 yrs. is under 5 per Cent. the Answer.

80. ANNUITIES, PENSIONS IN ARREARS.

$$\text{Theorem V. } \frac{U r^t - u}{r - 1} = A, \text{ the Amount.}$$

(2) Here is given $u = 100$, $t = 6$, and $r = 1,05$.

Then per Theorem

$$\frac{100 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 \times 1,05 - 100}{1,05 - 1} =$$

$$\frac{100 \times 1,3400956 - 100}{,05} = \frac{134,00956 - 100}{,05} = \frac{34,00956}{,05}$$

$$= 680,192 = 680\text{£. } 3\text{s. } 9\frac{1}{4}\text{d. the Amount.}$$

(3)

- (3) Here is given $n=70$, $t=7$, and $r=1,03$; which raised to the 7th Power $=1,2298733$. (see Table I.)

$$\text{Then per Theo. } \frac{70 \times 1,2298733 - 70}{1,03 - 1} = \frac{86,091131 - 70}{,03} = \frac{16,091131}{,03} = 536,37103 = 536 \text{ £. } 7 \text{ s. } 5 \text{ d. the Answer.}$$

- (4) Here is given $n=30$, $t=30$, and $r=1,035$; which raised to the 30th Power $=2,8067937$ per Table I.

$$\text{Then per Theo. } \frac{30 \times 2,8067937 - 30}{1,035 - 1} = \frac{84,20381 - 30}{,035} = \frac{54,20381}{,035} = 1548,680257 = 1548 \text{ £. } 13 \text{ s. } 7 \frac{1}{2} \text{ d. the Amount required.}$$

Theorem VI. $\frac{ar - a}{r^t - 1} = U$, the Annuity, &c.

- (5) Here is given $a=297,25248$, $t=4$, and $r=1,04$.

$$\text{Then per Theo. } \frac{297,25248 \times 1,04 - 297,25248}{1,04 \times 1,04 \times 1,04 \times 1,04 - 1} = \frac{309,1425792 - 297,25248}{1,1698586 - 1} = \frac{11,890}{,1698586} = 70 \text{ £. the Annuity.}$$

- (6) Here is given $a=680,1912$, $t=6$, and $r=1,05$; which raised to the 6th Power $=1,3400956$.

$$\text{Then per Theo. } \frac{680,1912 \times 1,05 - 680,1912}{1,3400956 - 1} = \frac{34,00956}{,3400956} = 100 \text{ £. the Answer.}$$

- (7) Here is given $a=536,37103$ (per Quest. 3.) $t=7$, and $r=1,03$; which raised to the seventh Power $=1,2298733$.

$$\text{Then per Theo. } \frac{536,37103 \times 1,03 - 536,37103}{1,2298733 - 1} = \frac{552,46219 - 536,37103}{,2298733} = \frac{16,0911632}{,2298733} = 70 \text{ £. the Answer.}$$

- (8) Here $a=1548,680257$, $t=30$, and $r=1,035$; which raised to the 30th Power $=2,8067937$ per Table I.

Then

Then per Theo. $\frac{1548,680257 \times 1,035 - 1548,680257}{2,8067937 - 1} =$
 $\frac{1602,884066 - 1548,680257}{1,8067937} = \frac{54,203819}{1,8067937} = 30 \text{ } \text{£}.$

the Answer.

Theorem VII. $\frac{ar + u - a}{u} = R^t.$

(9) Here is given $a = 297,25248$, $u = 70$, and $r = 1,04$.

Then per Theo. $\frac{297,25248 \times 1,04 + 70 - 297,25248}{70} =$
 $\frac{309,1425792 - 297,25248}{70} = \frac{81,8900992}{70} = 1,16985856$

$= R^t$, which proceed with as in Exam. IX. and the Number of Divisions will be 4, that is, 4 yrs. the Time required.

(10) Here $a = 680,1912$, $u = 100$, and $r = 1,05$.

Then per Theo. $\frac{680,1912 \times 1,05 + 100 - 680,1912}{100} =$
 $\frac{814,20076 - 680,1912}{100} = \frac{340,0956}{100} = 3,400956 = R^t,$

which proceed with as directed in the last Example, will give 7 yrs. the Answer.

(11) Here $a = 536,37103$, $u = 70$, and $r = 1,03$.

Then per Theo. $\frac{536,37103 \times 1,03 + 70 - 536,37103}{70} =$
 $\frac{622,46219 - 536,37103}{70} = \frac{86,09113}{70} = 1,2298733 =$

R^t , which proceed with as before directed, will give 7 yrs. the Answer.

(12) Here $a = 1548,680257$, $u = 30$, and $r = 1,035$.

Then per Theo. $\frac{1548,680257 \times 1,035 + 30 - 1548,680257}{30} =$
 $\frac{1632,884066 - 1548,680257}{30} = \frac{84,203809}{30} =$

$2,8067936 = R^t$, which Number look for under $3\frac{1}{2}$ per Cent. and you will find it to be even with 30 yrs. the Answer. (see Table 1.)

Theorem VIII. $\frac{ar}{u} - r^t = \frac{a-u}{u} = R^t.$

(13) Here is given $a=297,25248$, $u=70$, and $t=4$.
Then per Theorem, see the Appendix; by the Table, thus
 $297,25248 \div 70 = 4,246464 = R^t$. which in Table III.
even with 4 yrs. is under 4 per Cent. the Answer.

(14) Here $a=680,1912$, $u=100$, and $t=6$.
Then per Table $680,1912 \div 100 = 6,801912 = R^t$. even
with 6 yrs. you will find it to be under 5 per Cent.

(15) Here $a=536,37103$, $u=70$, and $t=7$.
Then $536,37103 \div 70 = 7,662729 = R^t$. then in Table III.
even with 7 yrs. you will find it to be under 3 per Cent.
the Answer.

(16) Here $a=1548,680257$, $u=30$, and $t=30$.
Then $1548,680257 \div 30 = 51,6226752 = R^t$. which even
with 30 yrs. you will find it to be under $3\frac{1}{2}$ per Cent.
the Answer.

81. PRESENT WORTH OF ANNUITIES.

Theorem. IX. $u - \frac{u}{r^t} : \div r - 1 = P$. the present Worth.

(2) Here is given $u=60$, $t=6$, and $r=1,04$. Which
being involved to the 6th Power $= 1,265319$, per
Table I.

Then per Theo. $60 - \frac{60}{1,265319} : \div 1,04 - 1 =$
 $\frac{60 - 47,41096}{,04} = \frac{12,58904}{,04} = 314,726 = 314 \text{ £. } 14 \text{ s.}$
 $6\frac{1}{4} \text{ d. the Answer.}$

(3) Here $u=1000$, $t=21$, and $r=1,045$. Which being
involved to the 21st Power $= 2,5202411$, per Table I.

Then per Theorem, $1000 - \frac{1000}{2,5202411} : \div 1,045 - 1 =$
 $\frac{1000 - 396,787434}{,045} = \frac{603,2125657}{,045} = 13404,72386 =$
 $13404 \text{ £. } 14 \text{ s. } 5\frac{3}{4} \text{ d. the Answer.}$

Theo-

Theorem X. $\frac{prt \times r - prt}{r^t - 1} = U$, the Annuity.

- (4) Here is given $p = 323,1608$, $t = 8$, and $r = 1,05$; which being involved to the 8th Power $= 1,4774554$, per Table I.

Then per Theorem,

$$\frac{323,1608 \times 1,4774554 \times 1,05 - 323,1608 \times 1,4774554}{1,4774554 - 1} = \frac{501,3389 - 477,46567}{,4774554} = 50\text{£. the Annuity.}$$

- (5) Here $p = 314,726$, $t = 6$, and $r = 1,04$; which being involved to the 6th Power $= 1,265319$.

Then per Theorem,

$$\frac{314,726 \times 1,265319 \times 1,04 - 314,726 \times 1,265319}{1,265319 - 1} = \frac{414,157939 - 398,228785}{,265319} = \frac{15,92815}{,265319} = 60\text{£. the Annuity required.}$$

- (6) Here $p = 13404,72386$, $t = 21$, and $r = 1,045$; which being involved to the 21st Power $= 2,5202411$, per Table I.

Then per Theorem,

$$\frac{13404,72386 \times 2,5202411 \times 1,045 - 13404,72386 \times 2,5202411}{2,5202411 - 1} = \frac{353,0337712 - 337,83136}{1,5202411} = \frac{1520,2411}{1,5202411} = 1000\text{£. the Annuity.}$$

Theo. XI. $\frac{u}{p + u - pr} = R^t$.

- (7) Here is given $u = 50$, $p = 323,1608$, and $r = 1,05$.

Then per Theorem,

$$\frac{50}{323,1608 + 50 - 323,1608 \times 1,05} = \frac{50}{33,84196} = 1,4774554 = R^t. \text{ which proceed with as in Example III. and the Number of those Divisions will be 8, that is, 8 yrs. the Answer.}$$

268 *Annuities, Leases, &c. taken in Reversion*

(8) Here $u=60$, $p=314,726$, and $r=1,04$.

Then per Theorem,
$$\frac{60}{314,726 + 60 - \frac{314,726 \times 1,04}{60}} = \frac{60}{374,726 - 327,315} = \frac{60}{47,411} = 1,265319 = R.$$
which proceed with, as in the last Example, and the Number of Divisions will be 6, which is 6 Years, the Answer.

(9) Here $u=1000$, $p=13404,72386$, and $r=1,045$.

Then per Theorem,

$$\frac{1000}{13404,72386 + 1000 - \frac{13404,72386 \times 1,045}{1000}} = \frac{1000}{13404,72386 - 14007,936433} = \frac{1000}{396,787426} = 2,5202411 = R.$$
which in Table I. under $4\frac{1}{2}$ per Cent. is even with 21 Years, the Answer.

Theorem XII. $\frac{u}{p} = \frac{u}{p} R. + R. - R. + 1.$

To perform the three following Examples by the Theorem, see the Appendix.

(10) Here is given $u=50$, $p=323,1608$, and $r=8$. Then by the fifth Table, thus

First $323,1608 \div 50 = 6,463216$. Then even with 8 Years you will find it under 5 per Cent. the Answer.

(11) Here $u=60$, $p=314,726$, and $r=6$.

Then $314,726 \div 60 = 5,24543$, which Number look for in Table V. even with the given Time, and the nearest Number is under 4 per Cent. the Answer.

(12) Here $u=1000$, $p=13404,72386$.

Then $13404,72386 \div 1000 = 13,40472386$, which Number even with the given Time, you will find under $4\frac{1}{2}$ per Cent. the Answer.

82. ANNUITIES, LEASES, &c. TAKEN IN REVERSION.

Theorem I. $u - \frac{u}{r} : \div r - 1 = p$, which change to A.

Then $\frac{A}{R.} = p.$

- (2) Here is given $n=1000$, $t=20$, and $r=1,05$. Which involved to the 20th Power $=2,6532977$ per Table I.

Then per Theo. I. $1000 - \frac{1000}{2,6532977} : \div 1,05 - 1 =$

$$\frac{1000 - 376,88948}{1,246,22104} \div ,05 = 623,11052 \div ,05 =$$

Now per Rule II. $a=124622104$, $t=5$, and r as before; which being involved to the 5th Power $=1,276816$.

Then per Theo. II. $\frac{1246,22104}{1,2762816} = 9767,60175 = 9767 \text{ £.}$

125. ,035.

Theorem, first $prt=A$, which change to P.

Then $\frac{prt \times r - prt}{r^t - 1} = U$, the Annuity.

- (3) Here $p=368,30419$, $t=2$, and $r=1,05$; which being involved to the 2d Power $=1,1025$ per Table I.

Then per Theo. I. $368,30419 \times 1,1025 = 406,05536 = A$.

Now per Rule II. $p=406,05536$, $t=6$, and r as before; which being involved to the 6th Power $=1,3400956$.

Then per Theorem II.

$$\frac{406,05536 \times 1,3400956 \times 1,05 - 406,05536 \times 1,3400956}{1,3400956 - 1}$$

$$= \frac{571,360652 - 544,153002}{,3400956} = \frac{272,0765}{,3400956} = 80 \text{ £. the}$$

Annuity required.

- (4) Here $p=9767,60175$, $t=5$, and $r=1,05$; which being involved to the 5th Power $=1,2762816$.

Then per Theorem I. $9767,60175 \times 1,2762816 = 12466,2103896 = A$.

Now per Rule II. $p=12466,2103896$, $t=20$, and r as before; which being involved to the 20th Power $=2,6532977$ per Table I.

Then per Theorem II.

$$\frac{12466,2103896 \times 2,6532977 \times 1,05 - 12466,2103896}{2,6532977 - 1}$$

$$\times 2,6532977 = \frac{34730,30572229 - 33076,56735457}{1,6532977}$$

$$= \frac{1,65382836}{1,6532977} = 1000 \text{ £. the Annuity.}$$

83. REBATE OR DISCOUNT.

Theorem XIII. $\frac{s}{r^t} = P$, the present Worth.

- (2) Here $s=743,2375$, $t=6$, and $r=1,04$; which being involved to the 6th Power $=1,265319$.

Then per Theo. $743,2375 \div 1,265319 = 587,391005 = 587 \text{ £. } 7s. 9\frac{3}{4}d.$ the Answer.

Theorem XIV. $pr^t = S$, the Sum owed.

- (3) Here $p=123,405208$, $t=4$, and $r=1,05$; which being involved to the 4th Power $=1,2155063 = r^t$.

Then per Theorem, $123,405208 \times 1,2155063 = 150 \text{ £.}$ the Debt.

- (4) Here $p=587,3391005$, $t=6$, and $r=1,04$; which being involved to the 6th Power $=1,265319$, per Table I.

Then per Theorem, $587,3391005 \times 1,265319 = 743,237 = 743 \text{ £. } 4s. 9d.$ the Answer.

Theorem XV. $\frac{s}{p} = r^t$.

- (5) Here $p=123,405208$, $s=150$, and $r=1,05$.

Then per Theorem, $150 \div 123,405208 = 1,2155063 = r^t$; which being continually divided by $1,05$ till nothing remains, and the Number of those Divisions will be 4, that is, 4 Years, the Answer.

- (6) Here $p=587,3391005$, $s=743,235$, and $r=1,04$.

Then per Theorem, $743,235 \div 587,3391005 = 1,265319$; which proceed with as directed in the last Example, and the Number of Divisions will give 6 Years, the Answer.

Theorem XVI. $\frac{s}{p} = R^t$.

- (7) Here $s=150$, $p=123,405208$, and $t=4$.

Then per Theo. $\frac{150}{123,405208} = 1,2155063 = r^4$.

$\therefore \sqrt[4]{1,2155063} = 1,1025$, and $\sqrt[4]{1,1025} = 1,05$ or 5 per Cent, the Answer.

(8) Here $s=743,235$, $p=587,3391005$, and $t=6$.

Then per Theo. $743,235 \div 587,3391005 = 1,265319 = r$.

Then $\sqrt[2]{1,265319} = 1,124864$.

And $\sqrt[3]{1,124864} = 1,04$ or 4 per Cent. the Answer. (See the Guide, page 208.)

84. PURCHASING FREEHOLD, OR REAL ESTATES.

Theo. XVII. $\frac{u}{r-1} = P$, the present Worth.

(2) Here $u=25$, and $r=1,045$.

Then per Theo. $\frac{25}{1,045-1} = \frac{25}{,045} = 555,5 = 555\text{£. } 11\text{s. } 1\frac{1}{4}\text{d. } ,28$, the Answer.

Theo. XVIII. $\frac{P+u}{p} = R$, the Rate per Cent.

(3) Here $p=10000$, and $u=500$.

Then per Theo. $\frac{10000+500}{10000} = \frac{10500}{10000} = 1,05$, or 5 per Cent. the Answer.

(4) Here $p=555,5$, and $u=25$.

Then per Theo. $\frac{555,5+25}{555,5} = \frac{580,5}{555,5} = 1,045$ or $4\frac{1}{2}$ per Cent. the Answer.

Theo. XIX. $P \times \overline{r-1} = U$, the Annuity.

(5) Here $p=10000$, and $r=1,05$.

Then per Theo. $10000 \times 1,05 - 1 = 10000 \times ,05 = 500\text{£.}$ the Answer.

(6) Here $p=555,5$, and $r=1,045$.

Then per Theo. $555,5 \times 1,045 - 1 = 555,5 \times ,045 = 25\text{£.}$ the Answer.

272 Purchasing Freehold Estates in Reversion

85. PURCHASING FREEHOLD ESTATES IN REVERSION.

Theorem XX. first $\frac{u}{r-1} = P$, which change to A.

Then $\frac{a}{R^t} = P$, the present Worth.

(1) Here $u = 500$, and $r = 1,05$.

Then per Theo. I. $\frac{500}{1,05-1} = \frac{500}{,05} = 10000 = P$,

which change to a .

Then per Theo. II. $a = 10000$, $t = 4$, and r as before ;
which involved to the 4th Power $= 1,2155063$.

Then per Theo. II. $\frac{10000}{1,2155063} = 8227,024 = 8227\text{£}. 5\frac{3}{4}d$.

the Answer.

Theorem XXI. first $pr^t = A$, which change to P.

Then $\frac{pr^t r - pr}{r} = U$, the Annuity.

(2) Here $p = 8227,024$, $t = 4$, and $r = 1,05$; which being
involved to the 4th Power $= 1,2155063$.

Then per Theo. I. $,8227,024 \times 1,2155063 = 10000, = A$.

Now (per Rule II.) $p = 10000$, and r as before.

Then per Theo. II. $\frac{10000 \times 1,05 \times 1,05 - 10000 \times 1,05}{1,05}$
 $= \frac{11025 - 10500}{1,05} = \frac{525,00}{1,05} = 500\text{£}. \text{ the Answer.}$

86. QUESTIONS FOR EXERCISE.

(1) First $486,5\text{£}. - 94 = 392,5\text{£}. \text{ the Annuity.}$

Now the present Worth of $1\text{£}. \text{ for } 7, 14, \text{ and } 21 \text{ Yrs. at}$
 $5 \text{ per Cent. is } 5,7863737, 9,8986409, \text{ and } 12,8211527.$
(per Table V.)

Then

| | | £. s. d. | Yrs. | |
|------------|--------------------|------------|------------------|----------|
| 5,7863737 | } $\times 392,5 =$ | 2271,1507 | } $= 2271\ 3\ 0$ | |
| 9,8986409 | | 3885,21655 | | 3885 4 4 |
| 12,8211527 | | 5032,30243 | | 5032 6 0 |

(2) First $15 + 5 = 20 \text{ yrs. and } 186\text{£}. 7s\ 6d. = 186,375\text{£}.$

Then the Worth of $1\text{£}. \text{ for } 20 \text{ yrs.} = 13,903253$

And ditto for 15 yrs. $= 11,1183868$

Diff. $\underline{2,4719385}$

∴ $2,4719385 \times 186,375 = 460,70753 = 460 \text{ £. } 14\text{s. } 1\frac{1}{2}\text{d.}$
the Fine required.

(3) First $7 + 14 + 10 = 31$ Years.

Now the present Worth of 1 £. at 5 per Cent. for 31, 21, and 7 yrs. is 5,7863734, 12,8211527, and 15,5928104.

Then

$$\left. \begin{array}{r} 15,5928104 \\ 12,8211527 \\ 5,7863734 \end{array} \right\} \times 50 = \left\{ \begin{array}{r} 779,64052 \\ 641,057635 \\ 289,31867 \end{array} \right\} \text{ Present Worth of } 50 \text{ £. for } \left\{ \begin{array}{r} 31 \\ 21 \\ 7 \end{array} \right. \text{ Years.}$$

$$\begin{array}{l} \therefore 289,31867 = 289 \text{ £. } 6 \text{ s. } 4\frac{1}{2} \text{ d.} \\ \text{Also } 641,057635 - 289,31867 = 351 \text{ £. } 14 \text{ s. } 9\frac{1}{2} \text{ d.} \\ \text{And } 779,64052 - 641,057635 = 138 \text{ £. } 11 \text{ s. } 7\frac{3}{4} \text{ d.} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ Val. of } \left\{ \begin{array}{l} \text{A's.} \\ \text{B's.} \\ \text{C's.} \end{array} \right.$$

(4) By Table V. the present Worth of 300 £. per Annum for 7 Years at 4 per Cent. is $6,002547 \times 300 = 1800,61641$.

Then

$$\left. \begin{array}{r} 1800,61641 + 150 = 1950 \text{ £. } 12 \text{ s. } 4 \text{ d.} \\ 6,002547 + 250 + 400 = 1900 \text{ £. } 10 \text{ s. } 3\frac{1}{4} \text{ d.} \\ 6,002547 \times 200 + 650 = 1850 \text{ £. } 8 \text{ s. } 2\frac{1}{2} \text{ d.} \\ 1800 \text{ £. } 0 \text{ s. } 0 \text{ d.} \end{array} \right\} \text{ Value of } \left\{ \begin{array}{l} \text{A's} \\ \text{B's} \\ \text{C's} \\ \text{D's} \end{array} \right\} \text{ Offer.}$$

Hence it appears that A's offer $\left\{ \begin{array}{r} 50 \\ 100 \\ 150 \end{array} \right\}$ is better by above than $\left\{ \begin{array}{l} \text{B's.} \\ \text{C's.} \\ \text{D's.} \end{array} \right.$

(5) First $54,85 - 7,35 = 47,5 \text{ £.}$ the Annuity.

Now as the Payment is Half-yearly, we shall have $n = 475 \div 2 = 23,75$, $t = 6$, and $r = 1,05$; (see the Guide p. 223) which raised to the 6th Power $= 5,0756921$, per Table V.

∴ $5,0756291 \times 23,75 = 120,54768735 = 120 \text{ £. } 10\text{s. } 11\frac{1}{2}\text{d.}$
the Answer.

(6) First $153 - 50 = 103 \text{ £.}$ for 12 yrs. at 5 per Cent.

Then per Table VI. 1 £. will purchase for 12 yrs. at 5 per Cent. an Annuity of ,1128254 per Annum.

Then $,1128254 \times 103 = 11,6210162 = 11 \text{ £. } 12\text{s. } 5\text{d.}$ advance Rent.

∴ $16 \text{ £. } + 11 \text{ £. } 12,5 = 27 \text{ £. } 12\text{s. } 5\text{d.}$ the Answer.

(7) First $44 - 20 = 24$ £. and $1,9487171 =$ Amount of 1 £. for 7 Years. $= 250$ £. 0s. 0d.

Then $1,9487171 \times 24 = 46,9292104 = 46$ 18 7

At 10 £. per Annum for 7 yrs. $= 70$ 0 0

Answer 366 18 7

(8) First $33,3 \times 50 = 166,6,6 = 166$ £. 13s. 4d. Value of the Annuity.

Then per Table VI. 1 £. for 12 yrs. at 4 per Cent. will purchase ,1065522, per Annum.

$\therefore 166,6,6 \times ,1065522 = 177,587 = 177$ £. 11s. $8\frac{3}{4}$ d. the Answer.

(9) First $237 - 10 = 227$ £. the Annuity.

Then the present Worth for 1 £. for 4, 12, and 14 Years, is 3,5459505, 8,8632516, and 9,8986409, per Table V.

$\therefore$

| | | £. | s. | d. |
|---|--------------|----|------|-------------------|
| $\left. \begin{array}{l} 3,5459505 \\ 8,8632516 \\ 9,8986409 \end{array} \right\} \times 227 =$ | 804,9307635 | = | 804 | 18 $7\frac{1}{4}$ |
| | 2011,9581132 | = | 2011 | 19 $1\frac{3}{4}$ |
| | 2246,9914843 | = | 2246 | 19 $9\frac{3}{4}$ |

Sum = 5063 17 $6\frac{3}{4}$

Discount $4 + 12 + 14 \times 5 = 30 \times 5 = 150$ 0 0

Answer, received 4913 17 $6\frac{3}{4}$

87. SUPERFICIAL MEASURE.

EXAMPLES.

PROBLEM I.

(1) Multiply $\begin{array}{r} Ft. \\ 17 \ 7 \\ \text{by} \ 6 \end{array}$

Answer 105 6

(2) Multiply $\begin{array}{r} Ft. \\ 47 \ 8 \ 0 \\ \text{by} \ 8 \ 4 \end{array}$

$\begin{array}{r} 15 \ 10 \ 8 \\ 381 \ 4 \ 0 \\ \hline \end{array}$

Answer 397 2 8

(3)

Superficial Measure.

275

(3) Multiply $\begin{array}{r} \text{Ft. } 7 \text{ } 10 \text{ } 0 \\ \text{by } 8 \text{ } 6 \end{array}$

$\begin{array}{r} 3 \text{ } 11 \text{ } 0 \\ 62 \text{ } 8 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 66 \text{ } 7 \text{ } 0 \end{array}$

(4) Multiply $\begin{array}{r} \text{Ft. } 64 \text{ } 7 \text{ } 0 \\ \times 4 \text{ } 8 \end{array}$

$\begin{array}{r} 43 \text{ } 0 \text{ } 8 \\ 258 \text{ } 4 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 301 \text{ } 4 \text{ } 8 \end{array}$

(5) Mult. $\begin{array}{r} \text{Ft. } 12 \text{ } 8 \text{ } 9 \text{ } 0 \text{ } 0 \\ \times 9 \text{ } 6 \text{ } 7 \end{array}$

$\begin{array}{r} 7 \text{ } 5 \text{ } 1 \text{ } 3 \\ 6 \text{ } 4 \text{ } 4 \text{ } 6 \text{ } 0 \\ 114 \text{ } 6 \text{ } 9 \text{ } 0 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 121 \text{ } 6 \text{ } 6 \text{ } 7 \text{ } 3 \end{array}$

(6) Mult. $\begin{array}{r} \text{Ft. } 9 \text{ } 11 \text{ } 6 \text{ } 0 \\ \times 11 \text{ } 8 \end{array}$

$\begin{array}{r} 6 \text{ } 7 \text{ } 8 \text{ } 0 \\ 109 \text{ } 6 \text{ } 6 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 116 \text{ } 2 \text{ } 2 \text{ } 0 \end{array}$

(7) $\begin{array}{r} \text{Ft. } 6 \frac{1}{2} | 64 \text{ } 0 \text{ } 10 \text{ } 0 \\ \quad \quad \quad 14 \text{ } 9 \\ \hline 3 \frac{1}{2} | 32 \text{ } 0 \text{ } 5 \text{ } 0 \\ \quad \quad 16 \text{ } 0 \text{ } 2 \text{ } 6 \\ \quad 896 \text{ } 11 \text{ } 8 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 945 \text{ } 0 \text{ } 3 \text{ } 6 \end{array}$

(8) $\begin{array}{r} \text{Ft. } 6 \frac{1}{2} | 124 \text{ } 4 \text{ } 0 \text{ } 0 \\ 4 \frac{1}{3} | \quad \quad 42 \text{ } 0 \text{ } 9 \\ \hline \frac{1}{2} | \quad \quad 5 \text{ } 2 \text{ } 2 \\ \quad \quad 2 \text{ } 7 \text{ } 1 \\ \quad \quad 14 \text{ } 0 \text{ } 0 \\ \quad 248 \text{ } 0 \text{ } 0 \\ \quad 496 \text{ } 0 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 5229 \text{ } 9 \text{ } 3 \end{array}$

(9) $\begin{array}{r} \text{Ft. } 16 \text{ } 7 \text{ } 10 \text{ } 0 \text{ } 0 \\ \quad \quad \quad 6 \text{ } 5 \text{ } 7 \\ \hline 0 \text{ } 9 \text{ } 8 \text{ } 6 \text{ } 10 \\ 6 \text{ } 11 \text{ } 3 \text{ } 2 \text{ } 0 \\ 99 \text{ } 11 \text{ } 0 \text{ } 0 \text{ } 0 \end{array}$

Ans. $\begin{array}{r} 107 \text{ } 7 \text{ } 11 \text{ } 8 \text{ } 10 \end{array}$

(10) $\begin{array}{r} \text{Ft. } 6' \frac{1}{2} | 474 \text{ } 6 \text{ } 8 \\ 6 \frac{1}{2} | \quad \quad 186 \text{ } 7 \text{ } 4 \\ \hline 1 \frac{1}{6} | 237 \text{ } 3 \text{ } 4 \text{ } 0 \text{ } 0 \\ 4'' \frac{1}{3} | \quad \quad 39 \text{ } 6 \text{ } 6 \text{ } 8 \text{ } 0 \\ \quad \quad 13 \text{ } 2 \text{ } 2 \text{ } 2 \text{ } 8 \\ \quad \quad 93 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0 \\ 8 \frac{1}{9} | \quad \quad 10 \text{ } 4 \text{ } 0 \text{ } 0 \text{ } 0 \\ \quad \quad 2844 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0 \\ \quad 8532 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0 \end{array}$

Answer $\begin{array}{r} 88557 \text{ } 4 \text{ } 0 \text{ } 10 \text{ } 8 \end{array}$

| | Ft. | ' | '' | ''' | ''' | v | | Ft. | ' | '' | ''' | ''' | v | vi | | | | |
|--------|-----|---------------|-----|-----|-----|---|----|-----|--------|-----|---------------|-----|----|----|----|---|---|---|
| (11) | 6 | $\frac{1}{2}$ | 24 | 11 | 8 | 6 | 7 | 0 | (12) | 4 | $\frac{1}{3}$ | 46 | 6 | 8 | 4 | 0 | 0 | 0 |
| | | | | | 8 | 6 | 7 | | | | | | | | 6 | 4 | 8 | 6 |
| | | | | | | | | | | | | | | | | | | |
| | | | 1 | 2 | 6 | 9 | 10 | 1 | | | $\frac{1}{8}$ | 15 | 6 | 2 | 10 | 0 | 0 | 0 |
| | | | 12 | 5 | 10 | 3 | 3 | 6 | | | $\frac{1}{3}$ | 1 | 11 | 3 | 4 | 3 | 0 | 0 |
| | | | 199 | 9 | 8 | 4 | 8 | 0 | | | | 0 | 7 | 9 | 1 | 5 | 9 | 0 |
| | | | | | | | | | | | | 0 | 1 | 11 | 3 | 4 | 2 | 0 |
| Answer | 213 | 6 | 1 | 0 | 11 | 4 | | | 279 | 4 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | Answer | 297 | 7 | 4 | 7 | 0 | 11 | 0 | | |

PROBLEM II.

- (1) Here $AB = 14,5$, and $AC = 9,75$ Chains, per Fig. 2.
Then per Theo. $14,5 \times 9,75 = 131,375 = 131 \text{ a. } 1 \text{ r. } 20 \text{ p.}$
the Area, required.
- (2) Here $AB = AC = 245$ yds. per Fig. 1.
Then $245 \times 245 = 60025$ Acres, or yds.
 $\therefore 4840 \overline{) 60025} (12,4 = 12 \text{ A. } 1 \text{ r. } 24 \text{ p.}$
- (3) Here $AB = 64 \text{ ft. } 6 \text{ inc.}$ and $AC = 47 \text{ ft. } 8 \text{ inc.}$ see
Fig. 3.
Then per Problem I. $64 \text{ ft. } 6 \text{ inc.} \times 47 \text{ ft. } 8 \text{ inc.} = 3074 \text{ ft. } 6 \text{ inc.}$
 $\therefore 9 \overline{) 3074 \text{ ft. } 6 \text{ inc.}} (= 341 \text{ yds. } 5 \text{ ft. } 6 \text{ inc. the Answer.}$

PROBLEM III.

- (4) Here $AB=28$, and $CD=20,5$ Chains, see Fig. 4.
Then per Theo. $\frac{28 \times 20,5}{2} = 14 \times 20,5 = 287$ Acres.
- (5) Here $AB=738$, and $CD=583$ Links.
Then $\frac{738 \times 583}{2} = \frac{430254}{2} = 215127$ Links, or
2,15127 Acres.
∴ As 2,15127 : 12 £. :: 1 : 5,5781 = 5 £. 11s. 6¾d. the
Answer.
- (6) Here $\frac{a+b+c}{2} = \frac{400+348+312}{2} = \frac{1060}{2} = 530$
=s. Then per Rule.
 $530 - \left\{ \begin{array}{l} 400 = 130 = e. \\ 348 = 182 = f. \\ 312 = 218 = g. \end{array} \right\}$ the three Remainders.

$$\therefore 530 \times 130 \times 182 \times 218 = 2733676400.$$

$$\text{And } \sqrt[2]{2733676400} = 52284,5.$$

Then $4840)52284,5(10,8 = 10 \text{ acr. } 3 \text{ r. } 8 \text{ p. the Answer.}$

PROBLEM IV.

(7) Here $AC=45$, $BF=17,25$, and $DE=14$, see Fig. 5.

$$\text{Then per Theo. } \frac{17,25 \times 14}{2} \times 45 = \frac{31,25}{2} \times 45 =$$

$$15,625 \times 45 = 703,125 \text{ ft. these } \div 9 = 78 \text{ yds. } 1 \text{ ft. } 18 \text{ inc. the Answer.}$$

Or thus,

R U L E.

If the Trapezium can be inscribed in a Circle, that is, if the Sum of any two opposite Angles be equal to two right Angles, or 180° ; then multiply any two adjacent Sides together, and the other two Sides together, and multiply by the Sum of these Products, half the Sine of the Angle included by either of the Pairs of Sides which are multiplied together, so shall this last Product be the Area.

$$\text{Thus } \frac{AD \times DC + AB \times BC \times s. \angle A, \text{ or } s. C}{2} = \text{the Area}$$

E X A M P L E.

If the Sides be $AD=4$, $DC=7,5$, $AB=6$, $BC=5,5$, and the Angle $C=74^\circ 40\frac{1}{4}'$.

Therefore the Angle $A=180^\circ - 74^\circ 40\frac{1}{4}' = 105^\circ 19\frac{3}{4}'$.

$$\text{Then by Rule } \frac{4 \times 6 + 7,5 \times 5,5}{2} \times ,9644229 (s. 74^\circ$$

$$40\frac{1}{4}') = 31,46429 + \text{Square Feet, the Area.}$$

Or thus,

R U L E.

From half the Sum of the four Sides subtract each Side severally; multiply the four Remainders continually together, and the square Root of the last Product will be the Area.

$$\text{Thus } \sqrt{\frac{a+b+c-d}{2} \times \frac{a+b+d-c}{2} \times \frac{a+d+c-b}{2} \times \frac{b+c+d-a}{2}} = \text{the Area.}$$

EXAMPLE.

Suppose the Sides be $AB=a=15,6$, $BC=b=13,2$, $CD=c=10$, and $AD=d=26$, what is the Area?

Then per Rule, first $\frac{15,6+13,2+10+26}{2} = \frac{64,8}{2} =$

$32,4$.

$\therefore \sqrt{32,4 - 15,6 \times 32,4 - 13,2 \times 32,4 - 10 \times 32,4 - 26}$
 $= \sqrt{16,8 \times 19,2 \times 22,4 \times 6,4} = 21,504$, the Area re-
 quired. (See Quest. 25, in Algebra.)

PROBLEM V.

(8) Here $AB=48$, $CD=41,57$, and $n=5$.

Then per Theo. $\frac{48 \times 41,57 \times 5}{2} = \frac{9976,8}{2} = 4988,4$ ft.

the Area required.

(9) First the Multiplier for an Hexagon is 2,598076 by the Table.

Then per Rule II. thus $2,598076 \times 30 \times 30 = 2338,2684$, the Area.

(10) Here the Multiplier for an Octagon is 4,828427 by the Table.

Then $4,828427 \times 24 \times 24 = 2781,173952$, the Area.

PROBLEM VI.

(11) Thus $3,1416 \times 7 = 21,9912$, or rather 22 the Circumference required. (per Rule I.)

(12) Thus $22 \div 3,1416 = 7$ (nearly) the Diameter, (per Rule II.)

(13) Thus $3,1416 \times 8000 = 25132,8$ Miles, the Diameter required.

PROBLEM VII.

(14) Here $C=6,2832$, $D=\frac{6,2832}{3,1416} = 2$.

Then per Theo. $\frac{6,2832}{2} \times \frac{2}{2} = \frac{6,2832}{2} = 3,1416$, the

Area. (per Rule I.)

(15) First $12 \times 12 = 144$, Square of the Diameter.

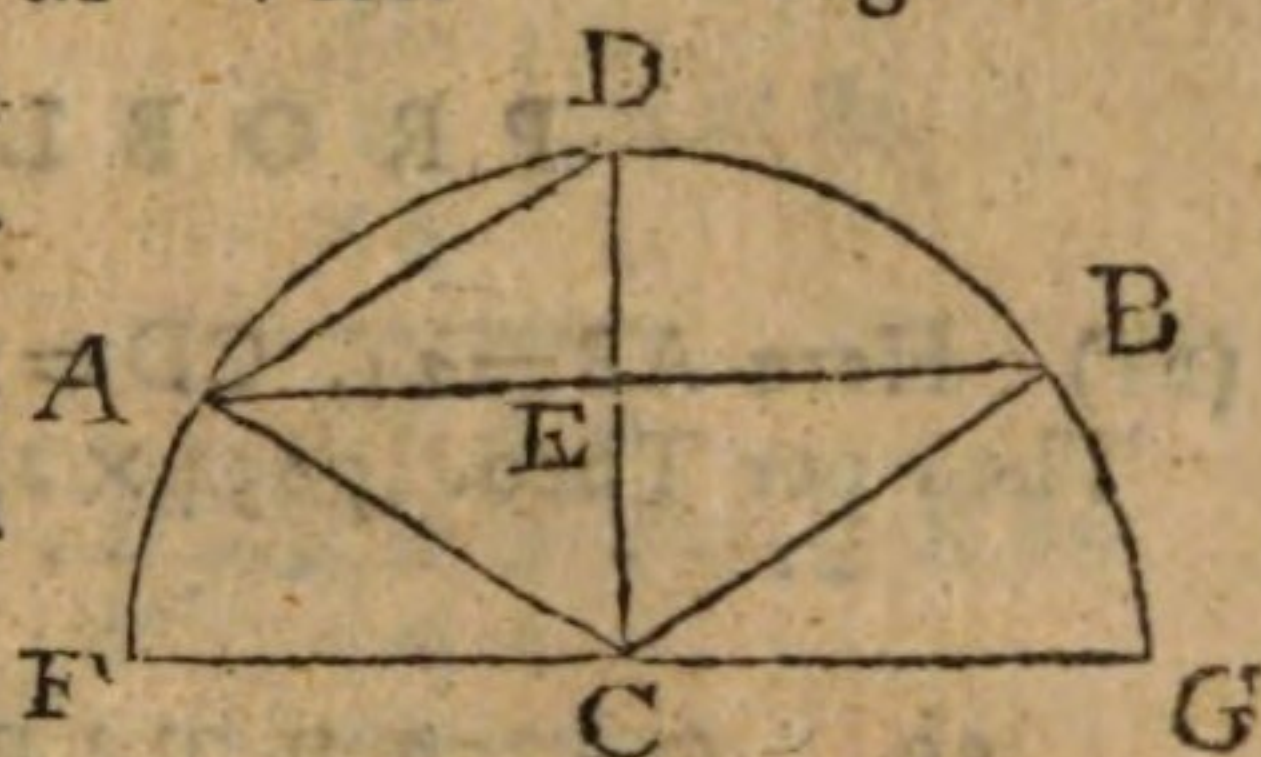
Then $,7854 \times 144 = 113,0976$, the Area. (per Rule II.)

PRO-

PROBLEM VIII.

Fig. 22.

- (16) Here $DE=9$, $ADB=29,5$, and $b=.01745329$.
Then per Theorem
 $.01745329 \times 29,5 \times 9 = 4,633848495$, the Length
of the Arch required.



PROBLEM IX.

- (17) Here $AC=55$, and $AB=59$; see Fig. 9.

Then per Theo. $55 \times \frac{59}{2} = 55 \times 29,5 = 1622,5$, the
Area required.

PROBLEM X.

- (18) First $.7854 \times 18 \times 18 = 254,4696$, the Area of the
whole Circle. (per Prob. VII.)

$$\therefore \frac{254,4696}{4} = 63,6174, \text{ Area of the Quadrant CADB ;}$$

See Fig. 22.

$$\text{And } \frac{9 \times 9}{2} = \frac{81}{2} = 40,5, \text{ Area of the Triangle ABC.}$$

(per Prob. III.)

Then from 63,6174 take 40,5, remains 23,1174, the
Area of the Segment ABD.

PROBLEM XI.

- (19) Here $AB=84$, $CD=72,5$, and $AC=3,5$; see
Fig. 10.

$$\text{Then per Theo. } \frac{84+72,5}{2} \times 3,5 = \frac{156,5}{2} \times 3,5 =$$

$$78,25 \times 3,5 = 273,875, \text{ the Area required.}$$

- (20) First, as $7 : 22 :: 24 : 75,4$, which $\div 2 = 37,7$, the
greatest Semicircle AB.

Again, as $7 : 22 :: 16 : 50,2$, which $\div 2 = 25,1$, the
lesser CD.

$$\text{Also } \frac{24-16}{2} = \frac{8}{2} = 4 = AB; \text{ see Fig. 10.}$$

$$\text{Then } \frac{37,7+25,1}{2} \times 4 = \frac{62,8}{2} \times 4 = 31,4 \times 4 = 125,6,$$

Area required.

PROBLEM XII.

(21) Here $AB=24$, $CD=18$, and $b=,7854$.

Then per Theo. $,7854 \times 24 \times 18 = 339,2928$, the Area.

88. OF ARTIFICERS WORK.

$$\begin{array}{r}
 \text{(1)} \quad \begin{array}{r} \text{Ft.} \\ 3 \text{ } 10 \text{ } 0 \\ \times 2 \quad 8 \\ \hline 2 \text{ } 6 \text{ } 8 \\ 7 \text{ } 8 \text{ } 0 \\ \hline 10 \text{ } 2 \text{ } 8 \\ \times 12 \\ \hline 122 \text{ } 8 \text{ } 0 \end{array} \\
 \hline
 \text{Content.}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r} \text{Ft.} \\ 6d \text{ } 122 \text{ } 8 \text{ } 0 \end{array} \text{ at } 8\frac{1}{2}d. \\
 \begin{array}{r} 2 \text{ } 122 \text{ } 8 \text{ } 0 \\ \frac{1}{2} \text{ } 122 \text{ } 8 \text{ } 0 \\ \hline 6' \text{ } 122 \text{ } 8 \text{ } 0 \\ 2 \text{ } 122 \text{ } 8 \text{ } 0 \\ \hline \end{array} \\
 \hline
 \text{£. } 4 \text{ } 6 \text{ } 10\frac{1}{2} \text{ Value.}
 \end{array}$$

$$\begin{array}{r}
 \text{(2)} \quad \begin{array}{r} \text{Ft.} \\ 6 \text{ } 6 \\ 5 \text{ } 3 \\ 4 \text{ } 9 \\ \hline \text{Sum } 16 \text{ } 6 \\ \times 4 \\ \hline 66 \text{ } 0 \\ 3 \text{ } 9 \\ \hline 247 \text{ } 6 \end{array} \\
 \hline
 \text{Content.}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r} \text{Ft.} \\ 4d \text{ } 247 \text{ } 6 \end{array} \text{ at } 1s. \text{ } 4d. \\
 \begin{array}{r} 82 \text{ } 4 \\ 0 \text{ } 8 \\ \hline 2,0 \text{ } 33,0 \text{ } 0 \\ \hline \end{array} \\
 \hline
 \text{£. } 16 \text{ } 10 \text{ Answer.}
 \end{array}$$

By Decimals, thus $6,5 + 5,25 + 4,75 = 16,5$, Sum of their Heights; which $\times 4$, (the Number of Windows in a Tier) $= 66$ Heights together.

Then $66 \times 3,75$ (the Breadth) $= 247,5$ the Area.

Now $16d. = ,06\text{£}$. $\therefore 247,5 \times ,06 = 16,499$, or 16£ . $10s.$ as before.

(3)

| | | | |
|-------|---|---|----------|
| Ft. | ' | " | |
| 6 | 6 | 0 | |
| × 3 3 | | | |
| <hr/> | | | |
| 1 | 7 | 6 | |
| 19 | 6 | 0 | |
| <hr/> | | | |
| 21 | 1 | 6 | Content. |
| <hr/> | | | |

| | | | |
|------------------|---|-------------------------------|--------|
| Ft. | ' | " | |
| 21 | 1 | 6 | at 8s. |
| × 4 | | | |
| <hr/> | | | |
| 1' | | 8 8 0 | |
| 6'' | | 0 0 8 = $\frac{1}{12}$ of 8s. | |
| | | 0 0 4 | |
| <hr/> | | | |
| 8 9 0 £. Answer. | | | |
| <hr/> | | | |

By Decimals, thus $3,25 \times 6,5 = 21,125$ ft.

$\therefore 21,125 \times 4 = 8,45 = 8£. 9s.$ as before.

(4) First $16 \times 2 = 32$ the Length; and $9 \times 3 = 27$ the Breadth.
Then $32 \times 27 = 864$, square Inches.

$\therefore 144)864(6$ ft. the Content required.

(5)

| | | | |
|----------|-------|-----|--|
| Ft. | ' | " | |
| 176 | 6 | 0 | |
| × 56 9 0 | | | |
| <hr/> | | | |
| 88 | 3 | 0 | |
| 44 | 1 | 6 | |
| 28 | 0 | 0 | |
| 1056 | 0 | 0 | |
| 880 | 0 | 0 | |
| <hr/> | | | |
| 9) | 10016 | 4 6 | |
| <hr/> | | | |

For the odd Parts, thus

| | | | |
|-----------------|--|-----------------|---------------|
| 3 Ft. | | $\frac{1}{3}$ | 6d. |
| | | 2 | |
| | | 2 | |
| | | $0 \frac{1}{2}$ | $\frac{2}{3}$ |
| | | $0 \frac{1}{2}$ | $\frac{2}{3}$ |
| | | $\frac{1}{8}$ | $\frac{2}{3}$ |
| <hr/> | | | |
| $5 \frac{1}{2}$ | | | |
| <hr/> | | | |

6d. $\frac{1}{40}$ 1112 8 4 6 at 6d.

| | | |
|-------------------|----|---|
| 27 | 16 | 0 |
| + $5 \frac{1}{2}$ | | |
| <hr/> | | |

27 16 $5 \frac{1}{2}$ £. the Answer.

By Decimals, thus $175,5 \times 56,75 = 10016,375$ Feet.

Then $9)10016,375(1112,93$ yds. at 6d. per yd.

$\therefore 6d. \frac{1}{40} 1112,93(27,823 = 27£. 16s. 3 \frac{1}{2}d.$ the Answer, as before.

(6)

$$\begin{array}{r} \text{Ft.} \quad 9 \quad 6 \quad 0 \\ \times 8 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \quad 4 \quad 6 \\ 76 \quad 0 \quad 0 \\ \hline 9)78 \quad 4 \quad 6 \end{array}$$

yds. 8 6 4 6 at 6s.

 $\times 3$

$$\begin{array}{r} 2 \quad 8 \quad 0 \\ 0 \quad 4 \quad 3 \\ \hline \end{array}$$

£. 2 12 3 the Answer.By Decimals, thus $9,5 \times 8,25 = 78,375$ Feet.Then $9)78,375$ (8,7083 yds. at 6s. or 3£. $\therefore 8,7083 \times 3 = 2,6125 = 2£. 12s. 3d.$ the Answer.

Ft. In.

(7) First $84 \text{ ft.} \times 9 \text{ ft. } 6 \text{ inc.} = 798 \text{ 0 Room}$ And $\left\{ \begin{array}{l} 6 \times 3,5 = 61,6 \\ 4 \times 4 = 16 \end{array} \right\} = 77 \text{ 6 Win. and Chim.}$ $9)720 \text{ 6 Diff.}$ 80 yds. 6 inc. $\therefore 80 \text{ yds. } 6 \text{ inc.} \times 2 = 160 \text{ yds. } 1 \text{ ft.}$ the Answer.

Ft. Inc.

(8) First $47 \text{ ft. } 7 \text{ inc.} - 4 \text{ ft.} = 43 \text{ 7 Breadth.}$ $\times 47 \text{ 7 Length.}$

$$\begin{array}{r} 9)2073 \quad 10 \quad 1 \end{array}$$

$$\begin{array}{r} 6d. \quad 4 \quad 0 \\ \hline 3 \text{ ft.} \quad \frac{1}{3} \\ 6 \text{ in.} \quad \frac{1}{6} \\ 4 \quad \frac{1}{9} \end{array}$$

yds. 230 3 10 1 at 6d. per yd.

$$\begin{array}{r} 5 \quad 15 \quad 0 \end{array}$$

 $2 = \frac{1}{3} \text{ of } 6d.$ $\frac{1}{4} \frac{1}{3} = \frac{1}{6} \text{ of } 2d.$ $\frac{2}{3} \div \frac{1}{9} \text{ of dit.}$ £. 5 15 $2\frac{1}{2}$ the Answer.

(9)

Superficial Measure.

283

| | | | | |
|-----|------------------------|---|--------------------|------------------------------------|
| | <i>Ft. In. Ft. In.</i> | | <i>Ft. In. Ft.</i> | |
| (9) | 129 6 × 16 3 | = | 2104 4 6 | Room. |
| | 7 × 3 9 | = | 13 1 6 | } $\frac{1}{2}$ Doors and Shutter. |
| | <u>2</u> | | | |
| | 7 3 × 4 6 | = | 32 7 6 | |

8,6 + 46 × 2 = 26 × 1,2, × 2 = 60 8 0 Cheek-board.

14 + 3,9 = 17,9 × 1,4 = 23 8 0 Door-linings.

| | | | | |
|----------------|--------------------|---|--------------------|----------|
| | <i>Ft. In. Ft.</i> | | <i>Ft. In. Ft.</i> | |
| To be Deducted | 3 9 × 3 | = | 2234 5 6 | |
| | | | 11 3 0 | Chimney. |

9) 2223 2 6 Wood-work.

| | | | |
|--------------------|------|-------------|----------------------------|
| <i>s. d.</i> | | | |
| 2 6 $\frac{1}{8}$ | yds. | 247 — 2 6'' | at 3s. 6d. |
| 1 0 $\frac{1}{20}$ | | 30 17 6 | } = 43£. 4s. 6d. the Cost. |
| | | 12 7 0 | |

| | | | | |
|------|-----------------|--|--------------------|---|
| | <i>Yds. Ft.</i> | | <i>d. Yds. Ft.</i> | |
| (10) | 34 6 | | 6 $\frac{1}{40}$ | 76 6 at 10 $\frac{1}{2}$ d. per yd. |
| | × 20 | | | |
| | <u>690 0</u> | | 3 $\frac{1}{2}$ | 1 18 |
| | 9) 690 0 | | 1 $\frac{1}{2}$ | 19 |
| | <u>76 6</u> | | ft. | 9 6 |
| | | | 3 $\frac{1}{3}$ | 0 $3\frac{1}{2} = \frac{1}{3}$ of 10 $\frac{1}{2}$ d. |
| | | | 3 $\frac{1}{3}$ | <u>3 $\frac{1}{2}$</u> |

| | | | | |
|------|----------------------|--|-----------------|-----------|
| | <i>Ft. In.</i> | | <i>£. s. d.</i> | |
| (11) | 6 $\frac{1}{2}$ 34 8 | | £. 3 7 1 | the Answ. |
| | × 14 6 | | | |
| | <u>17 4</u> | | | |
| | 485 4 | | | |
| | 9) 502 8 | | | |

$\frac{1}{5}$ 55 yds. 7 ft. 8 inc. the whole Area.

| | | | |
|-----------|--------------------------------------|---------------------|-----------------|
| | <i>£. s. d.</i> | | <i>£. s. d.</i> |
| Plastered | 44 6 $1\frac{3}{5}$ at 8d. per yd. = | 1 9 9 $\frac{1}{4}$ | |
| Whited | 66 9 $2\frac{1}{5}$ at 2d. per yd. = | 11 4 $\frac{1}{2}$ | |
| Answer | £. 2 1 1 $\frac{3}{4}$ | | |

(12)

(12)

| | Ft. | In. | |
|----|---------------|------|---------------|
| 6' | $\frac{1}{2}$ | 120 | 6 |
| | | | 12 9 |
| 3 | $\frac{1}{2}$ | 60 | 3 0 |
| | | | 31 1 6 |
| | | 1446 | 0 0 |
| | | | Sq. Ft. In. " |

1,00) 15,36 4 6 (15,36 4 6 the Answer.

(13) First $28 \times 20 = 560$
 And $14 \times 10 \times 2 = 280$ } = 280 the Diff.

1,00) 840 Sum.

8,4 sq. at 2£. 5s. or 2,25£.

$\therefore 2,25 \times 8,4 = 18,9 = 18£. 18s.$ the Answer.

(14) $\left. \begin{array}{l} 30,5 \times 20,5 \\ 5,25 \times 6 \\ 4,25 \times 4 \\ 8,5 \times 10 \end{array} \right\} \times 4 = \left\{ \begin{array}{l} 2501 \text{ Area of the 4 Floors.} \\ 126 = \text{dit. of 4 Fire-places.} \\ 68 = \text{dit. of ditto.} \\ 340 = \text{dit. of a Well-hole.} \end{array} \right.$

The whole Deductions = 534

1,00) 19,67 sq. at 8,5£.

$\therefore 19,67 \times 8,5 = 167,195 = 167£. 3s. 10\frac{1}{2}d.$ the Answer.

(15) First $1,25 \times 15 = 18,75$ Area of one Plank.

And $33,5 \times 60,5 = 2026,75$ ditto of the Room.

$\therefore 18,75) 2026,75 (108$ Planks, (nearly.)

(16) First $36 + \frac{36}{2} = 36 + 18 = 54$ Width of the Roof.

Then $64 \times 54 = 34,56$ Feet, the Area.

And 1,00) 34,56 (34,56 sqrs. at 12s. 6d. or ,625£.

Or $34,56 \times ,625 = 21,6 = 21£. 12s.$ the Answer.

Superficial Measure.

285

(17) First $30 \times 2 = 60$, and $70 \times 60 = 4200$ ft. the Area.
Then $1,00)42,00(42$ sqr. at 10s. 8d. per sqr.

$$\begin{array}{r} \text{s. d.} \\ 10 \ 8 \\ \times 7 \times 6 = 42. \\ \hline 3 \ 14 \ 8 \\ 6 \\ \hline \text{Answer } \pounds. \ 22 \ 8 \ 0 \end{array}$$

(18) $\begin{array}{l} \text{In. Ft.} \\ 10 = ,83 \times 2 = \end{array}$ $\begin{array}{l} 22,75 \\ 11,375 \end{array}$ } Flat and Half.
 $1,666 =$ Eves Boards.

$$\begin{array}{r} 35,7916 \text{ Whole Width.} \\ \times 32,75 \text{ Length.} \\ \hline \end{array}$$

$$1,00)1172,177081(11,72177081 \text{ sqr. at } ,75 \pounds.$$

$\therefore 11,72177081 \times ,75 = 8,7913281075 = 8 \pounds. \ 15 \text{ s. } 10 \text{ d.}$
(nearly) the Answer.

(19) First $470 \text{ ft. } \times 9 \text{ ft. } 6 \text{ inc.} = 4465 \text{ ft.}$

And 4465×6 half Bricks $= 26790$.

Then per Rule I. $816)26790(32$ rds. $229\frac{1}{3}$ ft. the Answer.

(20) First $840 \times 9 = 7560$ Feet.

$\times 5$ Half Bricks thick.

Per Rule II. $3)37800$ Rds. Ft.

$$272)12600(46 \ 88$$

Rds. Ft.

$46 \ 88$ at $4 \pounds. \ 19 \text{ s. } 6 \text{ d.}$ per Rod.

$\times 4$

$$\begin{array}{r} \pounds. \ \text{s.} \\ 46 \text{ --- } 1 \ 3 = \\ 68 \left| \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{array} \right| \\ 17 \left| \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{array} \right| \\ 3 \left| \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{array} \right| \end{array} \quad \begin{array}{l} 184 \ 0 \text{ at } 4 \pounds. \\ 44 \ 17 \text{ at } 19 \text{ s. } 6 \text{ d.} \\ 1 \ 4 \ 10\frac{1}{2} = \frac{1}{4} \text{ of } 4 \pounds. \ 19 \text{ s. } 6 \text{ d.} \\ 6 \ 2\frac{1}{2} \\ 1 \ 0\frac{1}{2} \text{ nearly.} \end{array}$$

$\pounds. \ 230 \ 9 \ 1\frac{1}{2} \pounds.$ the Answer.

(21)

(21) First $38 \div 4 = 9,5$ Height of the Gable.

And $40 \div 3 = 13,3$

$$24,5 \times 13,3 \times \left\{ \begin{array}{l} 4 = 1306,666 \text{ at } 2 \\ 3 = 980 \text{ at } 1\frac{1}{2} \\ 2 = 653,333 \text{ at } 1 \end{array} \right\} \text{ Bricks thick.}$$

$$24,5 \times 13,3 \div 2 = 166,375 \text{ at } \frac{1}{2}$$

816) 3106,375 (3,8068 Rods.

$\therefore 3,8068 \times 5,5 = 20,6 = 20\text{f. } 18\text{s. } 9\text{d.}$ nearly, the Answer.

QUESTIONS.

(1) First 14 ft. 3 inc. = 171 inc. and 3 square yds. = $36 \times 36 = 1296$ Inches.

$\therefore 171) 1296 (7\frac{1}{19}$ Inches, the Answer.

(2) First a Foot square = 144 Inches.

Then $144 \div 27 = 5,3$ Inches, Breadth of the first Cut.

And $58 - 5,3 = 52,6$ remaining Length.

Also $144 \div 52,6 = 2,734$ Breadth of 2d Cut.

Then $27 - 2,734 = 24,266$ remaining Breadth.

$144 \div 24,266 = 5,934$ Breadth of 3d Cut.

$52,6 - 5,934 = 46,732$ remaining Length.

$144 \div 46,732 = 3,0814$ Breadth of the 4th Cut.

$24,266 - 3,0814 = 21,1846$ remaining Breadth.

$144 \div 21,1846 = 6,7974$ Breadth of the 5th Cut.

$46,732 - 6,7974 = 39,9346$ remaining Length.

$144 \div 39,9346 = 3,6059$ Breadth of the 6th Cut.

$21,1846 - 3,6059 = 17,5787$, remaining Breadth.

$144 \div 17,5787 = 8,1917$ Breadth of the 7th Section.

$39,9346 - 8,1917 = 31,7429$ remaining Length.

$144 \div 31,7429 = 4,5364$ Breadth of the 8th Cut

$17,5787 - 4,5364 = 13,0423$ remaining Breadth.

$144 \div 13,0423 = 11,0411$ Breadth of the 9th Section.

Then $31,7429 - 11,0411 = 20,7018$ remaining Length.

Also $144 \div 20,7018 = 6,956$ Breadth of the 10th Section.

$\therefore 13,0423 - 6,956 = 6,0863$ Breadth remaining at the last. Q. E. F.

(3) First $10584 \div 6 = 1764$.

Then $\sqrt{1764} = 42$ long Rows.

$\therefore 42 \times 6 = 252$ short Rows, 42 in a Row.

(4) First $7 \times 2,5 = 17,5$ Inches, Area of an End.
And $17,5 \times 2 = 35$, double Area.

$\therefore 35 \div 3 = 11\frac{2}{3}$ Inches deep, the Answer.

(5) First $19 \times 11 = 209$, Area of an End.

Then $209 \div 4 = 52,25$ Area of an End of the Piece wanting.

$\therefore 52,25 \div 9 = 5,805$ Width required.

(6) First a yd. $= 36 \times 36 = 1296$ Inches. And $3s. 2d. = 38d.$
Then, as $6d. : 1296 \text{ inc.} :: 38d. : 8208 \text{ inc.}$ Area of the whole Trough.

Also $102 \times 21 \times 2 = \underline{4284}$ Area of the two Sides.

$\underline{3924}$ Area of the Bot. and Ends.

Then $102 \div 21 \times 2 = 144$.

$\therefore 3924 \div 144 = 27\frac{1}{4}$ Inches, the Breadth required.

(7) First $26 \div 40 \times 2 = 106$ Breadth of the Bot. and Sides.

Then $106 \times 74 = 7844$ Area of Bottom and Sides.

And $40 \times 26 \times 2 = 2080$ ditto of both Ends.

Also $26 \times 16 \times 3 = 1248$ ditto of the Stays.

$\therefore$ $114) 11172 (77,583 \text{ sq. Feet.}$

$28\frac{1}{4} \text{ lb.}) 77,583 (19,39583 \text{ cwt. at } 1,1\text{£.}$

$\therefore 19,39583 \times 1,1 = 21,335416 = 21\text{£. } 6s. 8\frac{1}{2}d.$ Value of the Cistern.

Now $3s 6d. = ,175\text{£.}$ Therefore $21,335416 \div ,175 = 21,60416$ Mason's Bill.

$7d. = ,02916) 21,160216 (725,5 \text{ sqr. Feet, Shop.}$

$22 \text{ ft. } 10 \text{ inc.} = 22,83) 725,5 (31,778 = 31 \text{ ft. } 9\frac{1}{4} \text{ Inches,}$
the Answer.

(8) First $27 \text{ ft. } 112 \text{ inc.} = 4000 \text{ sq. Inches.}$

Then $20 \times 16 \times 2 = \underline{640}$ Area of the two Ends.

$\therefore \underline{20 \times 2 \div 16} = \underline{56) 3360}$ ditto of Bot. and Sides.

$12) 60 \text{ Inches.}$

Answer 5 Feet.

$Yds. \text{ —}$

(9) First $26\frac{1}{2} \text{ ft.} = 5,5$

Then $5,5 \times 5,5 = 30,25$

And $6 \times 6 = 36$

Also $7 \times 7 = 49$

$\left. \begin{array}{l} 30,25 \\ 36 \\ 49 \end{array} \right\} 1 \text{ Per. } \left\{ \begin{array}{l} \text{Statue} \\ \text{Cheshire} \\ \text{Yorkshire} \end{array} \right\} \text{Measure}$

$\therefore \text{As}$

∴ As 30,25 : 110 :: 36 : 92 1 28 Chesh. } Measure,
 30,25 : 110 :: 49 : 67 3 25 Yorksh. } the Anf.
 (10) First $7 \times 4 = 28$ sqr. Feet, Area that each Plant takes.

∴ $3584 \times 28 = 100352$ sqr. Feet, in all.

Square Feet in an Acre = 43560) 100352 (2,303764 =
 2 acr. 1 rd. $8\frac{1}{2}$ pol. the Answer.

(11) First $18d. = ,075$) 100 (133,3 Feet, the Area.

Now suppose Fig. 4, in the Guide, to represent the Court.

Then $1333,3 \div \frac{88}{2} = 1333,3 \div 44 = 30,03 = C D$, the

Perpendicular.

Then $44 \times 44 = 1936$ Square of A D, or B D.

Also $30,03 \times 30,03 = 918,27$ Square of D C.

∴ $\sqrt[2]{2854,27} = 53,425 = A C$, or B C.

And $53,425 \times 2 = 106,85$ the Sum required.

(12) Here the last mentioned Fig. may represent the form of the Bath.

Then $3)125(41,6 = A B$, or A C, or B C.

And $2)41,6(20,83 = A D$, or B D.

Then $41,6 \times 41,6 = 1736,112$

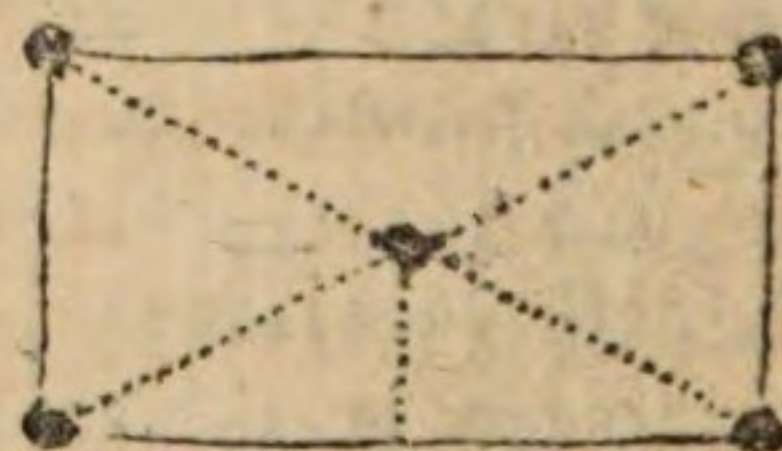
Also $20,83 \times 20,83 = 434,028$

$\sqrt[2]{1302,0832} = 36,084 = D C$.

∴ $\frac{36,084}{2} \times 41,6 = 751,75 = 75\frac{3}{4}$ the Area per Prob. III.

(13) Square Order. Quincunx Order.

Fig. 23.



First $6 \times 6 = 36$ sqr. Feet each in the Square Order.

And $3 \times 3 = 9$

$\sqrt[2]{27} = 5,19615$.

Then $5,19615 \times 6 = 31,1769$ sqr. Feet each Plant in the Quincunx Order.

Now an Acre = 43560 Feet.

∴ $43560 \times 10 = 435600$ sqr. Feet in 10 Acres.

There.

Plants.

Therefore $31,1769)435600(13972$ Quincunx. } Order.
 $36)435600(12100$ Square.

Answer 1872 Difference.

(14) First $10 + 2\frac{1}{2} + 1 = 13\frac{1}{2} = 2\frac{7}{2}$ whole Breadth.

Then $2\frac{7}{2} \times \frac{2}{9} = 3$ Feet, Rise of the Roof.

And $13,5 \div 2 = 6,75$ Half the Width.

Also $6,75 \times 6,75 = 45,5625$

$$3 \times 3 = 9$$

$$\sqrt[2]{54,5625} = 7,38664 \text{ Length.}$$

Then $13,5 \times 2 \times 7,38664 = 199,4393$ Feet, the Area, at $3\frac{1}{2}d.$ or $,0146\text{ } \mathcal{L}.$

$\therefore 199,4393 \times ,0146 = 2,9251 = 2\text{ } \mathcal{L}.$ 18s. 6d. Answer.

(15) See Example XXV. Sect. LIV.

(16) First, as $4,5 : 8 :: 9$ A to 16 B.

And $4 : 8,5 :: 16 : 34 = A's + C's.$

Then $34 - 9 = 25 = C's.$

$\therefore 9 + 16 + 25 = 50 = A's + B's + C's$, the Sum of the Sides.

Now $\sqrt[2]{9} = 3$ A's, $\sqrt[2]{16} = 4$ B's, and $\sqrt[2]{25} = 5$ C's, Area.

Also $\frac{3+4+5}{2} = \frac{12}{2} = 6$ Poles, the Area of Triangle.

Therefore $272,25 \times 144 = 39204$ Inches in a Pole.
 $\times 6$ Poles.

295224 Inches in 6 Poles.

6 Feet = $\times 72$ Inches, the Depth.

A gal. = 231 inc.) 16936128 (73316,57 galls.

$\therefore 63)73316,57(1163,755 = 1163$ Hhds. $47\frac{1}{2}$ galls. the Answer.

(17) Here suppose Fig. 5, in the Guide, to represent the Orchard; which contains 3,75 Acres.

Then AC = the Diagonal; and $430 + 360 = 790 = BF + DE$, the Sum of the Perpendiculars; half of which is 395 Links, and 3,75 Acres = 3,75000 Links.

$\therefore 395)3,75000(949\frac{2}{5}$ Links = AC, the Diagonal.

(18) First $3,1416 \times 16 = 50,2656$ Poles, the Circumference, (per Prob. VI. Rule I.)

Then $\frac{50,2656}{2} \times \frac{16}{2} = 25,1328 \times 8 = 201,0624$ Poles, the Area, (per Prob. VII.)

$\therefore 160)201,0624(1$ Acre, 41 Poles, the Answer.

(19) First $16,5 \div 2 = 8,25$ one Round.

$\therefore 3,1416)8,2500(2,626$ ft. $= 2$ ft. $7\frac{1}{2}$ Inches, the Answer.

(20) First $3,1416)130,0000(41,38$ ft. the Diameter.

Then $\frac{41,38}{2} \times \frac{130}{2} = 20,68 \times 65 = 1344,85$.

9)1344,85(149,428 yds. at 4d. or ,016.

$\therefore 149,428 \times ,016 = 2,490466 = 2$ f. 9s. $9\frac{1}{2}$ d. ,8, Answer.

(21) First $4840 \div 2 = 2420$ yds. half an Acre.

Then ,7854)2420(3081,23 yds. Square of the Diameter, (per Prob. VII. Rule II.)

$\therefore \sqrt{3081,23} = 55,5$ Diameter.

Therefore $55,5 \div 2 = 27,75 = 27\frac{3}{4}$ yds. the Answer.

(22) First $42 + 14,5 \times 2 = 71$, greater Diameter.

Then $71 \times 71 = 5041$; also $5041 \times ,7854 = 3959,2014$

And $42 \times 42 = 1764$; also $1764 \times ,7854 = 1385,4436$

Diff. of their Areas, are 2573,7578

Then 144)2573,7578(17,8733 ft. at 8d. or 6s.

$\therefore 17,8733 \times 6 = 11,9155 = 11$ s. 11d. (nearly) the Answ.

(23) First, as $3,25 : 5 :: 60 : 92,307$ Circumference.

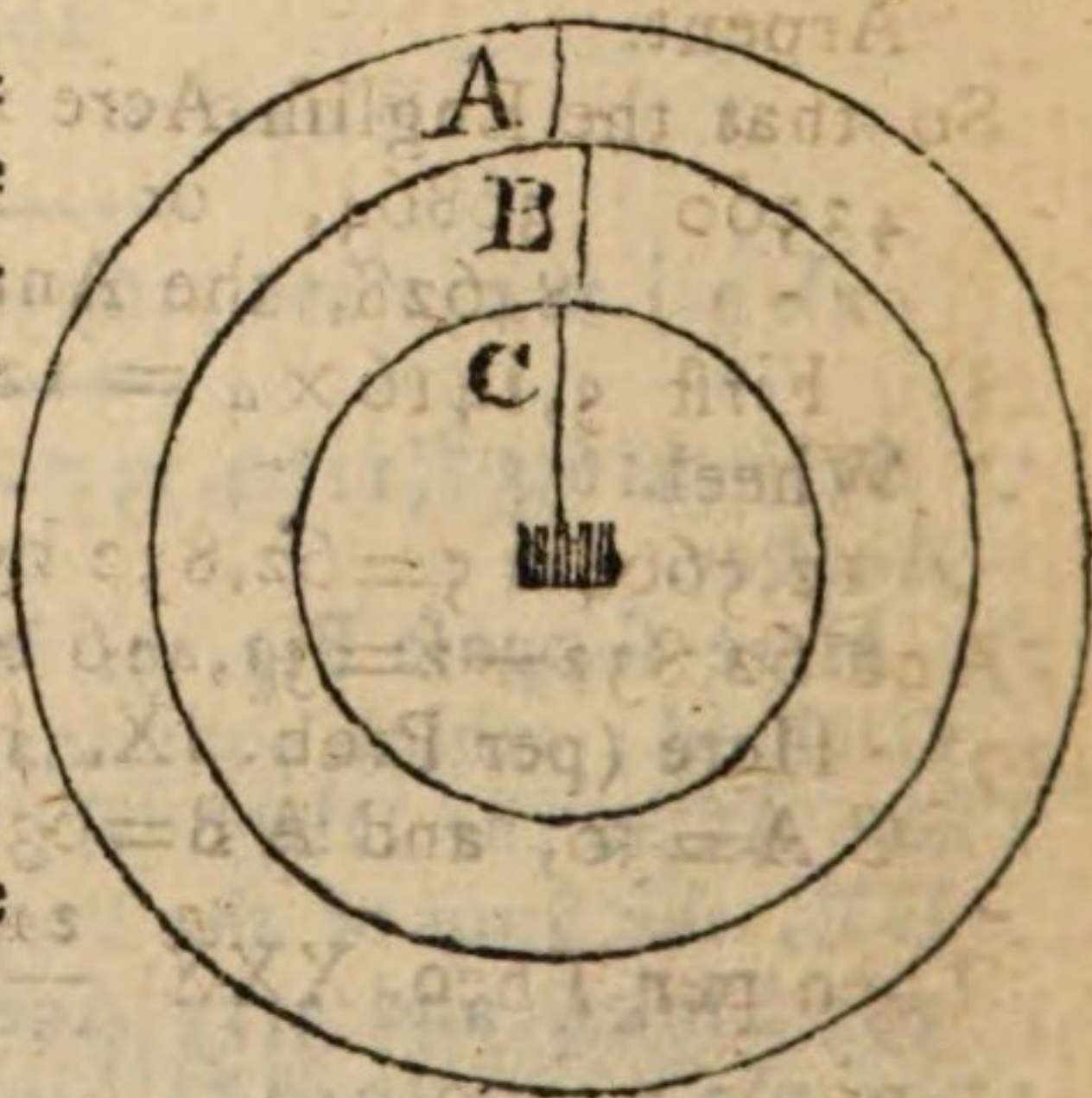
$\therefore 92,307 \div 3,1416 = 29,38$ Diameter.

And $29,38 \div 2 = 14,69$ Inches, the Answer. Fig. 24.

(24) First $36 \times 36 \times ,7854 = 1017,8784$. Area of the whole Stone and Spindle-Hole. (per Prob. VII. Rule II.)

And $5 \times 5 \times ,7854 = 39,27$, Area of the Circle circumscribing the Spindle-Hole.

$\therefore 1017,8784 - 39,27 = 978,6084$, Area of the whole Stone.



There-

Therefore,

$$\text{As } 20s. : 978,6084 :: \left\{ \begin{array}{l} 7 : 324,5129 \text{ A's} \\ 8 : 391,4433 \text{ B's} \\ 5 : 244,6521 \text{ C's} \end{array} \right\} \text{Area.}$$

Then $244,6521 + 39,27 = 283,9221$, this $\div ,7854 = 361,5$, whose Square Root $= 19,03$ inc. the Diameter, where C begins to grind.

And $283,9221 + 391,4433 = 675,3654$, this $\div ,7854 = 859,9$, whose Square Root $= 29,324$ inc. the Diameter, where B begins to grind.

(25) First $10 \times 10 = 100$; also $100 \times 2 = 200$, Area of the inside Circle.

Then $\sqrt[2]{200} = 14,142135$ Side of the inscribed Square.

Also $14,142135 \times 14,142135 = 200$, its Area.

Again $20 \times 20 = 400$ Area of the circumscribed Square.

Lastly $,7854 \times 400 = 314,16$ Area of the circular Section.

Hence $314,16 - 200 = 114,16$ Inches, too little.

And $400 - 314,16 = 85,84$ ditto, too much.

(26) First $2s. 4d. = ,116\text{£} : 1\text{ft.} :: 10\text{£} : 85,7143\text{ ft.}$ Area of the Semi-Circle, which $\times 2 = 171,4286$, Area of the whole Circle.

Then $,7854) 171,4286 (218,269$ Square of the Diameter, (per Prob. VII. Rule II.)

$\therefore \sqrt[2]{218,269} = 14,7739 = 14\text{ ft. } 9\frac{1}{4}\text{ inc.}$ the Answer.

(27) First $,18 \times 18 \times 100 = 32400$ French Feet, the Arpent.

And $16,5 \times 16,5 \times 160 = 43560$ English Feet, in an Arp.

Then $16 \times 16 = 256$; also $15 \times 15 = 225$.

$\therefore$ Recipe $256 : 32400 :: 225 : 36864$ English Feet, in an Arpent.

So that the English Acre is to the Arpent of France, as $43560 : 36864$, or $: 605 : 512$, nearly $13 : 11$, or $: 1 : ,84628$, the Answer.

(28) First $3,1416 \times 4 = 12,5664$ Circumference of the Wheel.

$\therefore 12,5664 \times 5 = 62,832$ by the greater.

And $62,832 \div 2 = 31,416$ by the less.

(29) Here (per Prob. IX. Fig. 9. in the Guide) is given $CA = 30$, and $AB = 63\text{ ft.} = 21\text{ yds.}$

Then per Theo. XXX. $\frac{21}{2} = 30 \times 10,5 = 315\text{ yds.}$ Answ.

(30) First 15 Inches = ,416 Yards.

Then $21 - ,416 = 20,583 = AB$, (see Fig. 22.)

And $20,583 \div 2 = 10,2915 = AE$.

Also $30 \times 30 = 900 = {}^2AC$.

$10,2915 \times 10,2915 = 105,9184 = {}^2AE$.

Diff. $794,0814 = {}^2CE$.

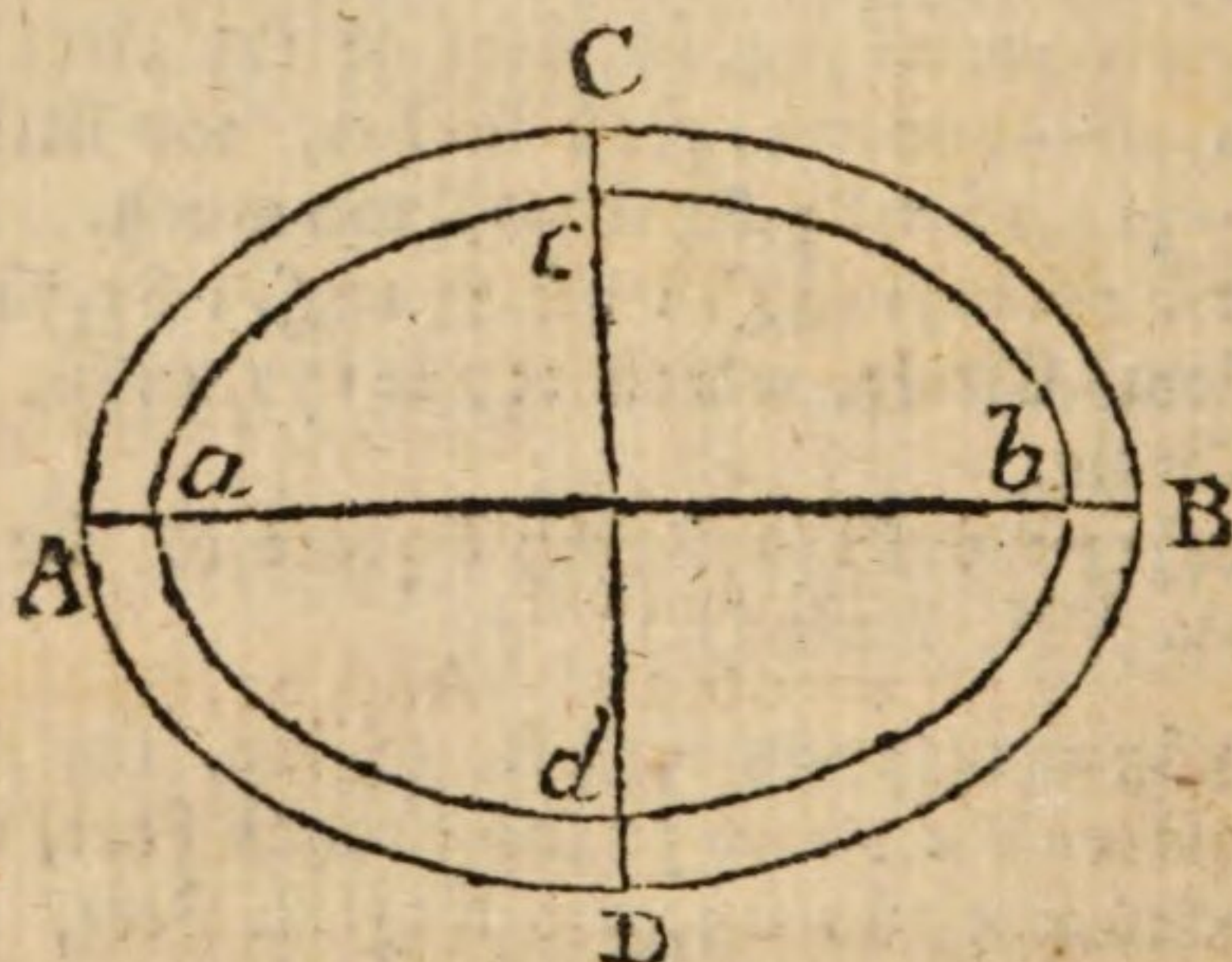
$\therefore \sqrt[2]{794,0814} = 28,18 = CE$, the Perpendicular.

Then per Prob. X. $30 \times \frac{21}{2} = 315$, Area of the Sector
ABCD.

And $10,2915 \times 28,18 = 290,01916$, Area of the Triangle
ABC.

$\therefore 315 - 290,01916 = 24,98083$, Area of the Segment
ABD.

Fig. 25.



(31) First $14 \times 2 = 28$ Inches, or 2,3 Feet.

Lin. Ft. Lin. Ft.

Then, as $100 : 66 :: 840 : 554,4 = ab$.

$100 : 66 :: 612 : 403,92 = cd$.

$\therefore 544,4 + 2,3 = 556,73 = AB$.

$403,92 + 2,3 = 406,253 = CD$; then per Prob. XXVI.

$556,73 \times 406,253 \times ,7854 = 177637,66 = \text{Area of } ABCD$.

$554,4 \times 403,92 \times ,7854 = 175877,17 = \text{ditto } abcd$.

Covered by the Wall $1760,49$ Diff.

Now an Acre = 4840 yds. $\times 9 = 43560$ ft.

$\therefore 43560 \overline{) 1760,49} (4,0375 = 4 \text{ Acres, } 6 \text{ Perches, the Answer, (see Prob. XII.)}$

(32) First $4 \times 10 = 40$ Feet, and $7 \times 7 = 49$ Square Side.

Then, as $4 : 49 :: 40 : 490$ Square of the Diameter.

$\therefore \sqrt[2]{490} = 22,136$ Inches, the Diameter required.

(33) First $6 \times 6 = 36$. And $3 \times 3 \times 4 \times 2 = 72$.

Then, as $36 : 3 :: 72 : 6$ Hours, the Answer.

(34) First $9 \times 9 = 81$. And $12 \times 12 \times 2 = 288$.

Then, as $81 : 22 \text{ lb.} :: 288 : 78\frac{2}{7} \text{ lb.}$ the Answer.

(35) First $1,25 \times 1,25 \times 20 = 31,25$ Feet.

And $\frac{7}{8} = ,875$; $\times ,875 \times 50 = 38,28125$ Feet.

$\therefore$ as $31,25 : \frac{1}{2} \text{ Ton, or } 1120 \text{ lb.} :: 38,28125 : 1372 \text{ lb.}$
at $3\frac{1}{2} d.$ per lb.

| | | |
|---------------|----------------|----------------------|
| $d.$ | $lb.$ | |
| 3 | $\frac{1}{80}$ | 1372 |
| | | |
| $\frac{1}{2}$ | $\frac{1}{6}$ | $17 \quad 3$ |
| | | $2 \quad 17 \quad 2$ |
| | | |

Answer £. 20 0 2

(36) First $16 \times 2 = 32$ Length. And $9 \times 3 = 27$ Breadth.

Then $32 \times 27 = 864$ Square Inches.

$\therefore 144)864(6$ Feet, the Answer.

(37) First $22,5 \times 22,5 = 506,25$. And $22,5 \times 2 = 45$.

Also $45 \times 45 = 2025$.

Then, as $506,25 : 3 \text{ bush.} :: 2025 : 12 \text{ bush.}$ the Answer.

(38) First $64,3 \times 64,3 = 9259,2$ Square Inches.

And $\frac{3}{8} = ,375$; $\times 9259,2 = 3472,2$ Solid Inches.

Then $3472,2 \times 4,36 = 15151,418$ Ounces, which $\div 1792$
(oz. in 1 Cwt.) $= 8,455$ Cwt.

$\therefore$ as $19,5 : 21 \text{ £.} :: 8,455 : 9,10538 = 9 \text{ £. } 2s. 1\frac{1}{2}d.$ the Answer.

89. MEASUREMENT OF SOLIDS.

PROBLEM XIII.

(1) Thus $2,5 \times 2,5 \times 2,5 = 15,625$ Feet, the Solidity.

(2) First $57 \times 42 \times 34 = 81396$ cubic Inches.

A gall. of Ale $= 282)81396(288,638$ gall. the Answer.

(3) First $45 \times 45 \times ,7854 = 15,90435$ Area of the Base,
(per. Prob. VII. Rule II.)

Then $15,90435 \times 8 = 127,2248$ Feet, the Content req.

PROBLEM XIV.

- (4) Thus $10,5 \times 7,75 \times = 81,375$, the Surface required.

PROBLEM XV.

- (5) First $12 \times 12 = 144$ Area of the Base.

Then $144 \times \frac{25}{3} = 144 \times 8,3 = 1200$ Feet, the Answer.

- (6) Now (per Prob. III. Rule III.) $5,5 \times 3 = 16,5$, Sum of the three Sides, half of which is $= 8,25$.

Then $8,25 - 5,5 = 2,75$ Difference.

And $8,25 \times 2,75 \times 2,75 \times 2,75 = 171,57421875$.

Then $\sqrt{171,57421875} = 130,986 =$ Area of the Base.

$\therefore 130,986 \times \frac{30}{3} = 130,986 \times 10 = 1309,86$, the solid Content required.

- (7) First $3,5 \times 3,5 \times ,7854 = 96,2115$ Area of the Base, (per Prob. VII. Rule II.)

Then $96,2115 \times \frac{6}{3} = 96,2115 \times 2 = 192,423$, the Solidity required.

PROBLEM XVI.

- (8) Thus $\frac{3,25 \times 3,25 \times 20}{2} = \frac{21125}{2} = 105,625$, the Surface required.

- (9) Thus $\frac{45 \times 20}{2} = \frac{900}{2} = 450$ Feet, the convex Surface required.

PROBLEM XVII.

- (10) First $15 \times 15 = 225 = A$. And $6 \times 6 = 36 = a$. Also 24.

Then per Theo. $225 \times 36 = 8100$.

And $\sqrt{8100} = 90$ the mean Prop.

$\therefore 225 + 36 + 90 \times \frac{24}{3} = 351 \times 8 = 2808$ Inches.

Therefore $144)2808 (= 19,5$ Feet, the Solidity.

Or thus, to the Squares of the $\left. \begin{array}{l} \text{Diameters} \\ \text{Circumference} \end{array} \right\}$ of the Ends, add their Product: multiply by the Altitude of the Frustum, and the Product which arises by $\left. \begin{array}{l} ,2618 \\ ,0265 \end{array} \right\}$ and this last Product will be the Content. (11)

(11) Now by the preceding Rule $20 \times 20 = 400$.
 And $3 \times 3 = 9$. Also $20 \times 3 = 60$, which are the Squares
 of the Diameters and the Product.
 Then $400 + 9 + 60 = 469$ their Sum.
 $\therefore 469 \times 60 = 28140$, which $\times 2618 = 7366,052$, the
 Content.

(12) First $66 \times 66 = 4356$. And $56 \times 56 = 3136$.
 Also $66 \times 56 = 3696$, which are the Square of the Cir-
 cumferences and their Product.
 Then $4356 + 3136 + 3696 = 11188$, their Sum.
 $\therefore 11188 \times 4 \times ,0265 = 1185,928$ the Content.

PROBLEM XVIII.

(13) First 3 ft. 4 inc. + 2 ft. 2 inc. $\times 4 = 40$ inc. + $26 \times 4 =$
 $66 \times 4 = 264$ Inches, Sum of the Perimeters.

Then $\frac{264 \times 10}{2} = \frac{2640}{2} = 1320$ Inches.

$\therefore 12) 1320 (= 110$ Feet, the Surface.

(14) First $32 + 8 = 40$, Sum of the Circumference.

Then $\frac{40 \times 7}{2} = \frac{280}{2} = 140$ Feet, the Content.

(15) First, as $30 : 6 :: 10 : 2$, Circumference of the Top
 of the Fruustum.

And $30 - 6 = 24$ the Slant Height.

Then $\frac{10 + 2 \times 24}{2} = \frac{288}{2} = 144$ Feet, the Surface req.

PROBLEM XIX.

(17) First $30 \times 16 \times \frac{12}{2} = 480 \times 6 = 2880$ ft. the Content.

PROBLEM XX.

(18) First $16 + 36 \times 2 = 16 + 72 = 88$, Sum.

Then $88 \times 20 = 1760$.

Also $1760 \times \frac{12}{6} = 1760 \times 2 = 3520$ cubic Feet.

PROBLEM XXI.

(19) First $16 \times 16 = 256$. And $13 \times 13 = 169$.

Also $16 \times 13 = 208$, which are the Areas of the Ends, and
 the Product of their Sides.

Then

Then $256 + 169 + 208 = 633$ Sum.

$\therefore 633 \times \frac{60}{3} = 633 \times 20 = 12660$, the Content required.

PROBLEM XXII.

(20) First $304 \times 20 = 6080$. And $300 \times 16 = 4800$.

Also $304 + 300 \times 20 + 16 = 604 \times 36 = 21744$.

Now $6080 + 4800 + 21744 = 32644$, their Sum.

Then $32644 \times \frac{5}{6} = 32644 \times .83 = 27186$, the Content required.

PROBLEM XXIII.

(21) First (per Prob. VI.) $3,1416 \times 7 = 21,9912$ the Circumference.

Then $21,9912 \times 7 = 153,9384$, the Surface required.

(22) Thus $3,1416 \times 12 \times 12 = 452,3904$, the Surface.

(23) First $7957,75 \times 3,1416 = 25000$ Miles, (nearly) the Circumference, (per Prob. VI. Rule I.)

Then $7857,75 \times 25000 = 198943750$ square Miles, the whole Surface required.

(24) First $3,1416 \times 42 = 113,0976$ the Circumference.

Then $113,0976 \times 9 = 1017,8784$, the Surface required.

PROBLEM XXIV.

(25) Thus $2,7 \times 7 \times 7 \times ,5236 = 179,5948$, the Solidity required, (per Rule II.)

(26) First, the Surface is 198943750 Miles, (per Prob. XXIII. See Ex. 30.)

Then $19843750 \times \frac{7957,75}{6} = 263857437760$ Miles, the

Solidity required, (per Rule I)

By Rule II. thus $7957,75^3 \times ,5236 = 263857624944$, the Solidity by this Rule.

The Difference arises by taking the Number ,5236 a little too great.

PROBLEM XXV.

(27) First $18 \times 3 - 4 \times 2 = 54 - 8 = 46$.

Then $46 \times 4 \times 4 \times ,5236 = 385,3696$, the Content.

PROBLEM XXVI.

(28) For an Oblate, thus

$$,5235 \times 33 \times 55 \times 55 = 52258,3875 \text{ solid Inches.}$$

Then $1728)52258,3875(30,242 +$ solid Feet.

For an Oblong, thus $,5235 \times 55 \times 33 \times 33 = 31355,0325$ solid Inches.

$$\therefore 1728)31355,0325(=18,1452 \text{ solid Feet.}$$

PROBLEM XXVII.

(29) First $,41888 \times 36 \times 36 \times 99 = 53743,97952$ solid Inc.

Then $1728)53743,97952(31,10184$ Feet, the Content required.

By Rule II. Thus $,7854 \times 36 \times 36 = 1017,8784$, Area of the greatest Circle, (per Prob. VII. Rule II.)

$$\therefore 1017,8784 \times 99 = 100769,9616.$$

$$\text{Which } \div 15 = 6717,99744 = \frac{1}{15}.$$

Therefore $6717,99744 \times 8 = 53743,97952$ solid Inches, as before.

PROBLEM XXVIII.

(30) By Rule I. Thus, first $42 \div 4 = 10,5$ Quarter Girth.

$$\text{And } 10,5 \times 10,5 \times 16 = 1764 \text{ Inches.}$$

$$\therefore 144)1764(12\frac{1}{4} \text{ Feet, the Content by this Rule.}$$

By Rule II. Thus, first $42 \div 5 = 8,4 = \frac{1}{5}$ of the Girth.

$$\text{And } 16 \times 2 = 32 = \text{twice the Length.}$$

$$\text{Then } 8,4 \times 8,4 \times 32 = 2257,92 \text{ Inches.}$$

$$\therefore 144)2257,92(15,68 \text{ Feet, the true Content.}$$

(31) By Rule I. Thus $64 \div 4 = 16$.

$$\text{Then } 16 \times 16 \times 30,5 = 7808 \text{ Inches.}$$

These $\div 144$, gives $44,2$ Feet, the Content by this Rule.

By Rule II. Thus $64 \div 5 = 12,8$. And $30,5 \times 2 = 61$.

$$\text{Then } 12,8 \times 61 = 9994,24 \text{ Inches.}$$

$$\therefore 144)9994,24(69,335 \text{ Feet, (very nearly) the true Content.}$$

Note, the first Rule differs from the Truth about $\frac{1}{4}$ of its Contents; that is, when it produces 4 for the Content, it should be above 5. The second Rule is about 50 times nearer to the Truth than the other, for it differs from the Truth only 1 Foot in 190; and it is full as easy in Practice, besides it hath in every Respect the Advantage of it. Therefore, I think it ought to be brought into general Use among the Measurers of Timber, who should certainly prefer Truth to such gross Errors as are always introduced by the other Method.

- (32) First $42 \times 30 \times 40 = 50400$.
 Then $144)50400(350$ Feet, the Content.
 $\therefore 5,0)35,0(7$ Tons, the Answer.

P R O B L E M XXXIX.

By R U L E I.

- (33) Thus $7854 \times 32 \times 32 \times 2 = 1608,4992$
 $7854 \times 26 \times 26 = 530,9304$

their Sum $= 2139,4296$
 which Multiply by $4\frac{2}{3} = 13,3$

gives $28525,728$ cubic Inc.

$\therefore 282)28525,728(101,155$ Ale } Gallons.
 $231)28525,728(123,488$ Wine }

By R U L E II.

- (34) First $36 \times 36 = 1296 =$ Square of the Head.
 And $40 \times 40 \times 2 = 3200 = 2$ ditto of the Bung.

their Sum $= 4496$

Now $40 - 36 = 4$. And $4 \times 4 = 16$, $\frac{2}{3}$ of which $= 6,4$.

Then $4496 - 6,4 = 4489,6$.

$\therefore 4489,6 \times 64 = 287334,4$.

Therefore

$287334,4 \times \begin{cases} ,00092837 = 266,75 & \text{Ale} \\ ,00113333 = 325,64 & \text{Wine} \end{cases}$ } Gallons.
 Or $1077,157)287334,4(266,75$ Ale } Gallons, as be-
 $882,35)287334,4(325,64$ Wine } fore.

Q U E S T I O N S.

- (1) First $6 \times 6 \times 6 = 216$ Solid Inches in $\frac{1}{2}$ a Foot solid.
 And $2)1728$

$\therefore 216)864($ solid Inches in $\frac{1}{2}$ a solid Foot.

Answer 4 Times as much as the first; or one is $\frac{1}{4}$ of the other.

- (2) First $25\ 6 \times 20\ 2 \times 14 = 7199\ 6\ 0$
 $\frac{1}{2} = 12\ 9 \times 10\ 1 \times 7 \times 2 = 1799\ 10\ 6$ which is just $\frac{1}{4}$
of the first; or as 4 is to 1, the Answer.

| | Ft. | Inc. | Ft. | Inc. | Ft. | Inc. | | Ft. | Inc. | | |
|-------------------------|-----|------|-----|------|-----|------|---|---------|------|----|---|
| (3) First | 17 | 7 | 13 | 10 | 9 | 6 | = | 2310 | 8 | 11 | |
| And | 5 | 6 | 2 | 1 | 9 | 6 | = | 108 | 10 | 3 | |
| | | | | | | | | | | | |
| | | | | | | | | Remains | 2201 | 10 | 8 |
| Fire-Place, and Windows | 27 | 4 | | | | | = | 108 | 0 | 0 | |
| | | | | | | | | | | | |
| | | | | | | | | Answer | 2309 | 10 | 8 |

Sol. Ft.

- (4) First $112,5 \times 32 \times 5,5 = 19800$ in the whole Hold.
Also $112,5 \times 4,5 \times 5,5 = 2784,375$ Gang-Way.

Remaining Capacity 17015,625

Then $3,3 \times 2,16 \times 3 = 21,626$, Content of 1 Bale.

$\therefore 21,626 \overline{) 17015,625}$ (787,2 + Bales, the Answer.

- (5) First a Fodder of Lead = 2184 lb.

Then $16 \overline{) 2184}$ (136,4 Square Feet.

$\therefore 4,25 \times 8 = 34$ Area of the Bottom.

Also $136,5 - 34 = 102,5 =$ Area of the Sides and Ends.

$8 \times 2 + 4,25 \times 2 = 16 + 8,5 = 24,5$ round.

Then $24,5 \overline{) 102,5}$ (4,183673 Feet, = 50,204 Inches.

$\therefore 50,204 - ,75 = 49,454$ Depth taken at.

Now 8 ft. = 96 inc. Length, and $4\frac{1}{2}$ ft. = 54 inc. Breadth.

Therefore $96 \times 54 \times 49,454 = 242126,784$ cubic Inches.

$\therefore 282 \overline{) 242126,784}$ (858 galls. = 16 hhds. 42 galls. the Answer.

- (6) First 18 inc. = 1,5 ft. Also 14 = 1,16.

Then $1,16 \times 1,5 = 1,75$ Breadth and Depth.

$\therefore 1,75 \overline{) 2,5}$ (1,42857 Length of the Piece cut off.

Therefore $18,5 - 1,42857 = 17,07143$ Feet, the Answer.

- (7) First $8,5 \overline{) 126,25}$ (14,853 ft. = 2138,8234 Inches, Area of an End.

$\therefore 38,5 \overline{) 2138,8234}$ (55,55 Inches deep, the Answer.

(8) First $18,5 \times 18,5 \times ,7854 \times 8 = 2150,4252$ cubic Inches in a Bushel.

Now 7 ft. 10 inc. = 94 inc. Length; 3 ft. 10 inc. = 46 inc. Breadth; and 4 ft. 2 inc. = 50 inc. Depth.

Then $94 \times 46 \times 50 = 216200$ cubic Inches in the Bin.

$\therefore 2150,4252 \mid 216200,0000 (100,5 \text{ bush.} = 12 \text{ qrs. } 4\frac{1}{2} \text{ bush.}$
the Answer.

(9) By the last Question a Bushel = 2150,4252 Inches.

Then $2150,4252 \times 9 = 19353,8268$ Inches, the Content.

$\therefore 19353,8268 \div 12 = 1612,8189$ Area of the Circle.

Which $\div ,7854 = 2053,49$ Square of the Diameter.

And $\sqrt{2053,49} = 45,3$ Inches, the Answer.

(10) First $28 \div 2 = 14$ Radius. And $14 \times 14 = 196$.

Also $7 \times 7 = 49$. Then $196 - 49 = 147$.

Then $\sqrt{147} = 12,1243557$ Perpendicular.

$\therefore 12,1243557 \times 7 = 84,8705$, Area of one Triangle.

And $84,8705 \times 6 = 509,223$, Area of the Base.

Also $14 \times 6 \times 134 = 11256$, ditto Sides.

Other Base 509,223

Yds. Ft. Inc.

A yd. = inc. 1296) 12274,446 (9 4 34 Superf.

Again $509,223 \times 134 = 68235,88$ solid Inches.

$\therefore 1728 \mid 68235,88 (39 \text{ Solid Feet, } 84 \text{ inc. the Answer.}$

(11) First 3 cubic Feet = 5184 cubic Inches.

As the Circumference is 44, the Diameter is 14, (per Prob. VI.)

Then $\frac{44}{2} \times \frac{14}{2} = 22 \times 7 = 154$, Area of an End, (per Prob. VII. Rule I.)

$\therefore 154 \mid 5184 (33,66$ Inches, the Answer.

(12) First $1,25 \times 1,25 \times ,7854 = 1,2271875$, Area of the Circle, (per Prob. VII. Rule II.)

And a Pint contains 28,875 cubic Inches.

$\therefore 1,227187 \mid 28,875 (23,5294$ Inches, the Answer.

(13) A Bushel contains 2150,4252 cubic Inches, (per Quest. VIII.)

And $13,5 \times 8 = 108$ Bushels, in $13\frac{1}{2}$ qrs.

Then $2150,4252 \times 108 = 232245,921$ cubic Inches.

$\therefore \sqrt[3]{232245,9216} = 61,468 +$, the Answer.

(14) First $20 \times 15 \times 8 = 2400$ Cubic Inches in the Stone.

And 290 Tons = 649600 lb.

$\therefore$ as 220 : 2400 :: 649600 : 7086545 Inches.

Then $1728)7086545(4101$ Feet, the Answer.

(15) A piece of Timber a Foot long, and 4 Feet round, is a solid Foot, customary Measure.

Also if a Circle be 4 Feet round, its Diameter will be

$$4 \div 3,1416 = 1,2732. \text{ (per Prob. VI.)}$$

Then a circular Piece of Timber 1 Foot in length, will contain 1,2732 Feet.

Therefore $1,2732 \times 50$ (Feet in a Load) = 63,66 ft.

$\therefore 63,66 - 50 = 13,66$ Feet in a Load, the Answer.

(16)

Sol. Inc.

$$20 \times 20 \times 20 \times \left\{ \begin{array}{l} ,7854 = 6283,2 \text{ Cyl.} \\ ,5236 = 4188,8 \text{ Globe.} \\ ,2618 = 2094,4 \text{ Cone.} \end{array} \right\} \text{ See Prob. XIII. XV. and XXIV.}$$

Now if the Diameter of a Circle be 20, the Circumference will be $3,1416 \times 20 = 62,832$. (per Prob. VI.)

Also if the Height (C D) of a Cone = 20, and the Diameter of the Base A B = 20.

Then $A D = \frac{20}{2} = 10$. (See Fig. 16. in the Guide.)

Then per Sect. LIV. Case IV. $20 \times 20 + 10 \times 10 = 400 + 100 = 500$.

$\therefore \sqrt{500} = 22,31614 = A C$, the Slope Side.

Then $62,832 \times 10 = 628,32$ Area of the two Bases.

And $62,832 \times 20 = 1256,64$

$$\begin{array}{lcl} \text{Cylinders} & 1884,96 & \text{Superficial Cent.} \\ 62,832 \times 20 = & 1256,64 & \text{Globe's ditto.} \\ \frac{62,832}{2} \times 22,31614 \left\{ = 1015,24 \right. & \text{Cone's ditto.} & \\ \text{their Sum} & = 4156,84 & \text{Inches, their Area.} \end{array} \left. \begin{array}{l} \text{See Pr. XVI. and XXIII.} \end{array} \right\}$$

A Yard = 1296 : 8 :: 4156,84 : 25,6 = 2s. $1\frac{1}{2}d$. the Answer.

(17) First $2170 \times 2170 \times 2170 \times ,5236 = 5350308686,8$
solid Miles, (per Prob. XXIV.)

Then a Mile $= 1760 \times 1760 \times 1760 = 545177600$ solid Yards.

$\therefore 5350308686,8 \times 545177600 = 29168684491287756800$
solid Yards, in the Moon.

Now in a solid Yard are $36 \times 36 \times 36 = 46656$ sol. Inches.

Therefore $29168684491287756800 \times 46656 =$
 $1360894143625521581260800$ solid Inc. in the Moon.

Then per Quest. VIII. a Quarter $= 2150,425 \times 8 =$
 $17203,4) 1360894143625521581260800 (79107034948$
 470144000 Quarters of Wheat, Moon would hold, if hollow. *Q. E. F.*

Again $2170 \times 3,1416 = 6817,272$ Circumference of the Moon, (per Prob. VI.)

Then per Prob. XXIII. $6817,272 \times 2170 = 14793480,25$
square Miles, the Surface of the Moon.

A Mile $= 1760 \times 1760 = 3097600$ square Yards.

$\therefore 14793480,25 \times 3097600 = 45824284391424$ square Yards of Stuff. *Q. E. F.*

(18) First $7970 + 60 \times 2 = 7970 + 120 = 8090$ Diameter of the Earth and Atmosphere.

Then $7970 \times 7970 \times 7970 \times ,5236 = 265078559622,8$ sol. Miles in the Globe of the Earth, (per Prob. XXIII.)

Also $8090 \times 8090 \times 8090 \times ,5236 = 277233177544,4$ solid Miles in the Earth and Atmosphere.

$\therefore 277233177544,4 - 265078559622,8 = 12154617921,6$
solid Miles in the Atmosphere.

And per the last Quest. a Mile $= 545177600$ solid Yards. Then

$12154617921,6 \times 545177600 = 66264254274148761600$
solid Yards in the Atmosphere. *Q. E. F.*

(19) First $30 \text{ In.} = 2,5 \text{ Feet.}$

Then $2,5 \times 2,5 = 6,25$ Area of the Base.

$\therefore 6,35 \times \frac{21}{3} = 6,25 \times 7 = 43,75$ solid Feet, (per Prob. XV.)

Then $21 \times \frac{2,5}{2} = 21 \times 1,25 = 26,25$ Area of one triangular Side, (per Prob. III)

$\therefore 26,25 \times 4 = 105$ Area of the four Sides.

Now

| | | £. | s. | d. |
|------------------|---|----|----|----|
| Now 43.75 at 7s. | = | 15 | 5 | 8 |
| And 105 at 8d. | = | 3 | 10 | 0 |

Answer Cost 18 15 8

(20) In order to complete the Cone, use this Analogy; as half the Difference of the Top and Bottom are to the Depth, so is half greater Diameter to the Altitude of the whole Cone.

$$\text{Thus } \frac{72-54}{2} = \frac{18}{2} = 9. \text{ Also } \frac{72}{2} = 36.$$

Then as 9 : 42 :: 36 : 168 Altitude.

$$\therefore 72 \times 72 \times .7854 \times \frac{168}{3} = 2288004,7616 \text{ Area of the whole Pyramid, (per Prob. XV.)}$$

$$\text{Again } \frac{168-42}{3} = \frac{126}{3} = 42 \text{ Altitude of the Piece wanting.}$$

$$\text{Then } 54 \times 54 \times .7854 \times 42 = 96189,5088 \text{ Area of the Piece wanting.}$$

$$\text{Therefore } 2288004,7616 - 96189,5088 = 131815,2528 \text{ cubic Inches.}$$

$$\therefore 282)131815,2528(467 \text{ Galls. } 3\frac{6}{41} \text{ pts. the Answer.}$$

$$(21) \text{ First } 16 + \frac{16}{5} = 16 + 3,2 = 19,2 \text{ Bottom Diameter.}$$

$$\text{And } 19,2 \times 8 = 153,6 \text{ the Height.}$$

$$\text{Also } 153,6 \div 3 = 51,2 \text{ cylindrical.}$$

$$153,6 - 51,2 = 102,4 \text{ a conical Frustum.}$$

$$\text{Now } 19,2 \times 19,2 \times .7854 = 289,529856 \text{ Area of the gr.}$$

$$\text{Also } 16 \times 16 \times .7854 = 201,0624 \text{ ditto lesser.}$$

$$\text{Sum of their Areas } \underline{490,592256}$$

$$\text{Again } 289,52985 \times 201,0624 = 58213,567719, \text{ whose square-Root} = 241,274; \text{ which added to } 490,592256 = 731,866256.$$

$$\therefore 731,866,256 \times \frac{102,4}{3} = 24981,034874546 \text{ Conical.}$$

$$\text{And } 201,0624 \times 51,2 = 10294,39488 \text{ Cylinder.}$$

Solid Content of the Pillar 35275,42975 Inches.

$$\therefore 1728)35275,42975 (20,41988 \text{ solid Feet, at } 3s. 6d. \text{ or } ,175 \text{ £.}$$

$$\text{Then } 20,41988 \times ,175 = 3,573479 = 3 \text{ £. } 11s. 5\frac{1}{2}d. \text{ Answ. Farther}$$

$$16 \times 3,1416 = 50,2656 \text{ Circumference of the Cyliind.}$$

$$19,2 \times 3,1416 = 60,3187 \text{ ditto Base.}$$

$$2)110,5843 (55,29215$$

$$\text{Then } \frac{19,2}{2} - \frac{16}{2} = 1,6, \text{ and } 1,6 \times 1,6 = 2,56$$

$$\text{Also } 102,4 \times 102,4 = 10485,76$$

$$\text{their Sum } \underline{10488,32}$$

$$\therefore \sqrt{10488,32} = 102,4125 \text{ Slope Height.}$$

$$\text{Then } 102,412 \times 55,29215 = 5662,6534 \text{ Con. Superf.}$$

$$\text{Also } 51,2 \times 50,2656 = 2573,5987 \text{ Cylinder.}$$

$$289,526 \text{ Bottom Area.}$$

$$201,0624 \text{ Top ditto.}$$

$$\text{their Sum } \underline{8726,8405} \text{ Inches.}$$

$$\therefore 144)8726,8405 (60,8 \text{ Feet, the Superficial Content.}$$

$$(22) \text{ First } 19,5 - 13,5 = 6 \text{ Diff. of the Sides.}$$

$$\text{And } 6 \times 6 = 36, \div 3 = 12 \text{ the } \frac{1}{3} \text{ of the sq. of their Dif.}$$

$$\text{Now } 16 \text{ ft. } 6 \text{ inc.} = 198 \text{ Inches, the Length.}$$

$$\text{Then } 19,5 \times 13,5 \times 12 \times 198 = 5499,5 \text{ cubic Inches.}$$

$$1728)5499,5 (31,539 \text{ solid Feet, at } 2s. 6d. \text{ or } ,125 \text{ £.}$$

$$\therefore 31,539 \times ,125 = 3,942375 = 3 \text{ £. } 18s. 10d. \text{ the Answer.}$$

$$(23) \text{ First } 3,1416 \times 26 \times 26 \times 2 = 4247,4432$$

$$\text{Also } 3,1416 \times 20 \times 20 = 1256,64$$

$$\text{Sum } \underline{5504,0832}$$

Then

Then $5504,0832 \times \frac{100}{3} = 183469,44$ cubic Inches, (per Prob. XXIX.)

$\therefore 289)183469,44(650\frac{1}{2}$ Ale = 20 Bar. London Beer.

$231)183469,44(794$ Brandy. Q. E. F.

(24) First 6 Feet = 72 Inches.

Then $3,1416 \times 72 = 226,19568$, the Circumference.

And $226,19568 \times 72 = 16286,0544$ Inches, superficial Content, (per Prob. XXIII.) at $3\frac{1}{2}d.$ per Inc. = 237£. 10s. 1d. 2, nearly the Answer.

(25) First $21 \times 21 \times ,7854 = 346,3614$, Area of the Base.

Then $346,3614 \times 31 = 10737,2034$ solid Feet.

$\therefore 10737,2034 \times 1728 = 18553887,4752$ cubic Inches.

$231)18553887,4752(80319,8$ Gallons.

A Tun = Galls. 252)80319,8(318 Tuns, 183,8 Gallons, the Answer.

90. SPECIFIC GRAVITY OF METALS.

CASE I.

(1) Thus $,0330946 \times 12096 = 400,3122816$ lb.

Adv. $\div 112 = 3,5742 +$ Cwt. the Answer.

(2) First $\frac{20}{5} = 4$. Also $4 \times 4 = 16$ sqr. of $\frac{1}{5}$ Girth.

And 40 Feet = 480 Inches.

Then $480 \times 2 \times 16 = 960 \times 16 = 15360$ cubic Inches, (per Prob. XX. in Mensuration.)

By Rule II. $,023763 \times 15360 = 345$ lb. (nearly) = $3\frac{9}{12}$ Cwt. the Weight required.

(3) First $7 \times 7 \times 7 \times ,5236 = 178,8748$ solid Inches, (per Prob. XXIV.)

Then $178,8748 \times ,2580647 = 46,387 +$ lb. the Answer.

(4) First $2580647)42,0000000(162,7499$ the Solidity.

Then $,5236)162,7499(=310,84778457$ Cube of the Diameter, (by Prob. XXIV.)

$\therefore \sqrt[3]{310,84778457} = 6,7743$, the Diameter required.

- (5) First $16 - 6 = 10$, Diameter of the Concavity.
 Then $16 \times 16 \times 16 \times ,5236 = 2144,6656$ Cont. of the whole.
 And $10 \times 10 \times 10 \times ,5236 = 523,6$ Cont. of the Concavity, (per Prob. XXIV.) —————
 Solidity of the Shell. $= 1621,0656$ Inches.

$\therefore 1621,0656 \times ,2580647 = 418,3398$ lb. the Weight req.

- (6) First $76 + 5 = 81$, the greater Diameter.
 Also 3 ft. 4 inc. $= 3,3$ the greater Breadth.
 $\therefore$ as $81 : 3,3 :: 76 : 3,127572$, its least Breadth.

Here the Chords and their Arches being nearly equal, in so small a Part of so large a Circle, differs very little from a Right-line: the Figure of the Key-stone may be reckoned a Prismoid, and measured accordingly, (see Prob. XXII.)

Now $3,3 \times 4 = 13,3$; $3,127572 \times 4 = 12,510288$.
 And $6,5519053 \times 8 = 52,4152426$, which are the Numbers to be added,

Then $13,3 + 12,510288 + 52,4152426 = 78,259864$, Sum.

$\therefore 78,259864 \times \frac{5}{6} = \frac{391,29932}{6} = 65,216553$ solid Feet.

Then $65,216553 \times 1728 = 112694$ cubic Inches, (nearly.)

$\therefore 112694 = 10475,3918842$ lb.

Therefore a Ton $=$ lb. 2240) $10475,3918842$ ($4,6765 +$
 $= 4$ Tons 13 cwt. 2 qrs. $3,3$ lb. the Answer.

- (7) First $63 \times 12 \times 12 = 9072$ solid Feet.

Then $9072 \times 1728 = 15676416$ cubic Inches.

$\therefore 15676416 \times ,0977286 = 1532034,1887$ lb.

And 2240) $1532034,1887$ (683 Tons 18 Cwt. 98 lb. the Answer.

C A S E II.

- (8) First 8 Tons 14 cwt. $= 19488$ lb.

Then $,0977286$) $19488,0000000$ ($199409,4$ Inches.

Now 1728) $199409,4$ ($115,4$ cubic Feet, at $6s.$ or $,3\text{£}$.

Then $115,4 \times ,3 = 34,62 = 34\text{£}$. $12s.$ $4\frac{3}{4}d.$ the Answer.

- (9) First $1300 \times 4 = 5200$ hhds. and a hhd. $= 282 \times 54 = 15228$ cubic Inches.

$\therefore 15228 \times 5200 = 79185600$ cubic Inches, displaced.

Then $79185600 \times ,037253 = 2949901$ lb. Averd.

$\therefore 2240$) 2949901 (1316 Tons 18 cwt. 1 qr. 17 lb. the Weight required.

(10) First $3 \times 3 = 9$ solid Inches, in the Gold Chain.

Then $9,962625 \times 9 = 89,663625$ its Weight in Air.

And $0,527458 \times 9 = 4,747122$ wt. of its Bulk of Water.

Weight of the Gold 84,916503 in Water, *Q. E. F.*

Now a solid Inch of Silver = 5,556769 oz. Troy.

Then as $5,556769 : 1 :: 14,5 : 2,6094$ inc. of Silver.

$\therefore 89,663625 - 14,5 = 75,163625$ oz. of Gold.

And $9,962625 \mid 75,163625$ (7,5546 Space taken up by the
2,6094 dit. of Silver. [Gold.]

Sum 10,164 by both.
9, ditto of Gold.

1,164 ditto of Silver.

$\therefore 9 \mid 1,164$ (,1293 inc. higher. *Q. E. F.*

Sol. Inc.

(11) First $10,36 \mid 63 \text{ } 00$ (6,08108, had it been Gold.)

Also $5,85 \mid 63 \text{ } 00$ (10,76923, if all Silver.

Then by Sect. XXIX.

$8,2245 \left\{ \begin{array}{l} 6,08108 \\ 10,76923 \end{array} \right\} \left\{ \begin{array}{l} 2,54473 \\ 2,14342 \end{array} \right.$

Sum 4,68815

$4,68815 \mid 2,54473$ (,5428, Part Gold, per oz.

$4,68815 \mid 2,14342$ (,4572, Part Silver.

Then

oz. dwts. grs.

$\left\{ \begin{array}{l} ,5428 \\ ,4572 \end{array} \right\} \times 63 = \left\{ \begin{array}{l} 34,1884 = 34 \quad 3 \quad 22\frac{1}{4} \text{ Gold.} \\ 28,8036 = 28 \quad 16 \quad 1\frac{3}{4} \text{ Silver.} \end{array} \right.$

91. CHRONOLOGY.

(1) Thus $1779 \div 4 = 444$, and 3 remains, so it is the 3d Year after Leap Year.

(2) Thus $1776 \div 4 = 444$, and 0 remains, so it is Leap Year.

(3) First 4) 1779 (444 + 1779 = 2223, Sum.

And 7) 2223 (317, and 4 remains.

Then $7 - 4 = 3 = C$, the Dominical Letter.

(4) First 4) 1776 (444 + 1776 = 2220, Sum.

And 7) 2220 (317, and 1 remains.

Then $7 - 1 = 6 = F$, for the Dominical Letter.

(5) The Dominical Letter for this Year is C, (per Prob. III. Ex. III.)

Now per the Verse, the 1st of May is B, viz. on Saturday.

Then B 1, B 8, B 15, C 16, D 17, E 18, F 19; so that the 19th of May happens on Wednesday.

(6) The Dominical Letter is F, per Ex. IV. and per the Verse, the first of June is E.

Then E 1, F 2, G 3, A 4; consequently the 4th of June must be on a Tuesday.

| | | | |
|-----|--------------|--------------|---------------|
| (8) | 1779 | 1779 | 1779 |
| | 9 | 1 | 3 |
| | 28) 1788 (63 | 19) 1780 (93 | 15) 1782 (118 |

| | | |
|---------------------|----------------|----------------|
| Rem. sol. Cyc. = 24 | lun. Cyc. = 13 | Indiction = 12 |
|---------------------|----------------|----------------|

| | | |
|--------------|--------------|---------------|
| 1776 | 1776 | 1776 |
| 9 | 1 | 3 |
| 28) 1785 (63 | 19) 1777 (93 | 15) 1779 (118 |

| | | |
|---------------------|----------------|---------------|
| Rem. sol. Cyc. = 21 | lun. Cyc. = 10 | Indiction = 9 |
|---------------------|----------------|---------------|

So that 24 and 21 are the solar Cycles, 13 and 16 lunar Cycles, or Golden Numbers; 12 and 9 the Indic. Cycles.

(8) The Golden Number is 10, (per Prob. IV.)

Then $10 \times 11 = 110$, which $\div 30 = 3$, and 20 remains.

$\therefore 20 - 11 = 9$, the Epact.

(9) The Golden Number is 13.

Then $13 \times 11 = 143$, which $\div 30 = 4$, and 23 remains.

$\therefore 23 - 11 = 12$, the Epact.

(10) The Epact is 9, (per Prob. V.) Number of the Month 3, and Day of the Month 21.

Now $9 + 3 + 21 = 33$,

Then

Then $33 - 30 = 3$ Days, the Moon's Age.

Also $30 - 3 = 27$ Days after the 21st of May, or June the 17th, the Day of the next new Moon.

(11) The Epact is 12, Number of the Month 1, and Day of the Month 24.

Now $12 + 1 + 24 = 37$.

Then $37 - 30 = 7$ Days, the Moon's Age.

Also $30 - 7 = 23$ Days, after the 24th of March, or the 16th of April, is the Day of the next new Moon.

(12) The Dominical Letter is F, (by Prob. II.)

On March 20th the Moon's Age is 4, (by Prob. VI.) so that the nearest new Moon to March 20th falls on March 16th, and the 14th Day of that Moon is March 26th.

Now March 1st is D, or Friday, (by the two Lines in Prob. III.) then the 26th is Tuesday, and the next Sunday or Easter-day, is on the 31st of March.

(13) The Dominical Letter is C, (by Prob. II.)

On March 21st the Moon's Age is 4, (by Prob. VI.) so that the nearest new Moon to March 21st falls on March 17th, and the 14th of that Moon is March 31st.

Now March 1st or Monday is D, (by the two Lines in Prob. III.) then the 31st will be on Wednesday.

∴ The next Sunday after will be on April 4th, which is Easter Sunday: from which the other moveable Feasts may be found.

Or by knowing what Day of the Month Shrove Sunday is on, all the rest may be found.

Shrove Sunday is always the first Sunday after the second Change of the Moon, which happeneth after New-Years-Day: and if that Day of the second Change be Sunday, then that Sunday is Shrove-Sunday.

EXAMPLE.

What Day of the Month is Shrove-Sunday in the Year 1779, being Feb. 12th.

First the Moon's Age is 4 Days old*, (per Prob. VI.)

Then by running back, I find the Day of her second Change will be Feb. 8th.

* In this case the Epact must always be used for the Year before; that is, for 1779 use the Epact for 1778, which is 20, (per Prob. V.)

Now by the Lines in Prob. III. the first of Feb. is D, or Monday, as the Dominical Letter is C, (per Prob. II.) Therefore D being the first, D is 8, E 9, F 10, G 11, A 12, B 13, and C 14, so that Shrove-Sunday will be on Feb. 14th, being the first Sunday after the second Change.

Now Shrove-Sunday being found as above, Febr. 14th.

Quadragesima, or the first Sunday in Lent, } Febr. 21.

Easter-Day, being 7 Weeks after Shrove-Sunday, must be } April 4.

Easter-Term, beginning a Fortnight after Easter-Wednesday, must be } April 21.

Rogation-Sunday, being 5 Weeks after Easter-Wednesday, must be } May 9.

Ascension-Day, being 39 Days after Easter, must be } May 13.

Easter-Term, ends the Monday after Ascension-day, } May 16.

Whit-Sunday, is 7 Weeks after Easter, must be } May 23.

Trinity-Sunday, the next after Whit-Sunday, must be } May 30.

Trinity-Term, begins Friday after Trinity-Sunday, } June 4.

Trinity-Term, ends Wednesday Fortnight afterwards } June 23.

And the Beginning and End of Hilary, and Michaelmas Terms, are fixed.

Note, the Septuagesima is a Fortnight, and Sexagesima a Week, before Shrove-Sunday.

Advent Sunday, is always that Sunday next the 30th of Nov. whether before or after it.

Epiphany is reckoned among the moveable Feasts; but it is always the 6th of January.

(14) 3 Moon's Age.

× 4

5) 12

Ans. 2 h. 24 m. in the Aftern.

(15) 7 Moon's Age.

× 4

5) 28

Ans. 5 h. 36 m. in the Aft.

(16) The Moon south's at b. m.
 At Lond. D bears N. E. or S. E. 2 24 p. m.
3 00

Sum 5 24

Ans. 24 min. past 5 in the Afternoon.

(17) The Moon South's at b. m.
 At Bristol Key D bears E. by S. and W. by N. 5 36 p. m.
6 45

Sum 12 21

Answer 21 min. past 12 at Night.

QUESTIONS.

(1) In the reign of William I. were 5 intercalary Days,
 (by Prob. I.)

Between Sept. 9, and Oct. 4, are 25 Days.

And $1087 - 1066 = 21$ yrs. wanting 25 Days.

Then $21 \times 365 + 5 - 25 = 1645$ Days, William I. reigned.

In the reign of William Rufus were 4 intercalary Days.

Between August 9, and Sept. 9, are 38 Days.

And $1100 - 1087 = 13$ yrs. all but 38 Days.

Then $365 \times 13 - 38 + 4 = 4711$ Days, William II. reigned.

Lastly, in the reign of William III. were 3 intercalary Days.

From Feb. 3, to March 8, are 33 Days.

And $1701 - 1689 = 12$ Years. and 33 Days.

Then $365 \times 12 + 33 + 3 = 4416$ Days, Will. III. reigned.

(2) In the reign of Richard I. were 2 intercalary Days.

Betwixt April 6, and July 7, are 92 Days.

And $1199 - 1189 = 10$ yrs. wanting 92 Days.

Then $365 \times 10 + 2 - 92 = 3560$ Days, Richard I. reigned.

In the reign of Richard II. were 5 intercalary Days.

From June 21, to Sept. 30, are 101 Days.

And $1399 - 1377 = 22$ Years and 101 Days he reigned.

Then $365 \times 22 + 101 + 5 = 8136$ Days, Rich. II. reigned.

In the reign of Richard III. were 1 intercalary Day.

From June 18, to August 22, are 65 Days.

And $1485 - 1483 = 2$ yrs. and 65 d. he reigned.

Then $365 \times 2 + 65 + 1 = 796$ Days, Richard III reigned.

Therefore $3560 + 8136 + 796 = 12492$ Days, the Answer.

(3) In the reign of Queen Mary, were only 1 intercalary Day.

But from 1553 till 1602, were 12 intercalary Days.

So that in the reign of Queen Elizab. were 11 intercal. Days.

From July 8, to Nov. being 4 mo. 9 d. are 132 Days.

Then $365 \times 5 + 132 + 1 = 1958$ Days, Mary reigned.

Now $1602 - 1553 = 49$ Years.

And from March 14, to July 8, are 115 Days.

Also $365 - 115 = 250$ Days.

Then $365 \times 49 + 250 + 12 = 18147$ Days, to the Beginning of James's reign.

$\therefore 18147 - 1958 = 16189$ Days, Elizabeth reigned.

In the reign of James I. were 6 intercalary Days.

From March 14, to the 27th is 13 Days.

And $1625 - 1602 = 23$ Years, but the Date altering at our Lady-Day, so that the intercal. was only 22 Years.

Therefore $365 \times 22 + 13 + 6 = 8049$ Days James I. reigned.

In the reign of Charles were 6 intercalary Days.

From March 27, to January 30, are 309 Days.

And $1648 - 1625 = 23$ Years.

Then $365 \times 23 + 309 + 6 = 8710$ Days Charles I. reigned.

$\therefore 16189 + 8049 + 8710 = 32948$ Days, the Answer.

(4) The Grant begun Decemb. 14th. 1109, and resumed Nov. 19th. 1219.

From Nov. 19, to Dec. 14, are 25 Days.

And $1219 - 1109 = 110$ yrs. wanting 25 Days.

In which Time are 27 Intercalary Days.

Then $365 \times 110 + 27 - 25 = 40152$ Days, the first Continuance.

The second Grant revived July 16th 1497, and ended May 10th 1524.

Then from May 10, to July 16, are 67 Days.

And $1524 - 1497 = 27$ yrs. wanting 67 Days, in which Time there are 7 intercalary Days.

There-

Therefore $365 \times 27 + 7 - 67 = 9795$ Days, last in Force.
 $\therefore 40152 + 9795 = 49947$ Days, the Grant was in Force.
 The first Grant resumed Nov. 19th 1219, and revived
 July 16th 1498.

From Nov. 19th, to July 16, are 239 Days.

And $365 - 239 = 126$ Days.

Also $1498 - 1219 = 279$ Years, wanting 126 Days; in
 which Period are 69 intercalary Days.

Therefore $365 \times 279 + 69 - 126 = 101778$ Days, super-
 feded. *Q. E. F.*

92. GEOGRAPHY.

- (1) Lat. $39^{\circ} 45'$ N. 57° S. and Long. 111° E. 80° W.
- (2) Jamaica and Cape of Good Hope.
- (3) Diff. $10^{\circ} 30'$ and 48° .
- (4) $469\frac{1}{4}$ Miles, Buda, in Hungary, Madrid, &c.
- (5) Decl. $19^{\circ} 15'$ N. Right Asc. $60^{\circ} 45'$, Mer. Alt. $57^{\circ} 15'$.
- (6) R. $30'$ P. 4, sets $37'$ P. 7, Amp. 30° N. Twil. beg. $21'$ P. 1, ends $30'$ P. 10.
- (7) Azim. 78° from the N. Alt. $12^{\circ} 30'$.
- (8) Port Royal bears W. from London, and London bears N. E. from Port Royal.
- (9) $30'$ P. 7, P. M. at Peking, Breakfast at Port Royal, Dinner at Rome, and Supper at the Ladrone Isles.
- (10) Antigua, Pagan, one of the Ladrone Isles, &c.
- (11) May 10th, and Sept. 2d.
- (12) All those Places whose Lat. is equal to $72^{\circ} 45'$ N.
- (13) Begins May 4th, and leaves them Aug. 9th, following.
- (14) Rising to N. Mexico, Ter. Fuego, &c. setting to Canady in the isle of Ceylon, and Part of Asia, &c. Noon at Porto, St. Vincent, &c.
- (16) Beg. in N. and S. America, the S. and Pacific Seas, Part of Tartary, the Land of Jessa, &c. Mid. all the Great S. Sea, the E. and W. Indies, Part of Asia, &c. End all Asia, and the East Indies, New Holland, the S. Sea, Part of N. America, &c.
- (17) 63° , (nearly.)

- (18) Antoei, Part of the S. Ocean, Peri. Part of the great Pacific Ocean, Antip. Part of New Zealand, or Part of the S. Ocean.

CELESTIAL GLOBE.

- (1) At $\frac{3}{4}$ p. 3, sets 20' p. 4, no Night.
 (2, 3, 4,) For these Examples, the best will be to answer them for the Year you are in; as you will then (by Parker's Ephemerides) have it in your Power to find the Node of the Moon, and her Place at Noon, likewise the Places of any of the Planets.
 (5) Decl. 29° N. Right Asc. 112, Lat. $6^{\circ} 30'$, Long. $19^{\circ} 45'$ Leo.
 (6) Aldabaran ris. at 1 A. M. sets at 4 P. M. comes to the Mer. $\frac{1}{2}$ p. 8, Amp. 26° N.
 (7) R. caſmically Sept. 24th, sets July 6th.
 (8) R. acronically Jan. 25th, sets Decem. 18th.
 (9) Jan. 13th.
 (10) Nov. 16th.
 (11) Obl. Decl. 120° , continues above the Horiz. 9 Hrs.
 (12) Azim. 10° , from the N. Alt. 8° .
 (13) $\frac{3}{4}$ p. 8.
 (14) $14^{\circ} 30'$ N.
 (15) 7' p. 12.
 (16) $\frac{1}{4}$ p. 3, A. M.
 (17) Lat. $44^{\circ} 30'$ N.
 (18) Lat. $22^{\circ} 30'$ N.
 (19) $44^{\circ} 30'$ S.
 (20) Arcturus on the Mer. nearly, Canis Minor setting, and the Stars Lyra, Altayr, Cor Hydræ, Cor Leonis, Caſtr. Cappella, &c. will be viſible.
 (21) Lat. $3^{\circ} 30'$ Long. $26^{\circ} 36'$ of ♊ , and Lat. 25° S. Long. $12^{\circ} 30'$ ♊ .
 (22) R. 3 h. 45 m. A. M. Amp. 14° , sets 2 h. 25 m. P. M. Decl. $9^{\circ} 15'$ S. Right Asc. $227^{\circ} 30'$.
 (23) Through the Tail of Capricorn, the Head of Indus, the Neck and Body of Pavo, the Neck of Apus, Tail of Centaurus, between the two Stars in the Back of Hydra, and then to the Ecliptic near Cor Leonis: Velocity $4^{\circ} \frac{1}{6}$ per Day.

A L G E B R A.

93. ADDITION.

EXAMPLES in CASE I.

| | | | | |
|-----|------------------|--|-----|---------------------------------|
| | (2) | | (3) | |
| To | $6a + 7b - 3c$ | | | $ab - 6b + 4x + 10y - 15z + 6$ |
| Add | $10a + b - 7c$ | | | $6ab - b + x + 4y - 9z + 3$ |
| | <hr/> | | | <hr/> |
| Sum | $16a + 8b - 10c$ | | | $7ab - 7b + 5x + 14y - 24z + 9$ |
| | <hr/> | | | <hr/> |

CASE II.

| | | | | |
|-----|------------|-----------------|-----|----------------------|
| | (5) | (6) | (7) | |
| To | $-4b - 6c$ | $-6b - 7c - 8x$ | | $6a - 6x + 7y - 10$ |
| Add | $2b + 9c$ | $4b + 9c + 5x$ | | $-6a + 6x - 4y + 13$ |
| | <hr/> | <hr/> | | <hr/> |
| Sum | $-2b + 3c$ | $-2b + 2c - 3x$ | | $* * + 3y + 3$ |
| | <hr/> | <hr/> | | <hr/> |

CASE III.

| | | | |
|-----|---------------------|-----------------------------------|--|
| | (9) | (10) | |
| To | $6b - 3x$ | $4a + 6b + 4c - 6$ | |
| Add | $3c - 4y$ | $3x - 7y + 4z$ | |
| | <hr/> | <hr/> | |
| Sum | $6b + 3c - 3x - 4y$ | $4a + 6b + 4c - 6 + 3x - 7y + 4z$ | |
| | <hr/> | <hr/> | |

| | | | | |
|-------|--|-----|----------------------|--|
| (11). | | To | $2a - 6be$ | |
| | | Add | $6x + 10$ | |
| | | | <hr/> | |
| | | Sum | $2a - 6be + 6x + 10$ | |
| | | | <hr/> | |

94. SUBTRACTION.

EXAMPLES.

| | | | | |
|------|-------|--------------------|-----|------------------|
| | (2) | (3) | (4) | |
| From | $3a$ | $2a - 4x + 7y - 7$ | | $6b - 4c + 4x$ |
| Take | $-3a$ | $6a + 4x + 7y + 4$ | | $7b + 7c - 9x$ |
| | <hr/> | <hr/> | | <hr/> |
| Rem. | $6a$ | $-4a - 8x * -11$ | | $-b - 11c + 13x$ |
| | <hr/> | <hr/> | | <hr/> |

95. MULTIPLICATION.

EXAMPLES in CASE I.

| | | | | |
|-------|------|---------|------------|------------|
| | (2) | (3) | (4) | (5) |
| Mul. | $-a$ | $a+c$ | $-a-b-c$ | $x+y+z$ |
| by | $-a$ | b | $-d$ | a |
| Prod. | aa | $ab+bc$ | $ad+bd+dc$ | $ax+ay+az$ |

CASE II.

| | | | |
|-------|--------|-------------|-------------|
| | (7) | (8) | (9) |
| Mul. | $-8x$ | $3a+7b$ | $12x+6y$ |
| by | $-4a$ | $5b$ | $4a$ |
| Prod. | $32ax$ | $15ab+35bb$ | $48ax+24ay$ |

CASE III.

| | | | |
|-------|---------|-------------|-----------------|
| | (11) | (12) | (13) |
| Mul. | $-6a$ | $6a-7c$ | $4x-5y-z$ |
| by | $7b$ | $4d$ | $6f$ |
| Prod. | $-42ba$ | $24ad-28cd$ | $24fx-30fy-6fz$ |

CASE IV.

| | | |
|-------|-----------------|----------------|
| | (15) | (16) |
| Mul. | $2a-4b$ | $aa+ab+bb$ |
| by | $2a-4b$ | $a-b$ |
| | $4aa-8ab$ | $aaa+aab+abb$ |
| | $-8ab+16bb$ | $-aab-abb-bbb$ |
| Prod. | $4aa-16ab+16bb$ | $aaa * * -bbb$ |

| | | |
|------|-------|-------------------|
| (17) | Mul. | $xx-2x$ |
| | by | $a+x$ |
| | | $axx-2ax$ |
| | | $xxx-2xx$ |
| | Prod. | $axx-2ax+xxx-2xx$ |

(18)

(18)

Mul. $7b+5d-4x+6$
by $2b-6x+4y-4$

$$14bb + 10bd - 8bx + 12b$$

$$-42bx - 30dx + 24xx - 36x$$

$$28by + 20dy - 16xy - 24y$$

$$-28b - 20d + 16x - 24$$

$$\text{Prod. } 14bb + 10bd - 50bx - 16b - 30dx + 24xx - 20x + 28by + 20dy - 16xy + 24y - 20d - 24$$

E c 3

(9)

(19)

$$\begin{array}{r}
 \text{Mul. } xx+xy+yy \\
 \text{by } \quad \quad \quad xx-xy+yy \\
 \hline
 x^4+x^3y+x^2y^2 \\
 \quad -x^3y-x^2y^2-xy^3 \\
 \quad \quad \quad x^2y^2+xy^3+y^4 \\
 \hline
 \text{Prod. } x^4 \quad * \quad +x^2y^2 \quad * \quad +y^4 \\
 \hline
 \hline
 \end{array}$$

(20)

$$\begin{array}{r}
 \text{M. } aaa-3aab+3ab-bbb \\
 \text{by } aa-2ab+bb \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 a^5-3a^4b+3a^3b-a^2b^3 \\
 -2a^4b+6a^3b-6a^2b^2+2ab^4 \\
 \quad \quad \quad a^3b^2-3a^2b^3+3ab^3-b^5 \\
 \hline
 \end{array}$$

$$\text{P. } a^5-5a^4b+9a^3b-4a^2b^3-6a^2b^2+2ab^4-a^3b^2+3ab^3-b^5$$

96. DIVISION.

EXAMPLES in CASE I.

$$\begin{array}{ll}
 (2) \quad d)ad+6d(a+6 & (3) \quad -d)-ad-bd(a+b \\
 (4) \quad a)aa+ab(a+b &
 \end{array}$$

CASE II.

$$\begin{array}{ll}
 (5) \quad -a)ab(-b & (6) \quad b)-ab-bd(-a-d \\
 (7) \quad -bc)abc+bcd+bcf(-a-d-f &
 \end{array}$$

CASE III.

$$\begin{array}{ll}
 (8) \quad 6a)24ab(4b & (9) \quad 7b)42db(6d \\
 (10) \quad 26x)8abx-18bx(4a-9 &
 \end{array}$$

CASE IV.

$$\begin{array}{l}
 (12) \quad 2b)ab+bb(= \frac{ab+bb}{2b} = \frac{a+b}{2}, \text{ here Unity, or } 1b \\
 \text{is the common Measure. (See Sect. XXXVIII. Case I.)} \\
 (13) \quad 20a)10ab+15ac(= \frac{10ab+15ac}{20a} = \frac{2b+3c}{4a}, \text{ here } 5 \text{ is} \\
 \text{the common Measure,}
 \end{array}$$

(14)

$$(14) \quad \begin{array}{r} a+b)aa+2ab+bb(a+b) \\ aa+ab \end{array}$$

$$\begin{array}{r} * \quad ab+bb \\ ab+bb \end{array}$$

$$\begin{array}{r} * \quad * \end{array}$$

$$(15) \quad \begin{array}{r} a+b)aa-bb(a-b) \\ aa+ab \end{array}$$

$$\begin{array}{r} * -ab-bb \\ ab-bb \end{array}$$

$$\begin{array}{r} * \quad * \end{array}$$

$$(16) \quad \begin{array}{r} 3a-6)6a^4-96(2a^3+4a^2+8a+16) \\ 6a^4-12a^3 \end{array}$$

$$\begin{array}{r} * \quad 12a^3-96 \\ 12a^3-24a^2 \end{array}$$

$$\begin{array}{r} * \quad 24a^2-96 \\ 24a^2-48a \end{array}$$

$$\begin{array}{r} * \quad 48a-96 \\ 48a-96 \end{array}$$

$$\begin{array}{r} * \quad * \end{array}$$

$$(17) \quad \begin{array}{r} 1-a)1*****(1+a+a^2+a^3+a^4, \&c. \\ 1-a \end{array}$$

$$*a$$

$$a-a^2$$

$$* -a^2$$

$$a^2-a^3$$

$$a^3$$

$$a^3-a^4$$

$$a^4$$

$$a^4-a^5$$

$$a^5$$

It often happens that the Operation may be continued without End, as in the last Example, and then you have an infinite Series for the Quotient; but by comparing the first three or four Terms you may find what Law the Terms observe: by which Means, without any more Division, you may continue the Quotient as far as you please.

Thus, the last Example may be continued as far as you please, by adding the Power of a .

(18)

$$\begin{array}{r}
 3x^2 - 4x + 5 \overline{) 18x^4 - 45x^3 + 82x^2 - 67x + 40} \quad (6x^2 - 7x + 8 \\
 \underline{18x^4 - 24x^3 + 30x^2} \\
 * \quad -21x^3 - 52x^2 - 67x \\
 \underline{-21x^3 - 28x^2 - 35x} \\
 * \quad -24x^2 - 32x + 40 \\
 \underline{-24x^2 - 32x + 40} \\
 * \quad * \quad *
 \end{array}$$

(19)

$$\begin{array}{r}
 4x - 5a \overline{) 48x^3 - 76ax^2 - 64a^2x + 105a^3} \quad (12x^2 - 4ax - 21a^2 \\
 \underline{48x^3 - 60ax^2} \\
 * \quad -16ax^2 - 64a^2x \\
 \underline{-16ax^2 + 20a^2x} \\
 * \quad -84a^2x + 105a^3 \\
 \underline{-84a^2x + 105a^3} \\
 * \quad *
 \end{array}$$

(20)

$$\begin{array}{r}
 3x + 4a \overline{) 81x^4 * * * - 256a^4} \quad (27x^3 - 36ax^2 + 48a^2x - 64a^3 \\
 \underline{81x^4 + 108ax^3} \\
 * \quad -108ax^3 \\
 \underline{108ax^3 - 144a^2x} \\
 * \quad 144a^2x \\
 \underline{144a^2x + 192a^3x} \\
 \quad -192a^3x - 256a^4 \\
 \underline{192a^3x - 256a^4} \\
 * \quad *
 \end{array}$$

(21)

$$(21) \quad 1+x) 1 * * * * (1-x+x^2-x^3+x^4, \&c.$$

$$\begin{array}{r}
 1+x \\
 \hline
 * -x \\
 \hline
 -x-x^2 \\
 \hline
 * \quad x^2 \\
 \quad x^2+x^3 \\
 \hline
 * -x^3 \\
 \quad x^3-x^4 \\
 \hline
 * \quad x^4 \\
 \quad x^4-x^5 \\
 \hline
 \quad \quad x^5 \&c. \\
 \hline
 \end{array}$$

(22)

$$2x-3a) 16x^4 * -72a^2x^2 * +81a^4(8x^3+12ax^2-18a^2x-27a^3)$$

$$16x^4-24ax^3$$

$$\begin{array}{r}
 * \quad 24ax^3-72a^2x^2 \\
 \quad 24ax^3-36a^2x^2 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 * \quad 36a^2x \quad * \\
 \quad 36a^2x+54a^3x \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 * \quad -54a^3x+81a^4 \\
 \quad 54a^3x+81a^4 \\
 \hline
 \end{array}$$

$$* \quad *$$

97. FRACTIONS.

$$(2) \quad \text{Thus } a-x+\frac{a^2-ax}{x}=\frac{ax-xx+a^2+ax}{x}=\frac{a^2-x^2}{x}, \text{ (by Sect. XXXVIII. Case III.)}$$

$$(3) \quad \text{Thus } a+b+\frac{x}{z}=\frac{az+bz+x}{z}.$$

(4)

$$(4) \text{ Thus } a - x + \frac{aa - ax}{x} = \frac{ax - xx + aa - ax}{x} = \frac{aa - xx}{x}.$$

$$(6) \text{ Thus } \frac{a^2 - x^2}{x} = -x + \frac{a^2}{x}.$$

$$(7) \text{ Thus } \frac{ax + bx + x}{x} = a + b + \frac{x}{x}.$$

$$(8) \text{ Thus } \frac{ax - xx + aa - ax}{x} = -x + \frac{aa}{x}.$$

$$(10) \text{ Thus } \frac{a}{b}, \frac{c}{d}, \frac{e}{f}, = \frac{adf}{bdf}, \frac{cbf}{bdf}, \frac{edb}{bdf}.$$

$$(11) \text{ Thus } \frac{b+c}{a+b} \text{ and } \frac{d-e}{b-d} = \frac{bb - bd + cb - cd}{ab - ad + bb - bd} \text{ and } \frac{ad - ac + bd - bc}{ab - ad + bb - bd}.$$

(13) First

$$\begin{array}{r} (a^2 + 2ab + b^2)a^3 - * \quad ab^2 \\ \hline a^3 + 2a^2b + ab^2 \end{array}$$

$$\text{Rem. } -2a^2b - 2ab^2) a^2 + 2ab + b^2 \left(-\frac{1}{2b} - \frac{1}{2a} \right)$$

$$\begin{array}{r} a^2 + ab \\ \hline ab + b^2 \\ \hline ab + b^2 \\ \hline * * \end{array}$$

Hence it appears that $-2a^2b - 2ab^2$, is the common Measure; by which $a^3 - ab^2$ being divided,

$$\text{viz. } -2a^2 - 2b^2) a^3 - * \quad -ab^2 \left(-\frac{a}{2b} + \frac{1}{2} \right)$$

$$\begin{array}{r} a^3 + a^2b \\ \hline -a^2b - ab^2 \\ \hline a^2b - ab^2 \\ \hline * * \end{array}$$

Then.

Then $\frac{a}{2b} + \frac{1}{2} = \frac{-a+b}{2ba}$, the new Numerator.

And $\frac{1}{2b} - \frac{1}{2a} = \frac{-a-b}{2ba}$, the Denominator.

Let both be $\times 2ba$, and we shall have $\frac{-a^2+ab}{-a-b}$ new N. Deno.

Or change the Signs of all the Quantities, it will be $\frac{a^2-ab}{a+b}$, the new Fraction required.

That is $\frac{a^2-ab}{a+b} = \frac{a^3-ab^2}{a^2+2ab+b^2}$.

(14) Thus $\frac{25za}{5xz+15az} = \frac{5a}{x+3a}$, by expunging z , and dividing by 5.

Also $\frac{a^3+b^3}{a^2-b^2} = \frac{a^2-ab+b^2}{a-b}$, by dividing by $a+b$, the common Measure.

A D D I T I O N.

EXAMPLES.

(2) Thus $\frac{a}{b} + \frac{c}{a} + \frac{d}{e} = \frac{ade+bce+ddb}{bde}$, the Sum.

(3) Thus $\frac{a-b+d}{b+a} + \frac{a+b-d}{d+a} = \frac{2a}{a+d}$, their Sum.

(4) Thus $\frac{2a-b}{d+c} + \frac{2b-a}{a+c} = \frac{a+b}{c+a}$, the Sum.

(5) Thus $\frac{a+b}{d} + \frac{2a+c}{d} = \frac{3a+b+c}{d}$, the Sum.

S U B T R A C T I O N.

EXAMPLES.

(2) Thus $\frac{x}{2} - \frac{x}{3} = \frac{3x}{6} - \frac{2x}{6} = \frac{x}{6}$, the Difference.

$$(3) \text{ Thus } \frac{a+x}{b} - \frac{a-x}{c} = \frac{ac+cx}{bc} - \frac{ab-bx}{bc} = \frac{ac+cx-ab+bx}{bc}, \text{ the Difference.}$$

$$(4) \text{ Thus } \frac{b^2+a^2}{c} - \frac{b^2}{c} = \frac{a^2}{c}, \text{ the Difference.}$$

$$(5) \text{ Thus } \frac{2b}{a+d} - \frac{a+b-d}{a+d} = \frac{b-a+d}{a+d}, \text{ the Diff.}$$

M U L T I P L I C A T I O N.

E X A M P L E S.

$$(2) \text{ Thus } \frac{a+b}{c} \times \frac{a-b}{d} = \frac{a^2-b^2}{cd}, \text{ the Product.}$$

$$(3) \text{ Thus } a + \frac{b}{c} \times \frac{d}{e} = \frac{ca+b}{c} \times \frac{d}{e} = \frac{acd+bd}{ce}, \text{ the Product.}$$

$$(4) \text{ Thus } \frac{3a-2b}{2d+c} \times \frac{4a+2b}{d} = \frac{12a^2-2ab-4b^2}{2ad-dc}, \text{ the Product.}$$

$$(5) \text{ Thus } 2a + \frac{b}{c} - 25 \times 36 + 4c = \frac{2ac+b-25c}{c} \times \frac{36+4c}{1} = \frac{6bac+3b^2-75bc+8ac^2+4bc-100c^2}{c} = 6ba-71b-8ac-100c + \frac{3b^2}{c}, \text{ the Product.}$$

D I V I S I O N.

E X A M P L E S.

$$(2) \text{ Thus } \frac{acd+bd}{cd} \div \frac{d}{e} = \frac{acde+bde}{cdd}, \text{ or } \frac{ace+be}{cd}, \text{ the Quotient.}$$

$$(3) \text{ Thus } \frac{a-b}{a} \div \frac{a+b}{a-b} = \frac{a^2-2ab+b^2}{a^2+ba}, \text{ the Quot.}$$

(4) Thus $a + \frac{b}{c} \div d + \frac{e}{f} = \frac{ac - b}{c} \div \frac{fd + e}{f} = \frac{fac - bf}{fac + ce}$,
the Quotient.

(5) Thus $\frac{ab}{c} \div \frac{1}{c} = ab$, the Quotient.

S U R D S.

98. ADDITION.

EXAMPLES in CASE I.

| | (2) | (3) | (4) | (5) |
|-----|---------------|----------------|------------------|------------------------|
| To | $7\sqrt{ab}$ | $7b\sqrt{ax}$ | $a\sqrt{bb+cc}$ | $b + {}^3\sqrt{aa-cc}$ |
| Add | $4\sqrt{ab}$ | $7b\sqrt{ax}$ | $4a\sqrt{bb+cc}$ | $b - {}^3\sqrt{aa-cc}$ |
| Sum | $11\sqrt{ab}$ | $14b\sqrt{ax}$ | $5a\sqrt{bb+cc}$ | $2b \quad *$ |

S U B T R A C T I O N.

EXAMPLES in CASE I.

| | (2) | (3) | (4) |
|-------|----------------|------------------|--------------------|
| From | $10a\sqrt{bc}$ | $6b\sqrt{aa+xx}$ | $b + a$ |
| Take | $6a\sqrt{bc}$ | $4b\sqrt{aa+xx}$ | $\sqrt{cc-aa}$ |
| Diff. | $4a\sqrt{bc}$ | $2b\sqrt{aa+xx}$ | $b + \sqrt{cc-aa}$ |

A D D I T I O N.

EXAMPLES in CASE II.

| | (2) | (3) |
|-----|----------------------------|-----------------------------------|
| To | $6d\sqrt{a}$ | ${}^3\sqrt{ac-ba}$ |
| Add | $2b\sqrt{ac}$ | $\sqrt{ac+ba}$ |
| Sum | $6d\sqrt{a} + 2b\sqrt{ac}$ | ${}^3\sqrt{ac-ba} + \sqrt{ac+ba}$ |

$$\begin{array}{r}
 (4) \quad \text{To } 6x\sqrt{aa-ba} \\
 \text{Add } 3y\sqrt{zx} \\
 \hline
 \text{Sum } 6x\sqrt{aa-ba} + 3y\sqrt{zx} \\
 \hline
 \end{array}$$

S U B T R A C T I O N.

E X A M P L E S in Case II.

$$\begin{array}{r}
 (2) \quad \text{From } a-b\sqrt{ccc+dc} \\
 \text{Take } b-2c\sqrt{ab+bb} \\
 \hline
 \text{Diff. } a-b\sqrt{ccc+dc} - b+2c\sqrt{ab+bb} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 (3) \\
 \sqrt{bc} \\
 \sqrt{ba} \\
 \hline
 \sqrt{bc} - \sqrt{ba} \\
 \hline
 \end{array}$$

M U L T I P L I C A T I O N.

E X A M P L E S.

$$\begin{array}{r}
 (2) \quad \text{Mul. } \sqrt{bc+dc} \\
 \text{By } \sqrt{ac} \\
 \hline
 \text{Prod. } \sqrt{abcc+adcc} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 (3) \quad \sqrt{xx+zz} \\
 \sqrt{xx-zz} \\
 \hline
 \sqrt{xxxx-zzzz} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (5) \quad \text{Mul. } 6cd\sqrt{b+da} \\
 \text{By } 3a\sqrt{ca} \\
 \hline
 \text{Prod. } 18acd\sqrt{bca+cdaa} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 (6) \quad 15\sqrt{az} \\
 5\sqrt{x} \\
 \hline
 25\sqrt{azx} \\
 \hline
 \end{array}$$

D I V I S I O N.

E X A M P L E S.

$$\begin{array}{r}
 (2) \quad \sqrt{ca}) bxxxx+dcxx (\sqrt{bx+dx}, \text{ the Quotient.} \\
 (3) \quad \sqrt{zz-xx}) \sqrt{zzzz-xxxx} (= \sqrt{zz+xx}, \text{ the Quot.} \\
 (5)
 \end{array}$$

- (5) $7\sqrt{yxx} \ 14zya\sqrt{yyxx+axx} (2zya\sqrt{y+a}, \text{ the Quot.}$
 (6) $20\sqrt{zcy} 60ab\sqrt{10acxy} (3ab\sqrt{5ax}, \text{ the Quotient.}$

70. INVOLUTION.

- (1) Thus the fourth Power of $x = x^4$.
 (2) The fifth Power of $ax+z$ $= ax+z$ 5 .
 (3) The third Power of $3x^3z^2 = 27x^9z^6$.
 (4) The sixth Power of $\frac{5ab}{2c} = \frac{15625a^6b^6}{64c^6}$.
 (5) Thus

$$a+b$$

$$a+b$$

$$aa+ab$$

$$ab+bb$$

$$a^2+2ab+b^2, \text{ the Square, or second Power.}$$

$$a+b$$

$$a^3+2a^2b+ab^2$$

$$a^2b+2ab^2+b^3$$

$$a^3+3a^2b+3ab^2+b^3 = \text{the Cube, or third Power.}$$

$$a+b$$

$$a^4+3a^3b+3a^2b^2+ab^3$$

$$a^3b+3a^2b^2+3ab^3+b^4$$

$$a^4+4a^3b+6a^2b^2+4ab^3+b^4, \text{ the fourth Power.}$$

$$a+b$$

$$a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4$$

$$a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5$$

$$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5, \text{ the fifth Power.}$$

$$a+b$$

$$a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5$$

$$a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6$$

$$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6 = 6\text{th P.}$$

(6) Thus

$$\begin{array}{r} a-b \\ a-b \\ \hline \end{array}$$

$$\begin{array}{r} a^2-ab \\ ab+b^2 \\ \hline \end{array}$$

$$a^2-2ab+b^2, \text{ second Power.}$$

$$\begin{array}{r} a-b \\ \hline \end{array}$$

$$\begin{array}{r} a^3-2a^2b+ab^2 \\ -ab+2ab^2-b^3 \\ \hline \end{array}$$

$$a^3-3a^2b+3ab^2-b^3, \text{ third Term.}$$

$$\begin{array}{r} a-b \\ \hline \end{array}$$

$$\begin{array}{r} a^4-3a^3b+3a^2b^2-ab^3 \\ a^3b+3a^2b^2-3ab^3+b^4 \\ \hline \end{array}$$

$$a^4-4a^3b+6a^2b^2-4ab^3+b^4, \text{ fourth Power.}$$

$$\begin{array}{r} a-b \\ \hline \end{array}$$

$$\begin{array}{r} a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4 \\ -a^4b+4a^3b^2-6a^2b^3+4ab^4-b^5 \\ \hline \end{array}$$

$$a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5, \text{ fifth Power.}$$

$$\begin{array}{r} a-b \\ \hline \end{array}$$

$$\begin{array}{r} a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5 \\ -a^5b+5a^4b^2-10a^3b^3+10a^2b^4-5ab^5+b^6 \\ \hline \end{array}$$

$$a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6 = 6^{\text{th}} \text{ P.}$$

These Examples are performed by the Theorem as follows :

Here m , Index of the proposed Power, being 6 ; the first Term a^m , of the general Expression is equal to a^6 ; the

second $ma^{m-1}b = 6a^5b$; the third $\frac{m \times m-1}{1 \times 2} \times a^{m-2}b$

$= 15a^4b^2$; the fourth $\frac{m \times m-1 \times m-2}{1 \times 2 \times 3} \times a^{m-3}b =$

$20a^3b^3$;

$20a^3b^3$; the fifth $\frac{m \times m-1 \times m-2 \times m-3}{1 \quad 2 \quad 3 \quad 4} \times a^{m-4}b$
 $= 15a^2b^4$; the sixth $\frac{m \times m-1 \times m-2 \times m-3 \times m-4}{1 \quad 2 \quad 3 \quad 4 \quad 5}$
 $\times a^{m-5}b = 6ab^5$; the seventh $\frac{m \times m-1 \times m-2 \times m-3}{1 \quad 2 \quad 3 \quad 4}$
 $\times \frac{m-4 \times m-5}{5 \quad 6} \times a^{m-6}b = b^6$; and the eighth $\frac{m \times m-1}{1 \quad 2}$
 $\times \frac{m-2 \times m-3 \times m-4 \times m-5 \times m-6}{3 \quad 4 \quad 5 \quad 6 \quad 7} \times a^{m-7}b =$ no-
 thing.

Therefore the sixth Power of $a+b = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$, as before.

71. EVOLUTION.

(2) $\begin{array}{r} x^2 - 2xy + y^2 \\ x^2 \end{array} (x-y, \text{ the Root.}$

$$\begin{array}{r} 2x-y) * -2xy + y^2 \\ \underline{-2xy + y^2} \end{array}$$

* *

(3) $\begin{array}{r} x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4 \\ x^4 \end{array} (x^2 - xy + y^2, \text{ the Rte.}$

$$\begin{array}{r} 2x^2 - xy) * -2x^3y + 3x^2y^2 \\ \underline{-2x^3y + x^2y^2} \end{array}$$

$$\begin{array}{r} 2x^2 - 2xy + y^2 * 2x^2y^2 - 2xy^3 + y^4 \\ \underline{2x^2y^2 - 2xy^3 + y^4} \end{array}$$

* * *

(4) $\begin{array}{r} x^3 - 6x^2y + 12xy^2 + 8y^3 \\ 3x^2) \quad \underline{-6x^2y} \end{array} (x-2y, \text{ the Root.}$

$$\begin{array}{r} x^3 - 6x^2y + 12xy^2 + 8y^3 \\ \underline{} \end{array}$$

* * * *

$$\begin{array}{r}
 (5) \\
 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4(2x-3y, \text{ Rt.} \\
 32x^3) * 96x^3y \\
 \hline
 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 - 81y^4 \\
 \hline
 * * * * *
 \end{array}$$

72. REDUCTION OF SIMPLE EQUATIONS.

(1) Given $20 - 3x - 8 = 60 - 7x$.

Then by transposing $7x$, we shall have $4x + 12 = 60$.

Therefore $x = \frac{60 - 12}{4} = \frac{48}{4} = 12$.

For $20 - 12 \times 3 - 8 = 60 - 12 \times 7 = -24$.

(2) Given $5x - 16 = 3x + 12$.

Here by transposing the $3x$ and 16 , we get $2x = 12 + 16$.

Therefore $x = \frac{12 + 16}{2} = \frac{28}{2} = 14$.

For $14 \times 5 - 16 = 14 \times 3 + 12 = 54$.

(3) Given $\frac{3x}{4} + 5 = \frac{5x}{6} + 2$.

Then (per Rule III) $18x + 120 = 20x + 48$.

Therefore $2x = 72$, whence $x = 72 \div 2 = 36$.

For $\frac{36 \times 3}{4} + 5 = \frac{36 \times 5}{6} + 2 = 32$.

(4) Given $\frac{7x}{8} - 5 = \frac{9x}{10} - 8$.

Then $70x - 400 = 72x - 640$, whence $2x = 240$, and $x = 120$.

For $\frac{120 \times 7}{8} - 5 = \frac{120 \times 9}{10} - 8 = 100$.

(5) Given $\frac{5x}{9} - 8 = 74 - \frac{7x}{12}$.

Then by multiplying both Sides of the Equation by 9 , and 12 , we shall have $60x - 864 = 7992 - 63x$.

Therefore $123x = 7992 + 864 = 8856$, and $x = 8856 \div 123 = 72$.

(6) Given $56 - \frac{3x}{4} = 48 - \frac{5x}{8}$.

Then $1792 - 24x = 1536 - 20x$.

Therefore $4x = 256$, and $x = 256 \div 4 = 64$, the Proof.

(7) Given $36 - \frac{4x}{9} = 8$.

Then by multiplying both Sides of the Equation by 9, we have $324 - 4x = 72$.

Therefore $4x = 324 - 72 = 252$.

Therefore $x = 252 \div 4 = 63$.

(8) Given $\frac{2x}{3} = \frac{176 - 4x}{5}$.

Then $10x = 528 - 12x$.

And $22x = 528$.

Therefore $x = 528 \div 22 = 24$.

(9) Given $\frac{45}{2x+3} = \frac{57}{4x-5}$.

Then multiplying by $2x+3$, we shall have $45 = \frac{114x+171}{4x-5}$; also multiplying by $4x-5$, we have

$180x - 225 = 114x + 171$.

Therefore $180x - 114x - 225 = 171$, that is $66x - 225 = 171$.

Therefore $66x = 171 + 225$, that is $66x = 396$, and $x = 396 \div 66 = 6$.

(10) Given $\frac{42x}{x-2} = \frac{35x}{x-3}$.

Then by multiplying by $x-2$, and $x-3$, we get $42x^2 - 126x = 35xx - 70x$.

Therefore $42x^2 - 35x^2 = 126x - 70x$, that is $7x^2 = 56x$.

Therefore by dividing by $7x$, we have $x = 8$.

(11) Given $\frac{xx-12}{3} = \frac{xx-4}{4}$.

Then by multiplying both Sides of the Equation by 3, and 4, we have $4x^2 - 48 = 3x^2 - 12$.

Therefore $4x^2 - 3x^2 = 48 - 12$, that is $x^2 = 36$.

Whence $x = \sqrt[2]{36} = 6$.

(12) Given $\frac{5xx}{16} = 8 + 12.$

Then by multiplying by 16, we get $5x^2 = 20 \times 16 = 320.$
Therefore $x^2 = 320 \div 5 = 64$, and $x = \sqrt{64} = 8.$

(13) Given $\frac{x+1}{2} + \frac{x+2}{3} = 16 - \frac{x+3}{4}.$

Then by multiplying by 2, 3, and 4, we get $12x + 12 + 8x + 16 = 384 - 6x - 18$, that is $20x + 28 = 366 - 6x.$
Therefore $20x + 6x = 366 - 28$, that is $26x = 338.$
Whence $x = 338 \div 26 = 13.$

(14) Given $ax + b^2 = \frac{ax^2 + ac^2}{a+x}.$

Then $\overline{ax + b^2} \times \overline{a+x} = ax^2 + ac^2$, that is $a^2x + ab^2 + ax^2 + b^2x = ax^2 + ac^2.$

Therefore $a^2x + b^2x = ac^2 - ab^2$, and $x = \frac{ac^2 - ab^2}{a+b}.$

(15) Given $\sqrt{\frac{5x}{3}} + 12 = 17.$

Then $\sqrt{\frac{5x}{3}} = 17 - 12 = 5$, and $\sqrt{5x} = 5 \times 3 = 15.$

Therefore (by Rule IV.) $5x = 225$, and $x = 225 \div 5 = 45.$

(16) Given $\sqrt{12+x} = 2 + \sqrt{x}$, then (by Rule IV.) $12+x = 4 + 4\sqrt{x+x}.$

Whence by transposition $4\sqrt{x} = 8$, and by division $\sqrt{x} = 2$, consequently $x = 4.$

(17) Given $\sqrt{x} + \sqrt{a+x} = \sqrt{\frac{2a}{a+x}}.$

Then $\sqrt{ax+xx} + a+x = 2a$, or $\sqrt{ax+xx} = a-x.$

Whence $ax + x^2 = a^2 - 2ax + x^2.$

Therefore $3ax = a^2.$

$\therefore x = \frac{a^2}{3} \quad \frac{a}{3}$

(18) Given $615x - 7x^3 = 48x$.

Then by transposition $615x - 48x = 7x^3$, that is $7x^3 = 567x$.

Whence divided by $7x$, we get $x^2 = 81$.

$\therefore x = 9$.

(19) Given $\sqrt{a^2 + x^2} = \sqrt[4]{b^4 + x^4}$.

Then by raising both Sides to the fourth Power, we have

$$(a^2 + x^2)^2 = b^4 + x^4, \text{ that is } a^4 + 2a^2x^2 + x^4 = b^4 + x^4.$$

Therefore $2a^2x^2 = b^4 - a^4$.

$$\therefore x^2 = \frac{b^4 - a^4}{2a^2} = \frac{b^4}{a^2} - a^2.$$

73. OF THE EXTERMINATION OF UNKNOWN QUANTITIES; OR, THE REDUCTION OF TWO OR MORE EQUATIONS, TO A SINGLE ONE.

(1) Given $\begin{cases} 5x + 8y = 106 \\ 4x - 5y = 5 \end{cases}$

Here by multiplying the first Equation by 4, and the second by 5, in order that the Coefficient of x may be the same in both, there arises

$$20x + 32y = 424$$

$$20x - 25y = 25$$

By subtracting the latter from the former we have $57y = 399$.

Hence $y = \frac{399}{57} = 7$.

And so by the first Equation $x = \frac{5 + 5 \times 7}{4} = \frac{40}{4} = 10$.

(2) Given $\begin{cases} 5x - 3y = 150 \\ 10x + 15y = 825 \end{cases}$

Here the first Equation being multiplied by 2, (in order that the Coefficients of x , in both Equations, may be the same) we have $10x - 6y = 300$. Let this Equation be subtracted from the second, and we shall have $21y = 525$.

Hence

Hence $y = \frac{525}{21} = 25.$

Therefore by the first Equation $x = \frac{150 + 25 \times 3}{5} = \frac{225}{5} = 45.$

(3) Given $\begin{cases} \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 62 \\ \frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 47 \\ \frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 38 \end{cases}$

Here the given Equations cleared of the Fractions, become

$$12x + 8y + 6z = 1488$$

$$20x + 15y + 12z = 2820$$

$$30x + 24y + 20z = 4560$$

Now, to exterminate z , let the second of these Equations be subtracted from the double of the first. And also, three Times the third, from four Times the second. Whence is had

$$4x + y = 156$$

$$10x + 3y = 420$$

Again, to exterminate y , from three Times the first take the second, and we shall then have $12x - 10x = 468 - 420$. From which $x = \frac{48}{2} = 24.$

Therefore $y = 156 - 24 \times 4 = 60.$

And $z = \left(\frac{1488 - 8y - 12x}{6} \right) = 120.$

(4) Given $\begin{cases} \frac{x}{4} + \frac{y}{5} = 15 \\ \frac{x}{6} + \frac{y}{9} = 9 \end{cases}$

Here our Equations cleared of Fractions will be

$$5x + 4y = 300$$

$$9x + 6y = 486$$

And

And if from double of the latter we take triple of the former, we shall have $18x - 15x = 972 - 900$, that is $3x = 72$.

Whence $x = \frac{72}{3} = 24$.

And $y = \frac{300 - 5x}{4} = 45$.

(5) Given $\begin{cases} \frac{x}{2} - 12 = \frac{y}{4} + 8 \\ \frac{x+y}{5} + \frac{x}{3} - 8 = \frac{2y-x}{4} + 27 \end{cases}$

Here the Equations cleared of the Fractions we shall have

$$4x - 96 = 2y + 64$$

$$\text{And } 12x + 12y + 20x - 480 = 30y - 15x + 1620$$

Which contracted, become

$$4x - 2y = 64 + 96 = 160$$

$$47x - 18y = 2100$$

From the last of which subtract nine Times the former, and we have $41x - 36x = 2100 - 1440$, that is $11x = 660$.

Hence $x = \frac{660}{11} = 60$.

And $y = \frac{4x - 160}{2} = \frac{240 - 160}{2} = \frac{80}{2} = 40$.

(6) Given $\begin{cases} x + y = 80 \\ x + z = 70 \\ y + z = 60 \end{cases}$

Here by subtracting the second Equation from the first (in order to exterminate x) we have $y - z = 10$, to which the third Equation being added, z will likewise be exterminated, there coming out $2y = 70$, or $y = 35$.

Whence $z = y - 10 = 25$, and $x = 80 - y = 45$.

(7) Given $\begin{cases} x + 100 = y + z \\ y + 100 = 2x + 2z \\ z + 100 = 3x + 3y \end{cases}$

To the double of the first, let the second Equation be added: so shall the x 's, on the contrary Sides destroy each other, and we shall have $300 + y = 2y + 4z$, or $y + 4z = 300$.

More-

Moreover, to the triple of the first let the third Equation be added, whence will be had $z+400=6y+3z$, or $2z+6y=400$.

Now if from the double of this last Equation (which $4z+12y=800$) the former (viz. $y+4z=300$) be subtracted, then will come out $11y=500$.

$$\text{Therefore } y = \frac{500}{11} = 45\frac{5}{11}, \text{ and } z = \frac{300-y}{4} = 75 - \frac{y}{4} \\ = 75 - 11\frac{4}{11} = 63\frac{7}{11}.$$

$$\text{And } x = y + z - 100 = 109\frac{1}{11} - 100 = 9\frac{1}{11}.$$

(8) Given $x-y=2$, and $xy+5x-6y=120$.

Then to exterminate x .

By the first Equation $x=2+y$; which Value being substituted in the last (according to Rule II.) it becomes $\overline{y+2} \times y + 5 \times \overline{y+2} - 6y = 120$, that is $y^2 + 2y + 5y + 10 - 6y = 120$, or $y^2 + y = 110$.

(9) Given $\begin{cases} x+y=s \\ x^2-y^2=d \end{cases}$ quere x and y .

Then $x=s-y$; the Square of which is $x^2=s^2-2sy+y^2$.

Also $x^2=d+y^2$.

Therefore $d+y^2=s^2-2sy+y^2$.

Hence $d=s^2-2sy$.

And $2sy=s^2-d$.

$$\text{So } y = \frac{s^2-d}{2s}.$$

$$\text{And } x = \frac{s^2+d}{2s}.$$

(10) Given $\begin{cases} x+y+z=12 \\ x+2y+3z=20 \\ \frac{x}{y}+\frac{y}{2}+z=6 \end{cases}$ To find x, y , and z .

Then we have $\begin{cases} 1. x=12-y-z \\ 2. x=20-2y-3z \\ 3. x=68-\frac{3}{2}y-3z \end{cases}$

From whence we have $\begin{cases} 12-y-z=20-2y-3z \\ 12-y-z=18-\frac{3}{2}y-3z \end{cases}$ these two Equations.

Therefore we have (by Rule I.) $y=4-z=2$, and $x=6$.
Some.

Sometimes the Equations are such, that the same Quantities in different Equations may have contrary Signs, and destroy each other; or to be otherwise affected, so as to shorten the common Progress very much.

$$\text{Thus } \begin{cases} x+y+z=26 \\ x-y=4 \\ x-z=6 \end{cases}$$

Then by Addition only, $3x=36$.

Hence $x=12$, $y=x-4=8$, and $z=x-6=6$.

104. QUADRATIC EQUATIONS.

(1) Given $x^2-4x=32$, to find the Value of x .

Here $x^2-4x+4=32+4=36$, (by Rule III.)

And $x-4=\sqrt{36}=6$, (by Rule IV.)

Therefore $x=6+4=10$.

(2) Given $12x^2-420x=-1200$, to find x .

Here $x^2-35x=-100$, (by Rule II.)

And $x^2-35x+306,25=206,25$, (by Rule III.)

Also $x-17,5=\sqrt{206,25}=14,361406$, (by Rule IV.)

Therefore $x=31,861406$, or $3,138594$.

(3) Given $x^2+60x=216$, to find x .

Here $x^2+15x=54$, (by Rule II.)

And $x^2+15x+56,25=110,25$.

Also $x+7,5=\sqrt{110,25}=10,5$.

Therefore $x=10,5-7,5=3$.

The fifteen following Equations, or Questions, are from
Mr. WARD'S MATHEMATICIAN'S GUIDE.

(1) Given $\begin{cases} a+e=s=240 \\ a-e=d=192 \end{cases}$ to find.

Here by Addition we get $2a=s+d$.

Therefore $a=\frac{s+d}{2}=\frac{240+192}{2}=\frac{432}{2}=216$.

Then by subtracting the second Equation from the first we get $2e=s-d$.

And $e=\frac{s-d}{2}=\frac{240-192}{2}=\frac{48}{2}=24$.

Now as a and e are found, the rest may be very easily found as follows

$$ae = \frac{ss - dd}{4}, \text{ or } 216 \times 24 = 5184 = p.$$

$$\text{And } \frac{a}{e} = \frac{s+d}{s-d}, \text{ or } 216 \div 24 = 9 = q.$$

$$\text{Also } a^2 + e^2 = \frac{s^2 + d^2}{2}, \text{ or } \frac{216 \times 216 + 24 \times 24}{2} = 47232 = z.$$

$$\text{Likewise } a^2 - e^2 = sd, \text{ or } \frac{216 \times 216 - 24 \times 24}{2} = 46080 = x.$$

$$(2) \text{ Given } \begin{cases} a + e = s = 240 \\ ae = p = 5184 \end{cases} \text{ to find } a, e, d, q, z, x.$$

Here by subtracting 4 Times the second Equation from the Square of the first, we have $a^2 - 2ae + e^2 = s^2 - 4p$.

Therefore $a - e = \sqrt{s^2 - 4p} = d$. Now by adding this

Equation to the first we get $2a = s + \sqrt{\frac{s^2 - 4p}{2}}$.

$$\text{Hence } a = \frac{s + \sqrt{ss - 4p}}{2} = 216.$$

And by subtracting we have $2e = s - \sqrt{ss - 4p}$.

$$\text{Hence } e = \frac{s - \sqrt{ss - 4p}}{2} = 24.$$

Therefore the rest may be very easily found by proceeding as in the last.

$$(3) \text{ Given } \begin{cases} a + e = s = 240 \\ \frac{a}{e} = q = 9 \end{cases} \text{ to find the rest.}$$

Here by subtracting the second Equation from the first, we have $e = s - qe$.

$$\text{Therefore } e = \frac{s}{q+1} = \frac{240}{10} = 24. \text{ Which taken from}$$

$$\text{the first Equation gives } a = s - \frac{s}{q+1} = 240 - \frac{240}{10} =$$

$$240 - 24 = 216.$$

From hence the rest may be very easily found.

- (4) Given $\begin{cases} a + e = s = 240 \\ a^2 + e^2 = z = 47232 \end{cases}$ to find the rest.

Here from the Square of the first Equation take the second, and we have $2ae = ss - z$ Which taken from the second Equation gives $a^2 - 2ae + e^2 = 2z - s^2$.

Therefore $a - e = \sqrt{2z - s^2} = d$.

This added to the first Equation gives $2a = s + \sqrt{2z - s^2}$.

Hence $a = \frac{s + \sqrt{2z - s^2}}{2} = 216$.

And subtracted gives $2e = s - \sqrt{2z - s^2}$.

Whence $e = \frac{s - \sqrt{2z - s^2}}{2} = 24$.

- (5) Given $\begin{cases} a + e = s = 240 \\ a^2 - e^2 = x = 46080 \end{cases}$ to find the rest.

Here the second Equation divided by the first gives

$$a - e = \frac{x}{s} = d.$$

Which added to the first, we have $2a = s + \frac{x}{s} = \frac{s^2 + x}{s}$.

Hence $a = \frac{s^2 + x}{2s} = 216$.

And the third subtracted from the first gives

$$2e = s - \frac{x}{s} = \frac{s^2 - x}{s}.$$

Hence $e = \frac{s^2 - x}{2s} = 24$.

- (6) Given $\begin{cases} a - e = d = 192 \\ ae = p = 5184 \end{cases}$ to find the rest.

Here to the Square of the first Equation add 4 Times the second, and we shall have $a^2 + 2ac + e^2 = d^2 + 4p$.

Therefore $a + e = \sqrt{dd + 4p} = s$.

This Equation added to the first gives $2a = d + \sqrt{dd + 4p}$.

Therefore $a = \frac{d + \sqrt{dd + 4p}}{2} = 216$.

Then by taking the first Equation from the fourth, we have $2e = \sqrt{dd + 4p} - d$.

Hence $e = \frac{\sqrt{dd + 4p} - d}{2} = 24$.

(7) Given $\left\{ \begin{array}{l} a - e = d = 192 \\ \frac{a}{e} = q = 9 \end{array} \right\}$ to find the rest.

Here by transposing e in the first Equation, and multiplying the second by e , we get $qe = d + e$.

Then $e = \frac{d}{q-1} = 24$.

For $9-1 \times e = qe - e$. And by adding this last Equation to the first we have $a = d + \frac{d}{q+1} = 216$.

(8) Given $\left\{ \begin{array}{l} a - e = d = 192 \\ a^2 + e^2 = z = 47232 \end{array} \right\}$ to find the rest.

Here by taking the square of the first from the second, we have $2ae = z - dd$. This added to the second Equation gives $a^2 + 2ae + e^2 = 2z - dd$. Therefore $a + e = \sqrt{2z - dd}$. Which added to the first Equation gives $2a = d + \sqrt{2z - dd}$.

Therefore $a = \frac{d + \sqrt{2z - dd}}{2} = 216$.

Then by taking the first Equation from the fifth we get

$2e = \sqrt{2z - dd} - d$. Hence $e = \frac{\sqrt{2z - dd} - d}{2}$.

(9) Given $\left\{ \begin{array}{l} a - e = d = 192 \\ a^2 - 2e = x = 46080 \end{array} \right\}$ to find the rest.

Here by dividing the second Equation by the first, we

have $a + e = \frac{x}{d}$. Then by adding this Equation to the

first, we get $2a = d + \frac{x}{d}$. Hence $a = \frac{dd + x}{2d}$.

And

And by taking this Equation from the third, we have

$$e = \frac{x - dd}{2d}.$$

(10) Given $\left\{ \begin{array}{l} ae = p = 5184 \\ \frac{a}{e} = q = 9 \end{array} \right\}$ to find the rest.

Here by multiplying the two Equations together, we have

$$aa = qp. \text{ For } \frac{ae}{1} \times \frac{a}{e} = \frac{aae}{e} = aa. \text{ Then } a = \sqrt{qp}, \\ = 216.$$

And by dividing the first Equation by the second, we shall

$$\text{have } ee = \frac{p}{q}.$$

$$\text{For } \frac{a}{1} \left(\frac{ae}{1} \right) \left(\frac{ae}{a} = ee. \text{ Hence } e = \sqrt{\frac{p}{q}} = 24.$$

(11) Given $\left\{ \begin{array}{l} ae = p = 5184 \\ a^2 + e^2 = z = 47232 \end{array} \right\}$ to find the rest.

Here to the second add twice the first, and we have $a^2 + 2ae + e^2 = z + 2p$.

$$\text{Therefore } a + e = \sqrt{z + 2p} = s.$$

Then by taking twice the first Equation from the second, we have $a^2 - 2ae + e^2 = z - 2p$.

Hence $a - e = \sqrt{z - 2p}$. This Equation added to the fourth gives $2a = \sqrt{z + 2p} + \sqrt{z - 2p}$.

$$\text{Therefore } a = \frac{\sqrt{z + 2p} + \sqrt{z - 2p}}{2} = 216.$$

Then by taking the sixth Equation from the fourth, we

$$\text{have } 2e = \sqrt{z + 2p} - \sqrt{z - 2p}.$$

$$\text{Hence } e = \frac{\sqrt{z + 2p} - \sqrt{z - 2p}}{2} = 24.$$

(12) Given $\left\{ \begin{array}{l} ae = p = 5184 \\ a^2 - e^2 = x = 46380 \end{array} \right\}$ to find the rest.

Here to the Square of the second Equation add 4 Times the Square of the first, and we shall have $a^4 + 2a^2e^2 + e^4 = xx + 4p^2$. Hence $a^2 + e^2 = \sqrt{x^2 + 4p^2} = z$.

This Equation added to the second will give $2a^2 = x + \sqrt{x^2 + 4p^2}$. Whence $a = \sqrt{x + \sqrt{x^2 + 4p^2}} = 216$. Then by subtracting the second Equation from the fourth, we have $2e^2 = \sqrt{x^2 + 4p^2} - x$.

$$\text{Hence } e = \sqrt{\frac{\sqrt{x^2 + 4p^2} - x}{2}} = 24.$$

$$(13) \text{ Given } \left\{ \begin{array}{l} \frac{a}{e} = q = 9 \\ a^2 + e^2 = z = 47232 \end{array} \right\} \text{ to find the rest.}$$

Here from the second Equation take the Square of the first, and we shall have $ee = z - qqee$, or $qqee + ee = z$.

$$\text{Hence } ee = \frac{z}{qq + 1}. \text{ For } qq + 1 \times ee = qqee + ee. \text{ There-}$$

$$\text{fore } e = \sqrt{\frac{z}{qq + 1}} = 24.$$

Then by taking this last Equation from the second, we

$$\text{have } aa = z - \frac{z}{qq + 1} = \frac{qqz}{qq + 1}. \text{ Hence } a = \sqrt{\frac{qqz}{qq + 1}} = 216.$$

$$(14) \text{ Given } \left\{ \begin{array}{l} \frac{a}{e} = q = 9 \\ aa - ee = x = 46080 \end{array} \right\} \text{ to find the rest.}$$

Here by comparing the first when multiplied by e , and squared with the second more ee , we have $qqee = x + ee$, or $qqee - ee = x$.

$$\text{Hence } ee = \frac{x}{qq - 1}.$$

$$\text{Therefore } e = \sqrt{\frac{x}{qq - 1}} = 24.$$

Then by adding the fourth Equation to the second, we get

$$aa = x + \frac{x}{qq - 1} = \frac{qqx}{qq - 1}. \text{ Hence } a = \sqrt{\frac{qqx}{qq - 1}} = 216.$$

(15) Given $\begin{cases} aa+ee=z=47232 \\ aa-ee=x=46080 \end{cases}$ to find the rest.

Here by addition (only) we have $2aa=x+z$.

$$\text{Hence } a = \sqrt{\frac{x+z}{2}}.$$

Then by subtracting the second Equation from the first, we get $2ee=z-x$.

$$\text{Therefore } e = \sqrt{\frac{z-x}{2}}.$$

The Reason of my proceeding (only) with the first Question throughout, is being confined, not having Room; but I hope this will not be the Pupil's Case.

P R O B L E M S.

(1) Here let x = the greater Number.

Then $x-20$ = the lesser.

Which added together gives $2x-20=70$.

Therefore $2x=70+20=90$.

And $x=90 \div 2=45$, the greater Number.

$\therefore 45-20=25$, the lesser.

For $45-25=20$.

And $45+25=70$, the Proof.

(2) Here let x = the greater Number.

Then $x-14$ = the lesser.

Therefore by the Quest. $\frac{x}{x-14}=3$.

Hence $x=3x-42$, and $2x=42$.

$\therefore x=42 \div 2=21$, the greater Number.

Likewise $21-14=7$, the lesser.

For $21-7=14$.

Also $21 \div 7=3$, the Proof.

(3) Suppose x to be the Number.

Then by the Quest. $\frac{x}{3} + \frac{x}{4} = 21$.

Therefore $7x=252$.

Hence $x=36$, the Number required.

For $\frac{36}{3} + \frac{36}{4} = 12 + 9 = 21$, the Proof.

(4) Let x be the Number.

Then per Quest. $\frac{x}{3} - \frac{x}{4} = 4$.

Hence $x = 48$, the Number required.

For $\frac{48}{3} - \frac{48}{4} = 16 - 12 = 4$, the Proof.

(5) Suppose $x =$ the Number required:

Then per Quest. $\frac{x}{3} - 4 = \frac{x}{4} - 25$.

Hence $4x - 48 = 3x - 3$.

Therefore by Transposition $x = 45$.

For $\frac{45}{3} - 4 = \frac{45}{4} - 25 = 15 - 4 = 11, 25 - 25$, the Proof.

(6) Let x represent the greater, and $x - 8$ the lesser Number.

Then per Quest. $16x - 64 = 208$.

Hence $16x = 272$, and $x = 17$, the greater.

Also $17 - 8 = 9$, the lesser Number.

For $17 - 9 = 8$, the Difference.

And $17 \times 17 - 9 \times 9 = 289 - 81 = 208$, the Proof.

The above Question may be solved by making use of two Letters. See Mr. WARD's 9th Question.

(7) Let $x =$ the greater Number, and y the lesser.

Then by the Quest. we shall have $x + y = 60$.

And as $x : y :: 9 : 3$.

Therefore $9y = 3x$, or $9y - 3x = 0$.

To which add the first Equation multiplied by 3, and we shall have $12y = 180$.

$\therefore y = 15$, the lesser Number.

And $60 - 15 = 45$, the greater.

For $45 + 15 = 60$.

And as $45 : 15 :: 9 : 3$, the Proof.

(8) Here let $x =$ the greater Number, and y the lesser.

Then per Quest. $xy = 108$.

And $\frac{3x}{y} = 4$.

Therefore by destroying x , we shall have $\frac{108}{y} = \frac{4y}{3}$.

Hence

Hence $4y^2 = 324$, and $y^2 = 81$.

$\therefore y = 9$, the lesser Number.

And $\frac{108}{9} = 12$, the greater.

For $12 \times 9 = 108$.

Also $\frac{12 \times 3}{9} = \frac{36}{9} = 4$, the Proof.

(9) Let $x =$ the greater Number, and y the lesser.

Then by the Question we shall have

$$1. \quad x + y + 8 = 2x$$

$$2. \quad x - y - 4.5 = \frac{y}{2}$$

Then by multiplying the second Equation by 2, we get

$$2x - 2y - 9 = y.$$

And by the first Equation $x = y + 8$.

Which substituted for x in the last Equation, we shall have

$y = 7$, the lesser Number.

Then $x = 7 + 8 = 15$, the greater.

For $15 + 7 + 8 = 15 \times 2 = 30$.

Also $15 - 7 - 4.5 = \frac{7}{2} = 3.5$, the Proof.

(10) For the Numbers sought, put x, y , and z .

Then by the Question we shall have

$$\left. \begin{array}{l} 1. \quad x + \frac{y+z}{2} = \\ 2. \quad y + \frac{x+z}{3} = \\ 3. \quad z + \frac{x+y}{4} = \end{array} \right\} 34 = a.$$

Then by clearing the Equations of the Fractions, we get as follows

$$4. \quad 2x + y + z = 2a$$

$$5. \quad 3y + x + z = 3a$$

$$6. \quad 4z + x + y = 4a$$

Now (to exterminate z) let the fourth Equation be subtracted from the fifth; also the sixth from four Times the fifth, whence is had

$$7. \quad 2y - x = a$$

$$8. \quad 11y + 3x = 8a$$

From which we have $2y - a = \frac{8a - 11y}{3}$.

There-

Therefore $17y = 11a = 374$.

Hence $y = \frac{374}{17} = 22$.

Then by the seventh, $x = 44 - 34 = 10$.

Also by the fourth, $z = 68 - 42 = 26$.

For $10 + \frac{22 + 26}{2} = 34$.

And $22 + \frac{10 + 26}{3} = 34$.

Also $26 + \frac{10 + 22}{4} = 34$, the Proof.

(11) Let $3x =$ the major Part of the first Division.

Then $100 - 3x =$ the minor Part.

And by the Question we shall have $x =$ the minor Part of the second Division.

Also $100 - x =$ the major Part.

Whence by the Question $100 - x = 200 - 6x$.

Hence $x = \frac{100}{5} = 20$, the minor Part of the 2d Division.

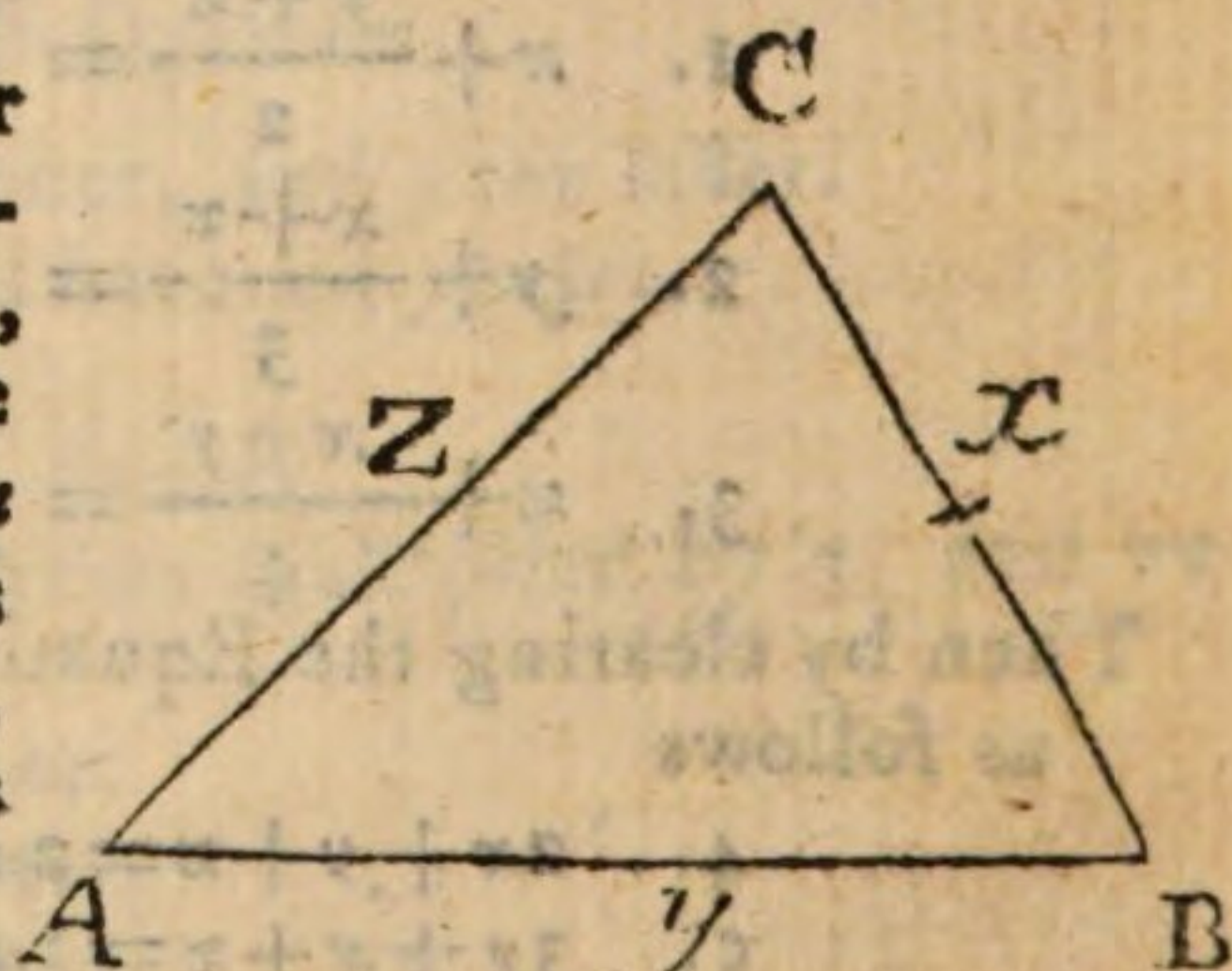
And $100 - 20 = 80$, the major Part.

Again $20 \times 3 = 60$, major } Part of first Division.

And $100 - 60 = 40$, minor }

For $60 = 20 \times 3$, and $80 = 40 \times 2$, the Proof.

(12) Here put s and c for the natural Sine and Cosine of the Angle ABC, Radius $= 1$, a for the Area of the Triangle, m for the Sum of the Cubes of the Sides, and x , y , and z , for the Sides, as in the Figure.



Then we shall have $sxy = 2a$, and $x^2 + y^2 = z^2 + 2cxy$ (per Trigonometry.)

Hence $xy \frac{2a}{s}$ which call p , and $x^2 + y^2 = z^2 + 2cp$, for

$2cp$ put b ; then $x^2 + y^2 = z^2 + b$.

Now $x = \frac{p}{y}$, $\therefore x^3 = \frac{p^3}{y^3}$, and therefore the Equa-

tion

tion $x^3 + y^3 + z^3 = m$, as given by the Quest. becomes

$$\frac{p^3}{y^3} + y^3 + z^3 = m, \text{ whence } z^3 = m - y^3 - \frac{p^3}{y^3}.$$

Again $z^2 = x^2 + y^2 - b$, whence $z^2 = \frac{p^2}{y^2} + y^2 - b$, con-

$$\text{sequently } z^6 = m - y^3 - \frac{p^3}{y^3} \Bigg|^2, \text{ and } z^6 = \frac{p^2}{y^2} + y^2 - b \Bigg|^3,$$

$$\text{hence } \overline{my^3 - y^6 - p^3}^2 = \overline{p^2 + y^4 - by^2}^3.$$

This Equation properly reduced will give $y=8$, from whence x is found to $=10$, and $z=6$, the Sides required.

Suppose the Sum of the Squares of the Sides had been given, instead of the Sum of the Cubes; then we should have had $x^2 + y^2 + z^2 = m$, and the other Equations as before, viz. $xy=p$, and $x^2 + y^2 = z + b$, hence $z^2 + b +$

$$z^2 = m; \text{ or } z = \sqrt{\frac{m-b}{2}}, \text{ which being now known,}$$

put $z^2 + b = n$, then $x^2 + y^2 = n$; but $xy=p$, therefore $x^2 + 2xy + y^2 = n + 2p$, and $x^2 - 2xy + y^2 = n - 2p$. Con-

sequently $x + y = \sqrt{n + 2p}$, and $x - y = \sqrt{n - 2p}$; therefore $x = \sqrt{n + 2p} + \sqrt{n - 2p}$.

(13) Let x represent A's Contribution.

Then $x + 10 = \text{B's}$ } by the Question.

And $2x + 10 = \text{C's}$ }

Therefore $4x + 20 = 76$.

$$\text{Hence } x = \frac{76 - 20}{4} = 14 \text{ A's.}$$

And $14 + 10 = 24$ B's.

Also $14 \times 2 + 10 = 38$ £. C's Contribution.

(14) Let $x =$ the Length.

Then $52 - x =$ the Breadth.

Now $52x - xx = 480$, per Question.

Therefore $xx - 52x = -480$.

And $xx - 52x + 676 = 676 - 480 = 196$.

$$\therefore x - 26 = 14.$$

Hence $x = 40$ in Length.

And $52 - 40 = 12$, in Breadth.

This Question may be solved by making use of two Letters. See Mr. WARD's second Question.

(15) Suppose $x =$ the Quantity sold at $12d.$ per lb.

Then as $\frac{3}{4} : \frac{2}{3} :: x : \frac{8x}{9} =$ the other at $15d.$ per lb.

Therefore by multiplying each Quantity by its Price, we

$$\text{have } 12x + \frac{40x}{3} = 19 \times 12 = 228.$$

$$\text{Hence } 36x + 40x = 684.$$

$$\text{From which } x = \frac{684}{76} = 9 \text{ lb. at } 1s. \text{ and } \frac{8x}{9} = 8 \text{ lb. at } 1s.$$

$3d$ per lb.

For as $9 : 8 :: \frac{3}{4}$ to $\frac{2}{3}$, the Proof.

(16) Suppose he had x Guineas.

$$\text{Then by the Quest. } x - \frac{x}{4} - \frac{x}{5} = 66.$$

$$\therefore 20x - 9x = 1320.$$

$$\text{Hence } 11x = 1320.$$

$$\text{And } x = 120 \text{ Guineas, had at first.}$$

(17) Let $x =$ the greater, and y the lesser Number.

$$\left. \begin{array}{l} \text{Then } xx + xy = 77 \\ \text{And } xy - yy = 12 \end{array} \right\} \text{ by the Question.}$$

Now by destroying xy , we get the following Equation

$$77 - xx = 12 + yy.$$

$$\text{Hence } xx = 65 - yy.$$

$$\therefore x = \sqrt{65 - yy}.$$

Then by substituting this for x , in the second Equation,

$$\text{we have } y\sqrt{65 - yy} - yy = 12.$$

$$\text{Or } \sqrt{65 - yy} = \frac{12 + yy}{y}.$$

$$\text{Then by squaring each Side, } 65 - yy = \frac{144 + 24y^2 + y^4}{y^2}.$$

$$\text{Or } 2y^4 - 41y^2 = -144.$$

$$\text{Therefore by } \div 2, \text{ we have } y^4 - 20,5y^2 = -72.$$

$$\text{And by completing the Square, we get } y^4 - 20,5y^2 + 105,0625 = 33,0625.$$

$$\text{Or } y^2 - 10,25 = \sqrt{33,0625} = 5,75.$$

$$\text{Then } y^2 = 16.$$

$$\text{Hence } y = 4, \text{ and } 65 - 16 = 49 = x^2.$$

$$\text{Therefore } x = 7.$$

(18) Let $x =$ the Number required.

Then by the Quest. $4xx - 40x + 100 - 1 = 7x$.

Or $4x^2 - 47x = -99$.

Therefore by $\div 4$, we have $x^2 - 11,75x = -24,75$.

Or $x^2 - 11,75x + 34,515625 = 9,765625$.

$\therefore x - 5,875 = \sqrt{9,765625} = 3,125$.

Hence $x = 3,125 + 5,875 = 9$, the Number required.

For $9 \times 2 - 10 = 8$.

Also $8 \times 8 - 1 = 9 \times 7 = 63$, the Proof.

(19) Here let $x =$ his part, and y her's.

Then $\left\{ \begin{array}{l} \frac{x}{5} = \frac{y}{4} + 10 \\ x + y = 1000 \end{array} \right\}$ by the Question.

Therefore by destroying the x 's we get $\frac{5y + 200}{4} = 1000y$.

Or $5y + 200 = 4000 - 4y$.

Then $y = 3800 \div 9 = 422\frac{2}{9}$ her } Share.

And $1000 - 422\frac{2}{9} = 577\frac{7}{9}$ his }

(20) Let $x =$ the least Number.

Then we shall have $x, x+2, x+4$, and $x+6$, for the Numbers.

But $x \times x+2 \times x+4 \times x+6 = x^4 + 12x^3 + 44x^2 + 48x$.

Therefore $x^4 + 12x^3 + 44x^2 + 48x = 945$, (by the Quest.)

Wherefore $105x^2 = 945$. Hence $x^2 = 9$. And $x = 3$, the least.

Therefore the Numbers are 3, 5, 7, and 9.

(21) Put $x =$ his Age in Months.

Then $x + \frac{x}{2} + \frac{x}{8} - 1 = 21 \times 21 = 441$, (by the Quest.)

Or $16x + 8x + 2x - 16 = 7056$. Or $26x = 7072$.

Hence $x = \frac{7072}{26} = 272$ m. or 22 yrs 8 m. his Age.

For $272 + \frac{272}{2} + \frac{272}{8} - 1 = 21 \times 21 = 441$, the Proof.

(22) Let x, y , and z , be the Numbers required.

Then per Quest. $x - y = 6$; or $y - 6 = x$, and $z - y = 15$; or $z = 15 + y$.

But $x : y :: z$; that is $y - 6 : y :: y : 15 + y$.

H h

The e

Therefore $yy = 90 - 9y + 1y$. Hence $9y = 90$. And $y = 10$, the second.

Then $10 - 6 = 4$, the first.

And $15 + 10 = 25$, the third Number.

(23) Suppose x to be the Number.

Then $x^6 - 19x^3 = 216$, (by the Quest.)

Put $x^3 = z$, $x^6 = z^2$, and it will be $z^2 - 19z + \frac{361}{4} =$

$216 + \frac{361}{4} = \frac{1225}{4}$, (per Rule p 332. in the Guide.)

And $z - \frac{19}{2} = \pm \frac{35}{2}$. Whence $z = \frac{19 \pm 35}{2} = 27,$

or $= -8$. But $x = \sqrt[3]{z}$. Therefore $x = +3$, or -2 .

(24) Let $x =$ the Length.

Then per Quest. $3 : 2 :: x : \frac{2x}{3} =$ the Breadth.

Therefore $\frac{x \times 2x}{3} = 60000$, or $2xx = 180000$.

Hence $xx = 90000$. And $x = 300$ Men in Rank.

Also $\frac{300 \times 2}{3} = 200$ Men in File.

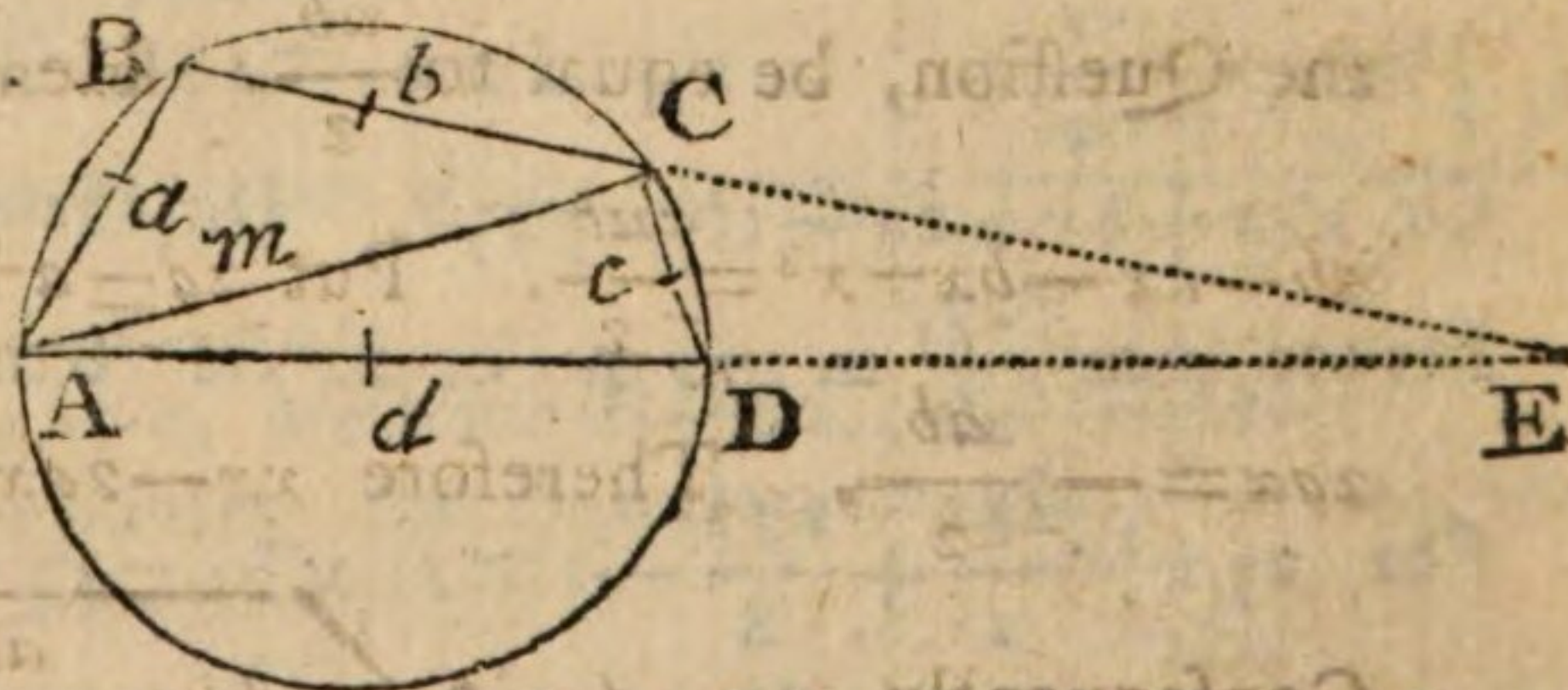
Now $\frac{300 - 1}{1} \times 2,75 = 822,25$ yds. Length.

And $\frac{200 - 1}{1} \times 2,75 = 547,25$ ditto Breadth.

4840) 4499763125 (92 acr. 3 rds. 35,2 p. the

Ground they stand on.

(25) Put $a =$
 $AB = 15,60,$
 $b = BC =$
 $13,2, c =$
 $CD = 10,$
 and $d = AD$
 $= 26.$



Also $m = AC$, and y the Co-sine of the Angle D, and $-y =$ the Co-sine of the Angle B.

Then (per Theo. p. 12. in Mr. ANT. TACKER'S MISCEL.) we shall have $a^2 + b^2 + 2ab = m^2$, and $d^2 + c^2 - 2cd = m^2$.

$$\therefore a^2 + b^2 + 2ab = d^2 + c^2 - 2dc. \text{ Hence } 2aby + 2dcy = \frac{d^2 + c^2 - a^2 - b^2}{2ab + 2ac}, \text{ whose}$$

Sine call s .

$$\text{Then } \frac{abs + dcs}{2} = 21,504, \text{ the Area of the Trapezium ABCD.}$$

$$\text{Or thus, suppose } DE = x. \text{ Then by sim. Triangles } a : d + x :: c : \frac{d + x \times c}{a} = 50 = CE. \therefore BE = \frac{ba + d + x \times c}{a}.$$

$$\text{Again } a : \frac{ba + d + x \times c}{a} :: c : x. \text{ Hence } x = \frac{ab + cd \times c}{a^2 - c^2} =$$

32,5. And the Area of the Trapezium ABCD = 21,504 Acres, as before (by Sect. 70. Prob. III. Rule III.)

(26) Suppose x = the Number of Servants.

Then by the Appendix we have $\frac{x + 1 - x}{x - 1} = 960799$, all the possible Variations of the Servants; which Equation solved, will give $x = 7$, the Number of Servants required.

(27) Put $AB = 100 = a$, $BC = 80 = b$, $AF = CD$, the Breadth of G the Walk $= x$.

Then $FG = a - x$, $GD = 6 - x$. And $FG \times$

$GD = a - x \times b - x = ab - ax - bx + x^2$, which mult by the Question, be equal to $\frac{ab}{2}$. Hence this Equation

$$ab - ax - bx + x^3 = \frac{ab}{2}. \text{ Put } 2d = a + b. \text{ Then } xx -$$

$$2dx = -\frac{ab}{2}. \text{ Therefore } xx - 2dx + dd = dd - \frac{ab}{2}.$$

$$\text{Consequently } x - d = \sqrt{dd - \frac{ab}{2}}. \text{ Hence } x = d +$$

$$\sqrt{dd - \frac{ab}{2}} = 90 - 64,031 = 25,969, \text{ the Breadth req.}$$

(28) Put x the side of the Square.

Then $xx + 284 =$ the whole Army.

And $x + 1 =$ the Number of Men designed for the Side of the Square.

Consequently $xx + 2x + 1 - 25 = xx + 284$. And by Transposition $2x = 308$. Hence $x = 154$ Men, the Side of the sq.

Wherefore $154 \times 154 + 284 = 24000$ Men, the whole Army required.

(29) Suppose $x =$ what he received.

Then per Quest. $\frac{x-94}{2} + \frac{x-94}{5} + \frac{x}{10} + 94 = x$.

This Equation cleared of the Fractions, we shall have

$$50x - 4700 + 20x - 1880 + 10x + 9400 = 100x, \text{ or } 20x = 2820.$$

Hence $x = 141\text{£}$. the Sum required.

(30) Put $BF = 9 = a$, $DE = 2 = b$, and $x = AD = DE$.

Then $DB = a + x$, and $AB = EB = a + x - b$.

Therefore by the 47th. of EUCLID's first Book, we

have $DB^2 = AB^2 + AD^2$, that is $a^2 + 2ax + x^2 = x^2 + a^2 + 2ax - 2ab + x^2 - 2bx + b^2$, or $x^2 - 2bx = 2ab - b^2$.

Then by Comp. the Square we get $x^2 - 2bx + b^2 = 2ab$.

Therefore $x - b = \sqrt{2ab}$. Hence $x = \sqrt{2ab} + b =$

$\sqrt{2 \times 9 \times 2} + 2 = 8 = AD$, the Breadth.

Also $9 + 8 = 17 = AB$, the Length.

(31) Here $AB = 13 = a$,

$AC = 14 = b$, $BC =$

$15 = c$, and $x = BD$.

Then by the 47th. of

Book I. of EUCLID.

we shall have AB^2

$- BD^2 = AC^2 -$

CD^2 . Or $a^2 - x^2$

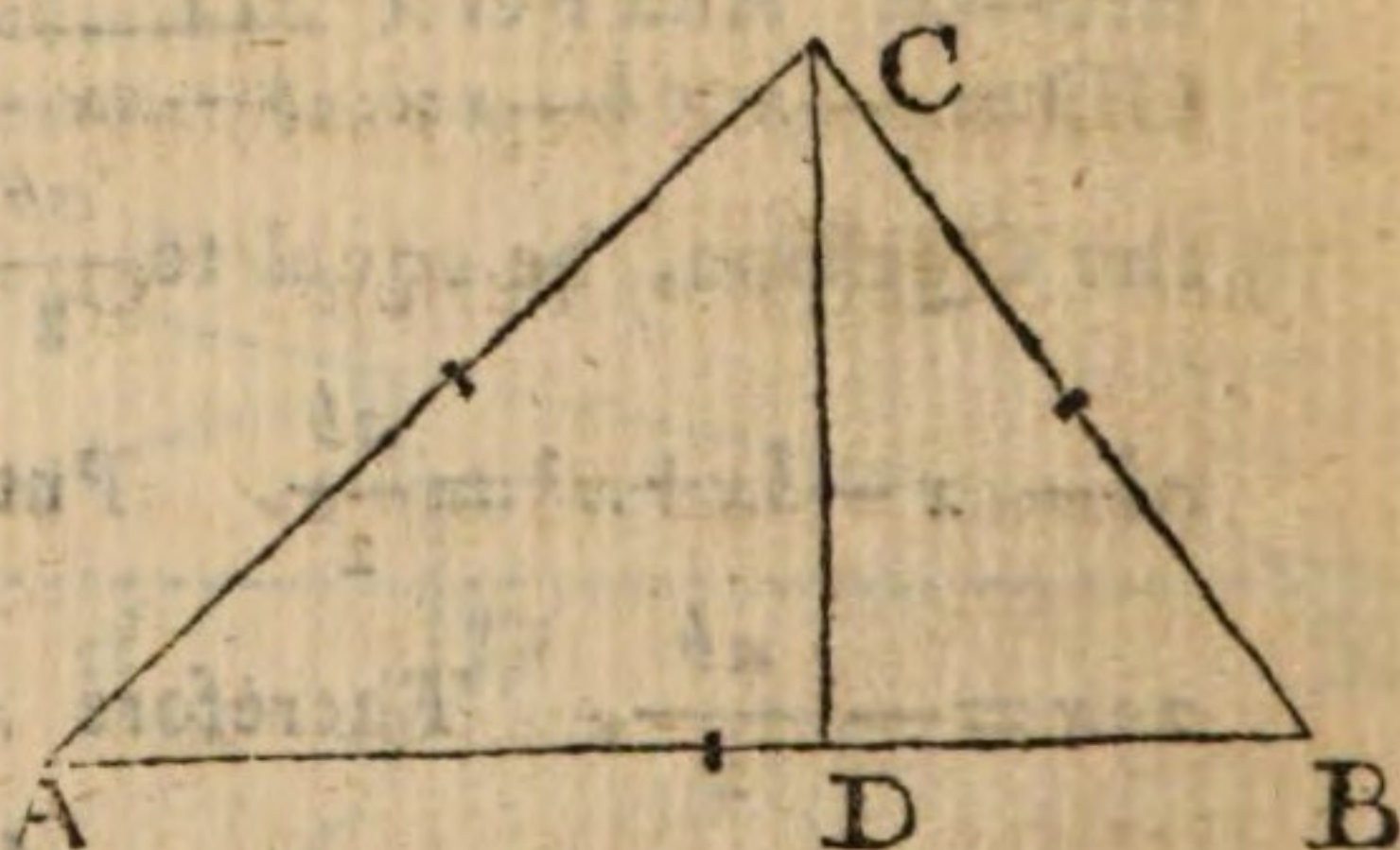
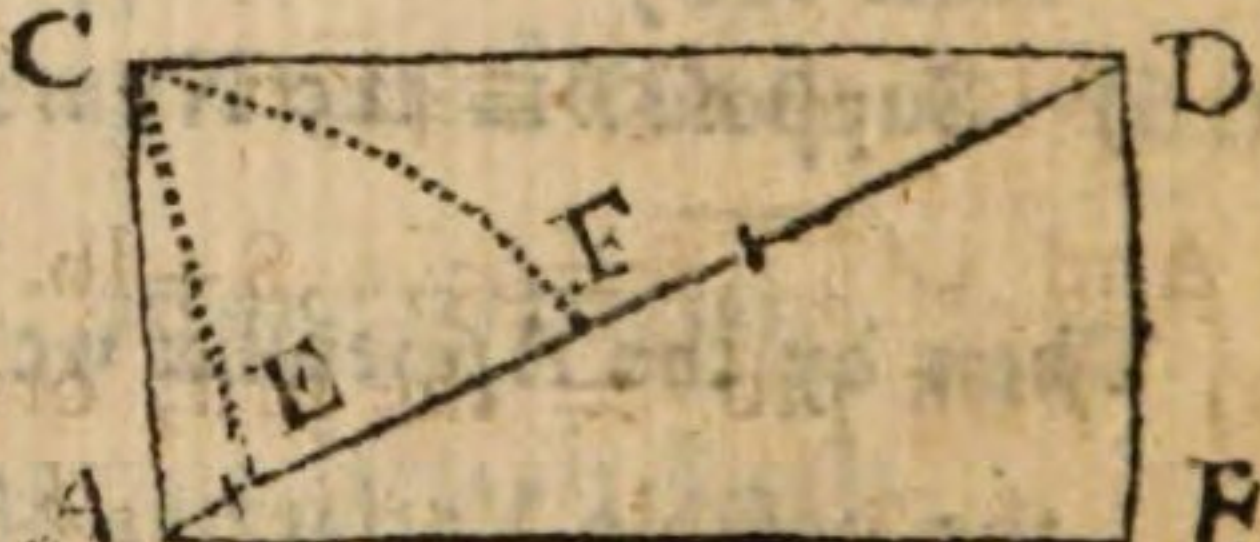
$= b^2 - c^2 + 2cx - x^2$,

that is $a^2 = b^2 - c^2 + 2cx$.

And by Transposition we get $2cx = a^2 - b^2 + c^2$. Hence x

$$= \frac{a^2 - b^2 + c^2}{2c} = \frac{169 - 296 + 225}{30} = \frac{198}{30} = 6,6 = BD.$$

(32) And $15 - 6,6 = 8,4 = DC$. Q. E. F.



(32) Let x = the Breadth fought.

$$18 + 2x = \text{Length AB or DC.}$$

$$12 + 2x = \text{AD or BC.}$$

$$\text{Hence } 4x^2 + 60x + 216 = 18$$

$$\times 12 \times 2 = 432. \text{ Or } 4x^2 +$$

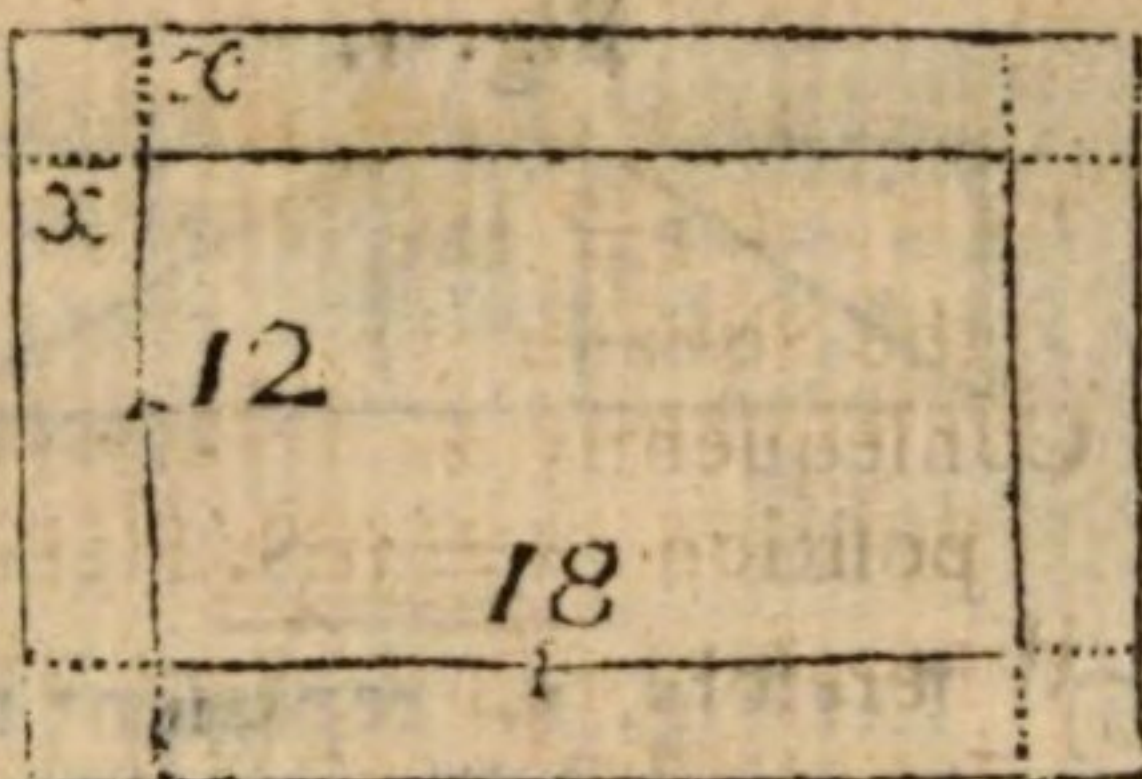
$$60x = 432 - 216 = 216.$$

$$\text{Which } \div 4 \text{ gives } x^2 + 15x = 54.$$

$$\text{Now by Comp. the Square, we have } x^2 + 15x + 56,25 = 110,25.$$

$$\text{Or } x + 7,5 = 10,5.$$

$$\text{Therefore } x = 10,5 - 7,5 = 3, \text{ the Breadth required.}$$



QUESTIONS, ANSWERED BY COMMON ARITHMETIC, ALGEBRA, AND GEOMETRY.

(1) First $76 \times 56 = 4256$.

And $\sqrt[2]{4256} = 65,238 + \text{lb.}$ the true Weight required.

(2) First 1 lb. = the first, or least Weight.

lb.

Then $2 + 1$ = 3 first. } Weight.

And $3 + 1 \times 2 + 1$ = 9 second. }

Also $9 + 3 + 1 \times 2 + 1$ = 27 third. }

Sum of which = 40 the whole Weight.

(3) First 6s. $\frac{1}{4}d.$ = 289 Farthings.

Then $\sqrt[2]{289} = 17$ Persons. Or 17 qrs. = $4\frac{1}{4}d.$ what each spent.

(4) Here x = the Number of Spots. $n = 7$, $r = 12$, and $w = 52$.

$$\text{Then per Theo. } x = 13n + w + r = 13 \times 7 + 52 + 12.$$

$$\text{Therefore } x = 103 + 52 = 51 \text{ Spots.}$$

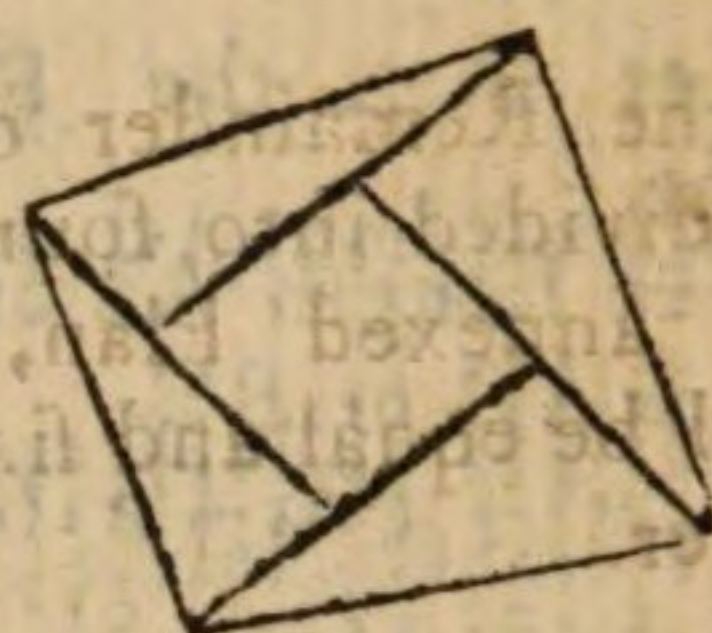
(5) Supposing the Oranges to be laid in Rows, and upon each other; then the Solution will be thus $2,5 \times 2,5 \times 2,5 \times 2000 = 31250$.

$$\therefore \sqrt[3]{31250} = 31,498 \text{ Inches, Inside of the Box.}$$

Fig. 1.



Fig. 2.



(6) Let Fig. 1. represent the Plank as it is to be cut, and the second Fig. as they are placed to make the Square.

(7) The Hour hand goes only $\frac{1}{12}$ the Circumference in an Hour, the other goes the whole, or $\frac{12}{12} = 1$.

Then $\frac{12}{12} - \frac{1}{12} = \frac{11}{12}$, Minute-hand gains in an Hour.

Therefore if $\frac{11}{12}$ cir. : 1 h. :: 1 cir. : $\frac{12}{11}$ h. = 1 h. $5\frac{5}{11}$ min.

$\therefore$ 1 h. $5\frac{5}{11} \times 5 = 27\frac{3}{11}$ min. past 5, the Time required.

For when the Hands are in Opposition, the Minute must be $\frac{6}{11}$ a head of the Hour hand.

Hence as 11 : 1 h. or 60 min. :: 6 : $32\frac{8}{11}$ min. past 12.

And for the next Conjunction it will be as 11 : 1 h. ::

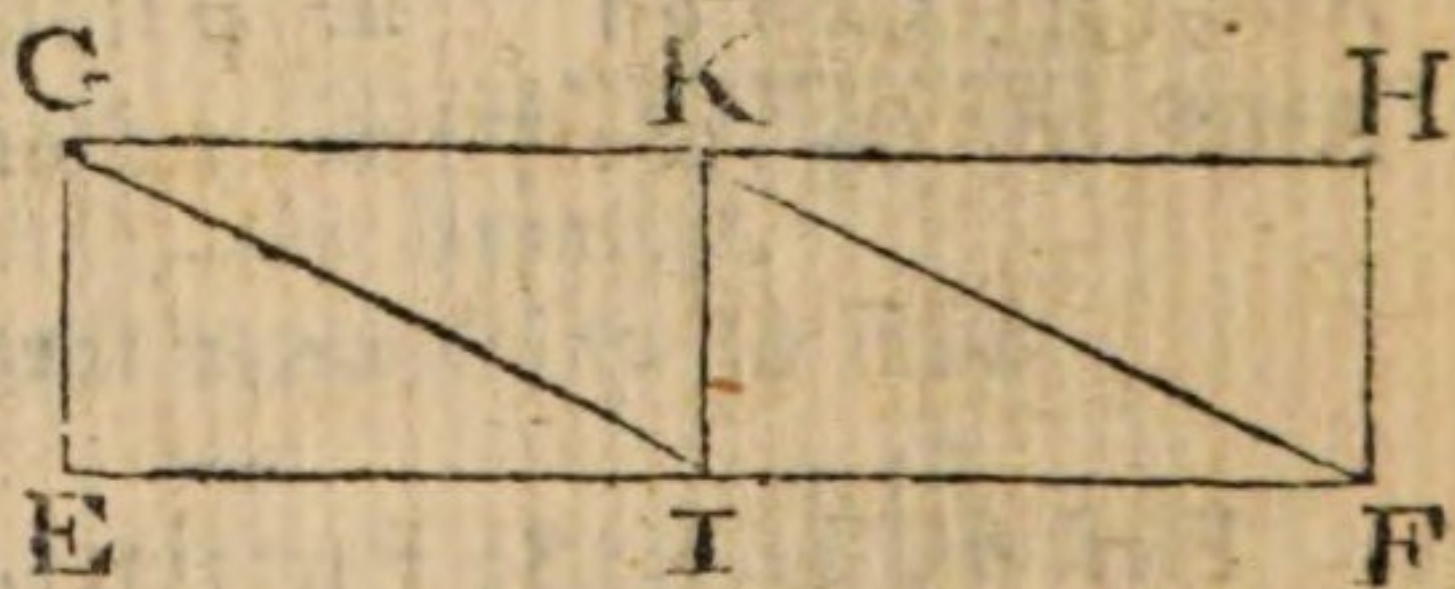
6×2 (12) : $\frac{12}{11} = 1$ h. $5\frac{5}{11}$ min. as above.

As 11 : 1 :: 12 + 12 (24) : $\frac{24}{11} = 2$ h. $11\frac{9}{11}$, the next Conjunction.

Also 11 : 1 :: (12 $\times$ 5260) : $\frac{60}{11} = 5$ h. $27\frac{3}{11} = 27\frac{5}{11}$ min. past 5, the Answer as above.

Fig. 1.

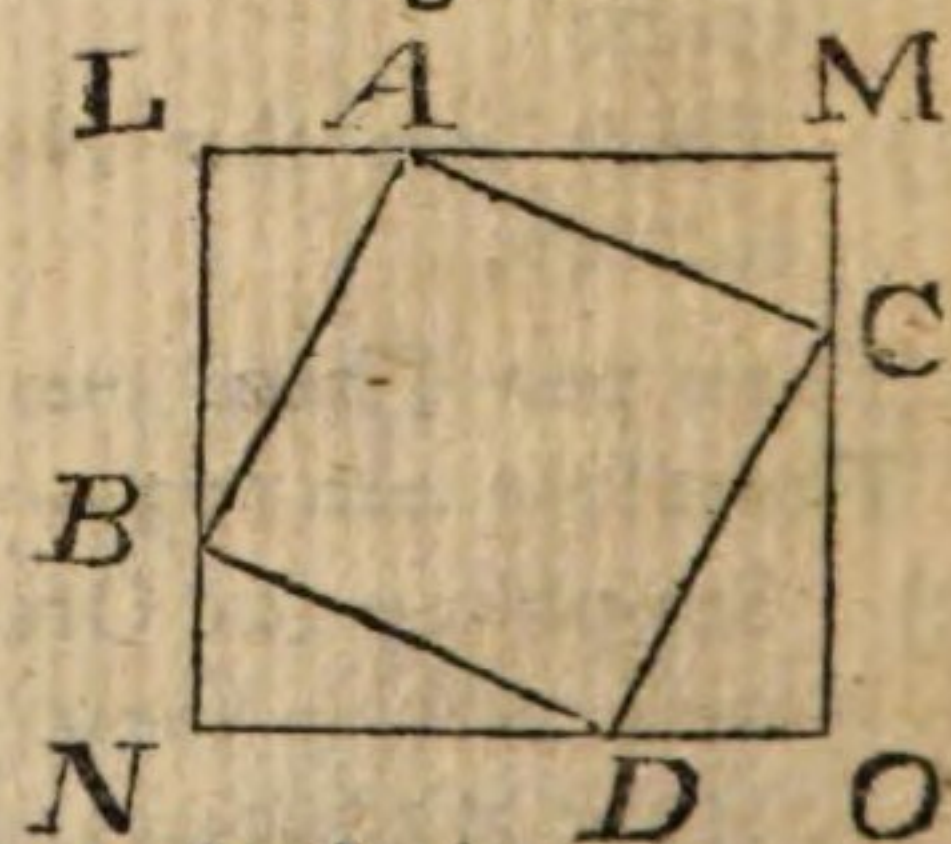
(8) Let ABCD, Fig. 2. represent the given Table = 27 Inches, and the Parallelogram EFGH Fig. 1. represent the Plank = 4 ft.



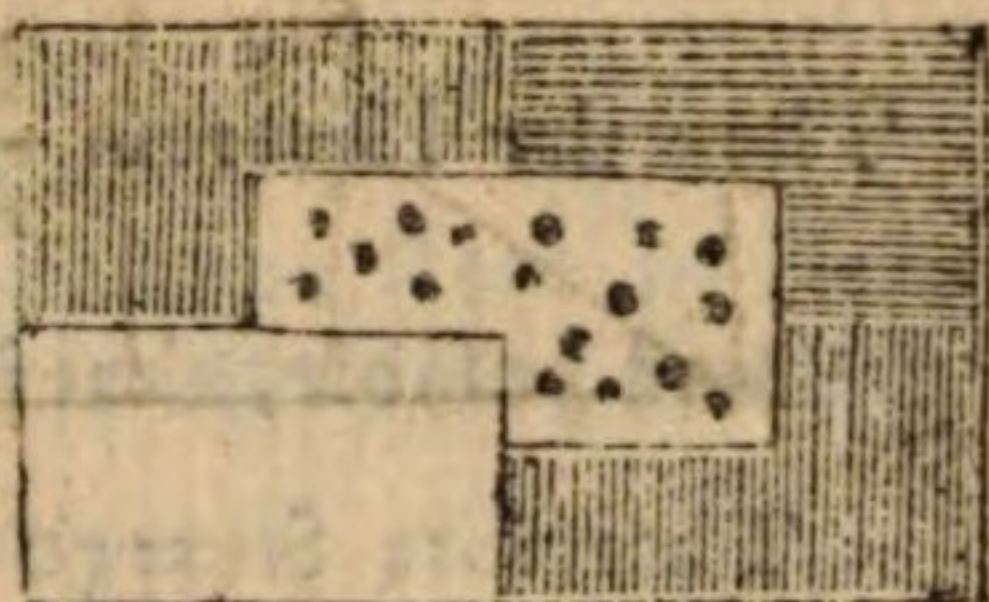
which as (per Quest.) must be cut into four equal Parts, viz. EGI, KIG, IKF, and FHK.

Fig. 2.

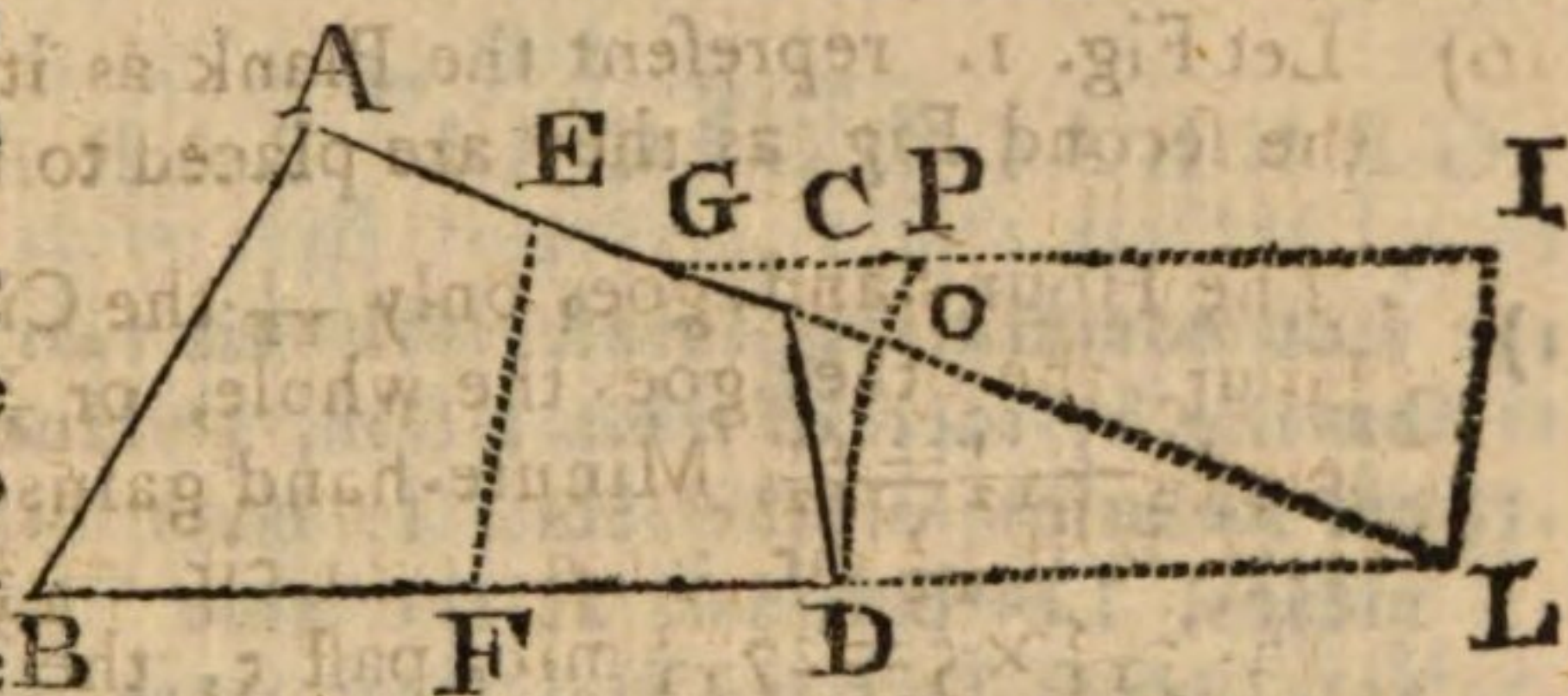
Then by properly applying the four equal Parts in the Parallelogram EFGH, to the given Table ABCD, then will EGI be = to NBD, KIG = LAB, IKF = MCA, and HFK will be = to ODC, which makes LMNO, a complete Square, each Side thereof = 36 Inches, which was required.



- (9) If the Remainder of the Land be divided into four Parts, as in the annexed Plan, these parts will be equal and similar to each other.



- (10) Let ABCD be the given Trapezium, and let AC and BD be produced to L; then upon L, make

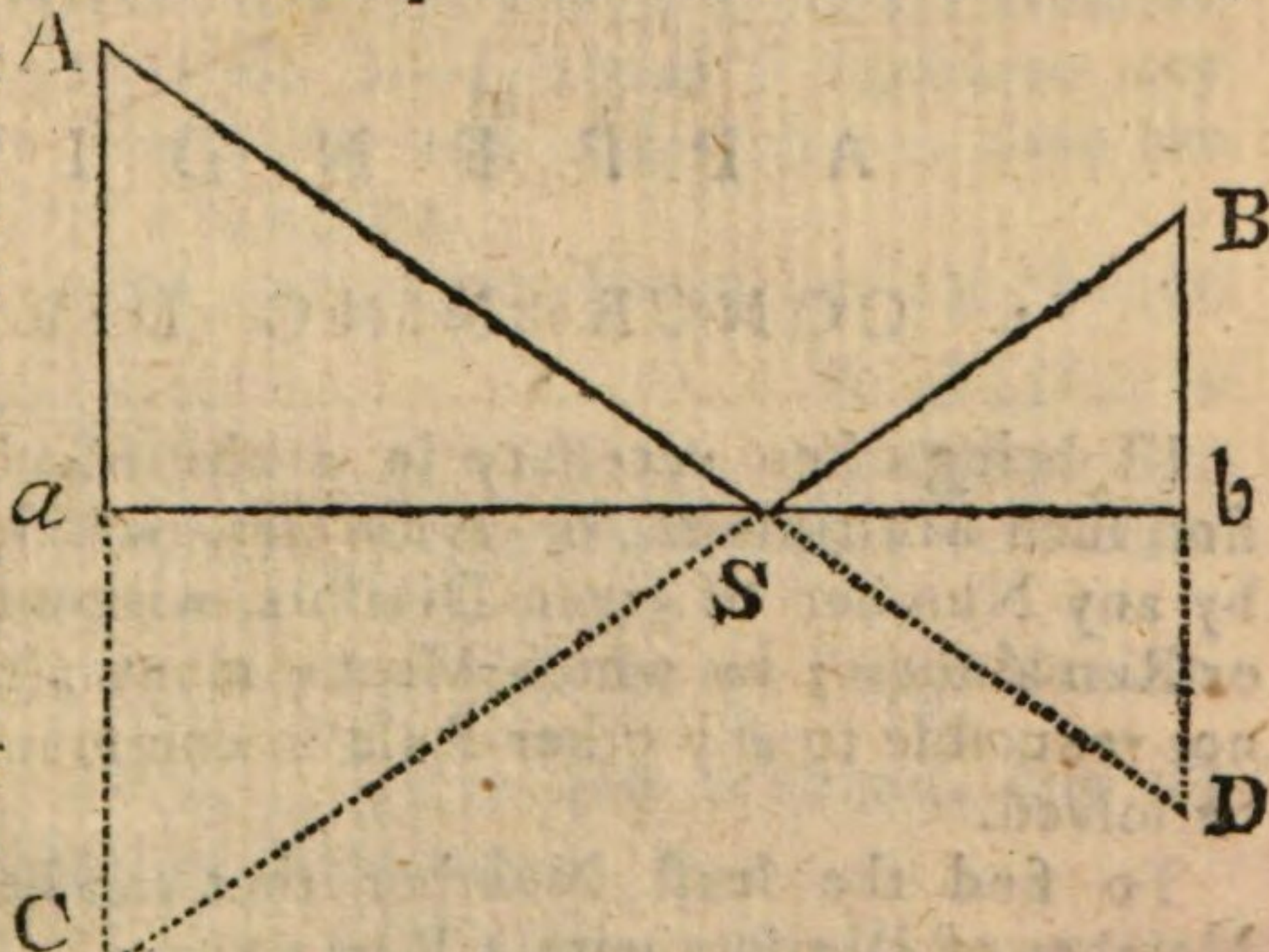


the Parallelogram DPIL = the Triangle DCL + half the given Trapezium, and making the Angle LDO = LOD, produce IP to meet AL in G; then take EL, a mean Proportional between PI, and $2GL$, and draw EF parallel to PD, so will EF be the Line required.

Demonstration, from the Similarity of Triangles, we have $\frac{1}{2}GL \times LI : GL^2 :: PI \times LI : EL^2$, consequently $EL^2 = 2GL \times PI$.

Therefore when the Triangle LEF = the Parallelogram PDLI, then EL will be a mean Proportional between $2GL$, and PI, and EF is the shortest Line possible. Q. E. D. (See Theo. X. p. 112 of SIMPSON'S GEOM.)

- (11) Thus let the Line Aa be continued to C, so that $Ca = Aa$. Then draw CB, and (S) where intersect the Line ab is the Situation of the Place required.



$\therefore AS + CS =$ the whole Distance travelled.

For example thus, suppose $ab = 50$, $Aa = 40$, and $Bb =$ required the Situation of a Place S, so that $SA + S$ may be the least Distance possible.

Then the Theorem $\frac{Bb \times ab}{Aa + Bb} = Sb$, when $AS + SB$ is the

least possible, that is $\frac{30 \times 50}{40 + 30} = \frac{1500}{70} = 21,42857 = Sb$.

Therefore $Sa = 50 - 21,42857 = 28,57143$.

GENEALOGICAL PARADOXES.

- (1) Lot committing Incest with his two Daughters, and having by each of them a Son, produced the Paradoxical Relation of the Sons to each other; for by that means, Lot became, at the same Time, their Father, and Grand Father; and they were Brothers, and first Cousins to each other; also each Mother was Aunt to the other's Son. See Gen. xix. ver. 31. to the End.
- (2) The Lord God formed the first Man, Adam, of the Dust of the Ground; Gen. ii. ver. 7. and from Adam made he a Woman, and called her Name Eve. Gen. ii. ver. 22. Now Abel (who was the second Son of Adam, and Eve his Wife) was murdered by his Brother Cain; therefore he got the Maidenhead of his Grandmother (the Earth) and was begot before his Father (Adam) who was made of the Earth, therefore was not begotten: and was born before his Mother (Eve) who was made of Man, therefore was not born.

A P P E N D I X.

I. CONCERNING DIVISORS.

IT being often necessary in arithmetical Calculations, to find such Multipliers, or Numbers, which may be divided by any Number of given Divisors, without any Remainder, or Remainders; by which Means many pleasant Questions, not reducible to any other Rule in common Arithmetic, may be solved.

To find the least Number that can be divided by any Number of Divisors with a Remainder.

RULE.

R U L E.

Multiply all the Prime * Numbers, and the Root of such as are Square or Cube Numbers, continually; the Product will be the Number required.

E X A M P L E S.

- (1) Required the three least Numbers, which divided by 20 shall leave 19 for a remainder; but, if divided by 19, shall leave 18, if divided by 18, shall leave 17; and so on, always leaving one less than the Divisor, to Unity.

G. DIARY, 1747.

By Algebra thus, suppose x = the least Number possible. And let a, b, c, d , &c. represent the Quotients respectively produced by dividing x , by 2, 3, 4, 5, &c. Then will the Remainders be 1, 2, 3, 4, &c. Hence $x = 2a + 1 = 3b + 2 = 4c + 3 = 5d + 4$, &c. to $20A + 19$.

Now $a = b + \frac{b+1}{2}$. Hence $\frac{b+1}{2} = m$, a whole Number.

$\therefore b = 2m - 1$, and $a = 3m - 1$.

Again $c = m - 1 + \frac{m}{2}$. Hence $\frac{m}{2} = n$, a whole Numb.

And therefore $m = 2n$, $c = 3n - 1$, and proceeding in this Manner we get $A = 11639628B - 1$; hence the Number required $x = 232792560B - 1$, where B represents any whole positive Number; and when $B = 1$, then the Number required $x = 23279$.

The Number required $x = 232792560 - 1 = 232792559$

$$2d. \quad \underline{232792560 \times 2 - 1} = 465585119$$

$$3d. \quad 232792560 \times 3 - 1 = 698377679$$

And thus may be performed other Numbers ad infinitum. Or thus by the Rule,

First 1, 2, 3, 5, 7, 11, 13, 17, and 19, are Prime Numbs.

Also $\sqrt[2]{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt[2]{9} = 3$, and $\sqrt[3]{16} = 2$, and all the rest are composite Numbers.

* A Prime Number, is such a Number as cannot be produced by the Multiplication of two, or more Integers.

∴ $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 \times 13 \times 2 \times 17 \times 19 = 232792560$, the least Number that can be divided by the given Divisors without a Remainder, as before.

- (2) What's the least Number that can be divided by the Nine Digits, without a Remainder.

LADIES DIARY, 1719.

Divisors 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now $\sqrt{4} = 2$; 6 may be cancelled, being composed 2×3 ; and 3, 5, and 7 are Prime Numbers.

And $\sqrt[3]{8} = 2$. Also $\sqrt[2]{9} = 3$.

Then per Rule $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 = 2520$.

- (3) A country Girl to Town did go,
Some Walnuts there to sell;
A Gentleman she chanc'd to meet,
And thus it her befell:
My pretty Maid, says he to her,
What Number have you here?
I can't tell, Sir, said she to him,
But this I'll make appear;
I told them o'er, ere I came out,
By six's, five's, four's, three's, two's;
And every time I number'd them,
One remained overplus:
I told them o'er by seven's at last,
And there were no remains:
If you can find the Number out,
Pray take it for your pains.

First the least Number that can be divided by 1, 2, 3, 4, 5, 6, without a Remainder, will be $1 \times 2 \times 3 \times 2 \times 5 = 60$, per Rule.

Then $60 \div 1 = 61$, will leave 1, when $\div 2, 3, 4, 5, 6$.

But $7 \nmid 61$ (8, and 5 Remains.

Also $60 \times 2 + 1 = 121$

$60 \times 3 + 1 = 181$

$60 \times 4 + 1 = 241$

} None of which are divisible by 7, without a Remainder.

But $60 \times 5 + 1 = 301$. is the least Number which admits of the Conditions of the Question.

Then to find the next least Number which admits of the same Conditions, we shall find (by proceeding as above) to be $60 \times 12 + 1 = 721$.

Also.

Also $723 - 301 = 420$, the common Difference of all the Numbers answering the Conditions of the Question.

$\therefore 301, 721, 1141, 1561, \&c.$ ad infinitum, will answer the Conditions of this Question. (See Quest. 18. p. 60.)

- (4) To find the least Number of Guineas, which being divided by 6, 5, 4, 3, and 2, shall leave 5, 4, 3, 2, and 1, respectively remaining.

LADIES DIARY, 1748.

First $1 \times 2 \times 3 \times 2 \times 5 = 60$, the least Number which will divide by 2, 3, 4, 5, and 6, as by the last Question.

$\therefore 60 - 1 = 59.$ Q. E. F.

2. COMBINATIONS.

Combination of Quantities, is the Manner of finding how many different Ways they may be varied, being taken 1 and 1, 2 and 2, 3 and 3, &c. as the Number of Combination of three Quantities, viz. a, b , and c , viz. ab, ac, bc . If three Quantities are to be combined, and their Number is only three, as a, b , and c , then the Number of Combinations will be only one, viz. abc ; and if there are four Quantities a, b, c , and d , and three to be taken, then the Combinations will be four, viz. abc, abd, bcd, acd ; and if the Number of Quantities to be combined be called q , and u , the Number of them to be taken, then the Number of Combinations will be

will be $\frac{q-u+1}{1} \times \frac{q-u+2}{2} \times \frac{q-u+3}{3} \times \frac{q-u+4}{4} \times \frac{q-u+5}{5}, \&c.$ For suppose the Number of

Quantities to be combined be 6, and the Number of them taken be 4, then the Number of Combinations will be

$$\frac{6-4+1}{1} \times \frac{6-4+2}{2} \times \frac{6-4+3}{3} \times \frac{6-4+4}{4} = \frac{1}{1} \times \frac{4}{2}$$

$\times \frac{5}{3} \times \frac{6}{4} = 15.$ The Number of all the possible Combinations beginning from the Combinations of every two, will be $2^q - q - 1$; as when the Number of Quanties be 5, then the Number of possible Combinations will be $2^5 - 6 = 26$. So that if u represents any Number of Quanties, then will

$\frac{u^{u+1} - u}{u-1}$ express the possible Number of all the Variation, as

if $n = 7$, then $\frac{7^7 - 7}{7 - 1} = \frac{5764801 - 7}{6} = \frac{5764794}{6} = 960799$. (See Quest. 26. in Algebra.)

To find the different Combinations in any Number, or Quantities.

R U L E.

Having placed the given Quantity by itself, decrease it gradually by an Unit, so often as there are Quantities in the Combination; placing one above another, with a Sign of Multiplication between them, which Numbers must be multiplied into one another for a Dividend: then placing an Unit with the like Number of Places, increasing by Unity till you arrive at the Number to be continued; which multiply continually for a Divisor, and the Quotient will be the Number of Combinations sought.

E X A M P L E S.

(1) How many different Ways may 11 Halfpence. huffed in a Hat turn up?

First, as a Halfpenny has two Faces. $\therefore 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, or $2^{11} = 2048$, the different Combinations required.

Now to find the different Chances for any Number of Heads, or Tails, let a represent the Heads, and b the Tails; then by the Theorem in Page 328, we shall have $a^{11} + 11a^{10}b + 55a^9b^2 + 165a^8b^3 + 330a^7b^4 + 462a^6b^5 + 462a^5b^6 + 330a^4b^7 + 165a^3b^8 + 55a^2b^9 + 11ab^{10} + b^{11}$ are all the different Combinations, or Ways 11 Halfpence can turn up, viz. $1 + 11 + 55 + 165 + 330 + 462 + 462 + 330 + 165 + 55 + 11 + 1 = 2048$, as before.

Here it is to be observed that a^{11} , or all Heads, hath but one Way of turning up; the same for b^{11} Tails: but 10 Heads and 1 Tail, and the contrary, may come up 11 different Ways each. Also nine Heads and two Tails, or the contrary may each come up $\frac{11 \times 10}{2} = 55$ different Ways.

Likewise eight Heads and three Tails, or the contrary may come up $\frac{55 \times 9}{2} = 247.5$ different ways, &c.

It may be also observed, that, by this Theorem, the Unciæ, or Coefficients, do only increase until the Indices of the two Letters become equal, or change Places, and then the rest decrease in the same Order.

- (2) A famous General having serv'd his King,
Long Time in Wars, and had victorious been;
For which his Service, with a pleasant Smile,
Ask'd of his King, one Farthing for each File
Of ten Men in a File, which he could then
Make with a Body of one Hundred Men.

The King, considering his brave Actions past,
And seeming modesty of his Request;
Gives his Consent. To what will it amount
In sterling Money? Take your Pen and count.

$$\text{Thus } \frac{100}{1} \times \frac{99}{2} \times \frac{98}{3} \times \frac{97}{4} \times \frac{96}{5} \times \frac{95}{6} \times \frac{94}{7} \times \frac{93}{8} \times \frac{92}{9} \times \frac{91}{10} = \frac{62815650955520472000}{3628800} = 17310309456440 \text{ Farthings, or } 18031572350 \text{ £. } 9 \text{ s. } 2 \text{ d. } 2 \text{ E. F.}$$

- (3) Two Gamesters one Day, at Dice they did play;
And being full merry with Wine;
Said B unto A, what Odds will you lay,
I cast not the six Aces this Time?
Says A then to B, ten to one I'll lay thee,
With six Dice, the six Aces you cast not.
Pray Youths, shew, and here let them know,
For the Odds on the Cast, Sirs, they do not.

First $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656 = 6^6$ different Combinations.

And $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ Variations.

Then $46656 - 720 = 45936$ Chances against A.

But as A laid 10 to 1. $\therefore 7200$ Chances to B.

Therefore A's Chance to that of B, $:: 45936 : 7200$, or
 $: 6,38 : 1$. 2. E. F.

- (4) Two gamesters met the other Day,
The one call'd B, the other A;
But having neither Cards, nor Dice,
They got to hotch-cap in a trice;
With sixteen Halfpence fair and flat,
All which they hussled in a Hat:

Says A, to B, all these are mine,
 And I will lay a Pint of Wine,
 That in two Trials there will be
 Nine Heads or Tails, as here you see.—
 No Matter which, but on they play'd,
 'Till Silver, Brass, and Gold were laid:
 But as to B, his Chance was bad,
 For he got broke of all he had.
 What were the Odds? I pray declare.
 Ye ingenious Youths, and place it here.

First $2^{16} = 65536$, Number of different Chances on 16 Halfpence.

Now let a represent the Heads, and b the Tails.

Then (by Theo. in p. 328.) we shall have $a^{16} + 16ab^{15} + 120a^{14}b^2 + 560a^{13}b^3 + 1820a^{12}b^4 + 4368a^{11}b^5 + 8008a^{10}b^6 + 11440a^9b^7$, &c.

Then $11440 \times 2 = 22880$ Chances for Nine Heads, or Tails to come.

$\therefore 65536 - 22880 = 42656$ Chances, not come up the first Time: viz. 22880 to 42656, that they come up nine Heads or Tails the first Toss.

Therefore as $65536 : 22880 :: 42656 : 14892\frac{7}{4}$.

Then $22880 + 14892\frac{7}{4} = 37772\frac{7}{4}$ for } Nine Heads.
 Also $65536 - 37772\frac{7}{4} = 27763\frac{5}{4}$ against } or 9 Tails.

M A G I C A L S Q U A R E S.

By a Magical we understand a Square divided into several other small equal Squares, filled with Terms of an Arithmetical Progression, so transplanted, that all the same Line, or Rank, whether taken perpendicularly, horizontally, or diagonally, make the same Sum.

E X A M P L E.

- (1) The Numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, being given, to form them in a Magic Square; viz. counting each Rank perpendicularly, horizontally, or diagonally, that these Ranks may be equal to each other.

Suppose it done, and represented in its proper form, by the following symbols, placed as follows,

$$a \quad b \quad c$$

$$d \quad e \quad f$$

$$g \quad h \quad m$$

Then by the $\begin{cases} a + e + m = 15 \\ b + e + h = 15 \\ c + e + g = 15 \end{cases}$ Quest. we required e the middle Num. have

Their Sum $= a + b + c + 3e + m + h + g = 45$.

Again $a + b + c = 15$ } added together we get $a + b + c + m$

Also $m + h + g = 15$ } $+ h + g = 30$; this taken from the fourth Equation gives $3e = 15$.

Then $e = \frac{15}{3} = 5$, the middle Number.

Or by Numbers, thus

First the Sum of the progression Numbers are $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$.

Then $3 =$ the Number of Rows.

$\therefore 45 \div 3 = 15 =$ the Sum of each Side or Rank.

And $15 \div 3 = 5 = e$, the middle Number, as before.

Again, to find the corner Figures; and first to find the Figure represented by a . Beginning with 1, I find the corner Letter a , or any other corner Letter cannot be 1; for if a was $= 1$, then m must $= 9$, $b + c = 15 - 1 = 14$; as also $d + g = 15 - 1 = 14$. But there remains no two Numbers after 5, 1, and 9, whose Sum is 14, but 6, and 8. Then if any of these Figures were b , the other would be c ; and then no Figures would remain for the Value of either d or g ; wherefore a is not equal to 1, nor any corner Letter $=$ to 1, or 9.

Now 3 cannot be $= a$; for if it were, then m should be $= 7$; and $b + c = 15 - 3 = 12$; as also $d + g = 12$; but there remains no two Numbers after 5, 3, and 7, whose Sum is 12, but 8 + 4, which cannot answer to b and c , and d and g ; wherefore a , or any other corner Letter, is not $= 3$; neither is m , nor another corner Letter $= 7$; from what has been said, it is plain, that (if the Question proposed is capable of being solved) the corner Letters are all even Numbers; wherefore if $a = 2$, m will be $= 8$, and c must be either 4, or 6. Let

$c = 4$,

$c=4$, then $g=6$, $b=9$, $d=7$, $f=3$, and $h=1$; and so the Square will be completed as was required.

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

But if $c=6$ (a being $=2$;) then $g=4$, $b=7$, $d=9$, $f=1$, and $h=3$, and then the Squares will stand thus

| | | |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

Or they may be found mechanically: thus, set them all down progressively, about which draw a Square corner-ways; thus



Then set the four angular Figures at the Corners, and put the outermost alternately: that is, place 1 between 8 and 6, 9 between 4 and 2, 3 between 4 and 8, and 7 between 2 and 6; thus

| | | |
|---|---|---|
| 2 | — | 6 |
|---|---|---|

| | | |
|---|---|---|
| 2 | 7 | 6 |
|---|---|---|

| | | |
|---|---|---|
| — | 5 | — |
|---|---|---|

| | | |
|---|---|---|
| 9 | 5 | 1 |
|---|---|---|

| | | |
|---|---|---|
| 5 | — | 8 |
|---|---|---|

| | | |
|---|---|---|
| 4 | 3 | 8 |
|---|---|---|

There are many Ways of constructing Magic Squares, as also many surprising properties thereto relating, and is as itself very curious and entertaining; yet it cannot be denied, but that it is of very little or no Use in any other Parts of the Mathematics: therefore, whoever would see more of these Matters, may consult the DIARIAN REPOSITORY, Page 103 to 108, also from p. 223 to 225, where they will find this Subject more largely treated of.

F I N I S.

Vyse, Charles

The key to the tutor's guide or the arithmeticians repository

London 1775

Math.p. 629

urn:nbn:de:bvb:12-bsb10083052-0